HOW TO ELIMINATE GERRYMANDERING

Part 3: Representing, electing and ranking

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September 27, 2007

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Lecture 3: Why blatant political gerrymandering is unavoidable in today's system . . . *and what to do about it*.

The original gerrymander

Gerrymandering

Political gerrymandering:

> *[The] practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage by diluting the opposition's voting strength.*

–from *Black's Law Dictionary* as quoted by Justice Antonin Scalia.

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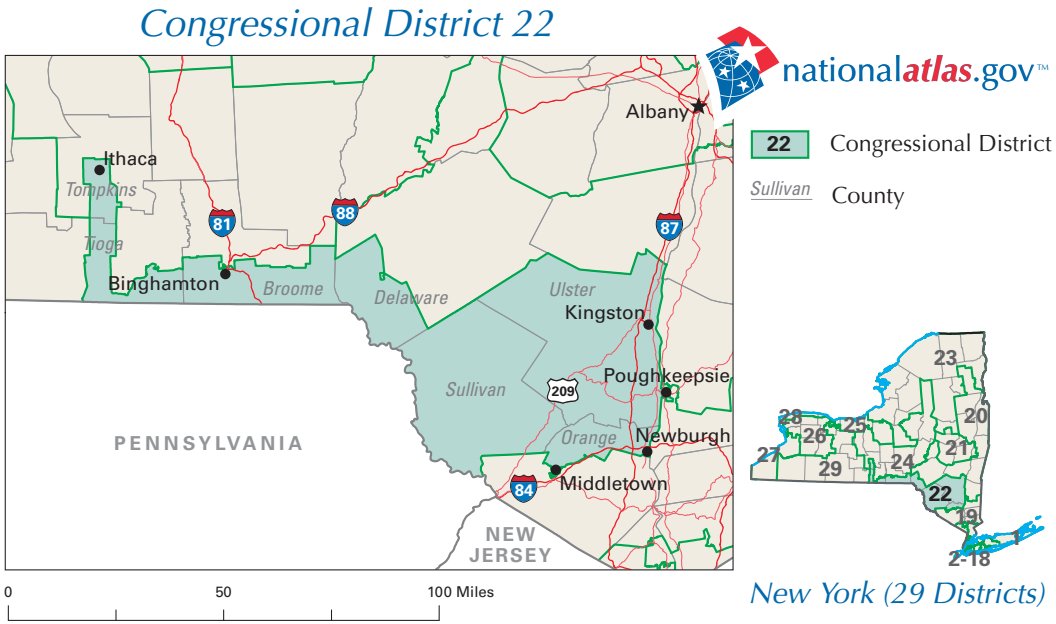
2 Fair majority voting

3 Biproportionality

Ithaca's gerrymander



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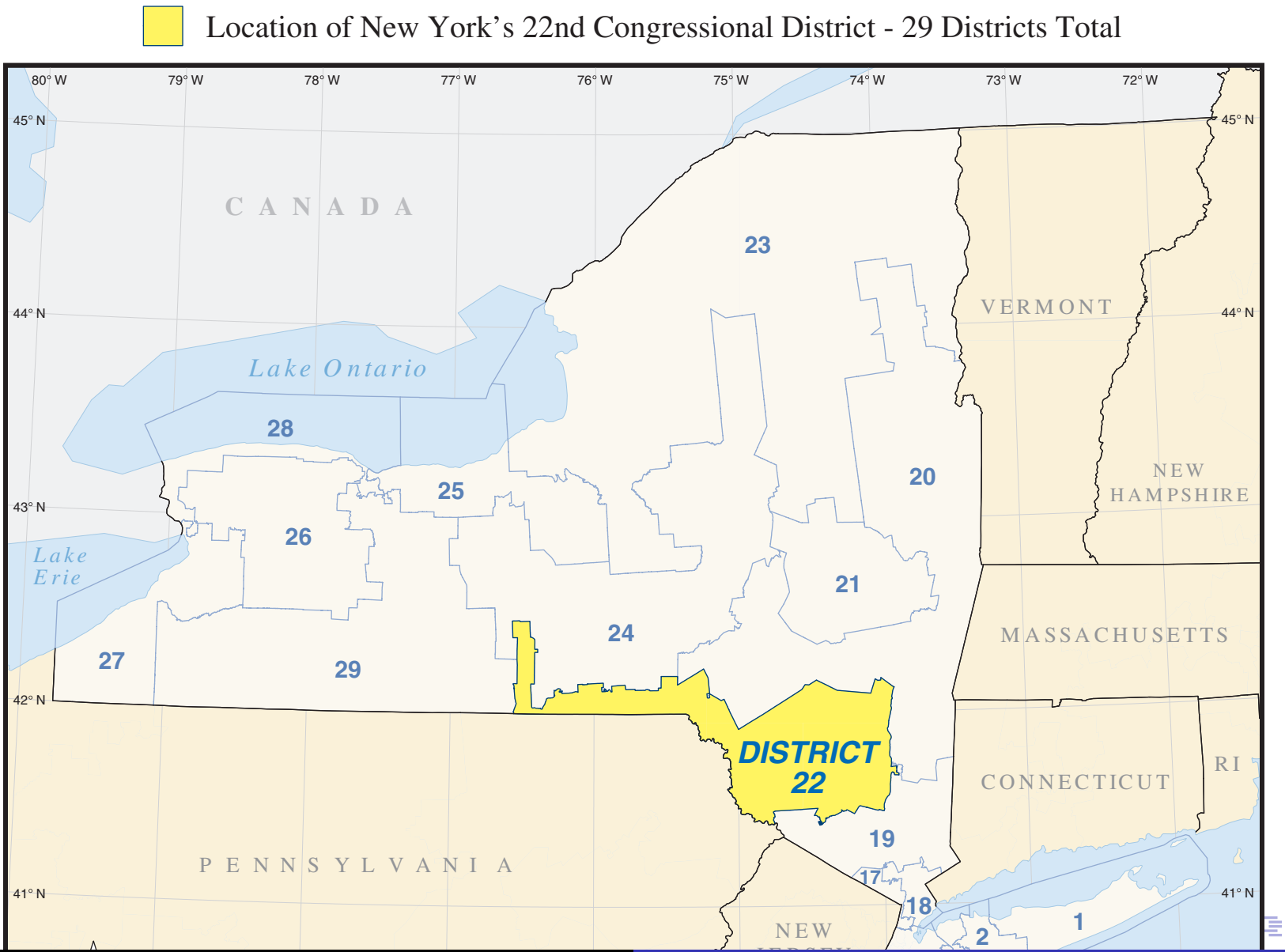

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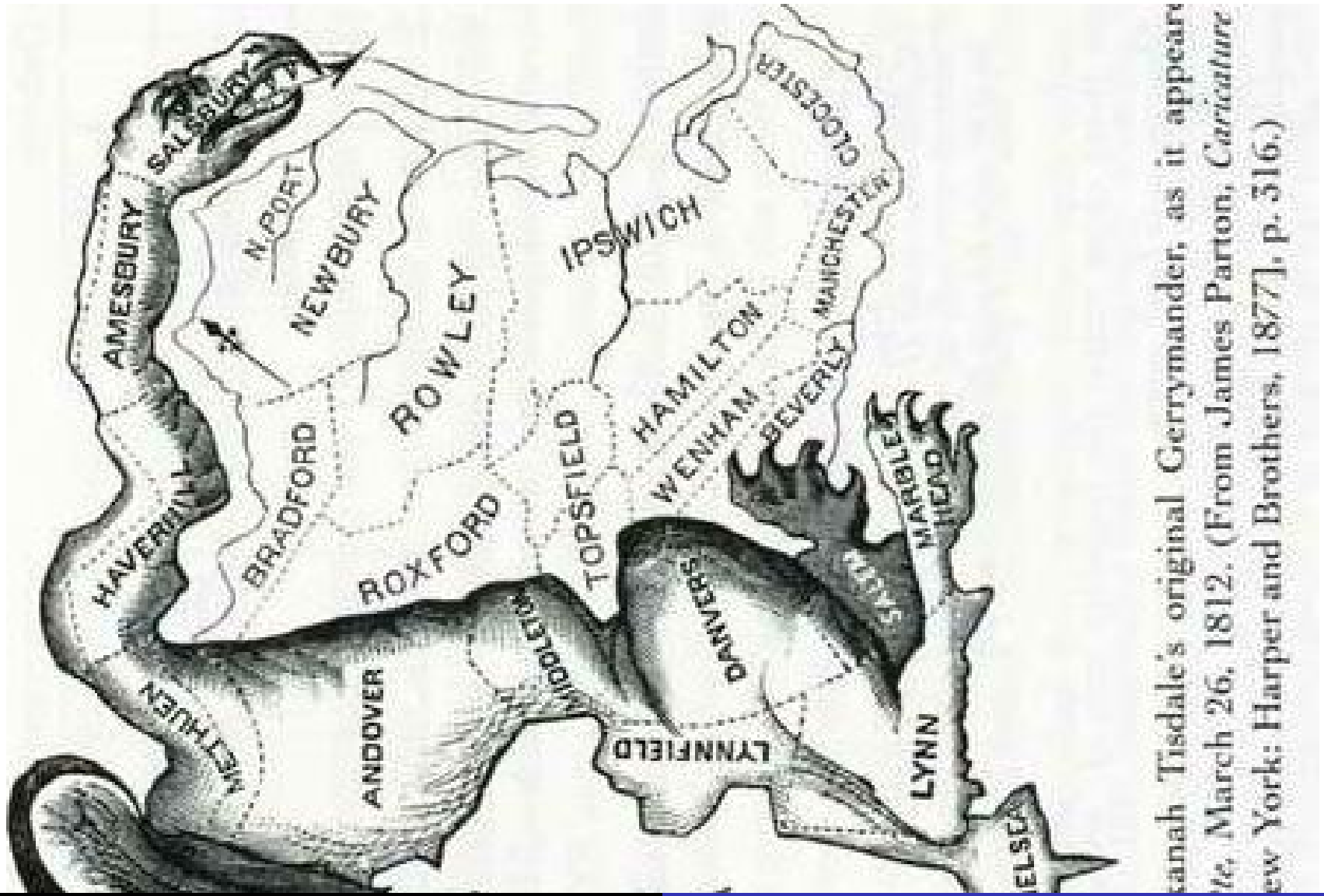
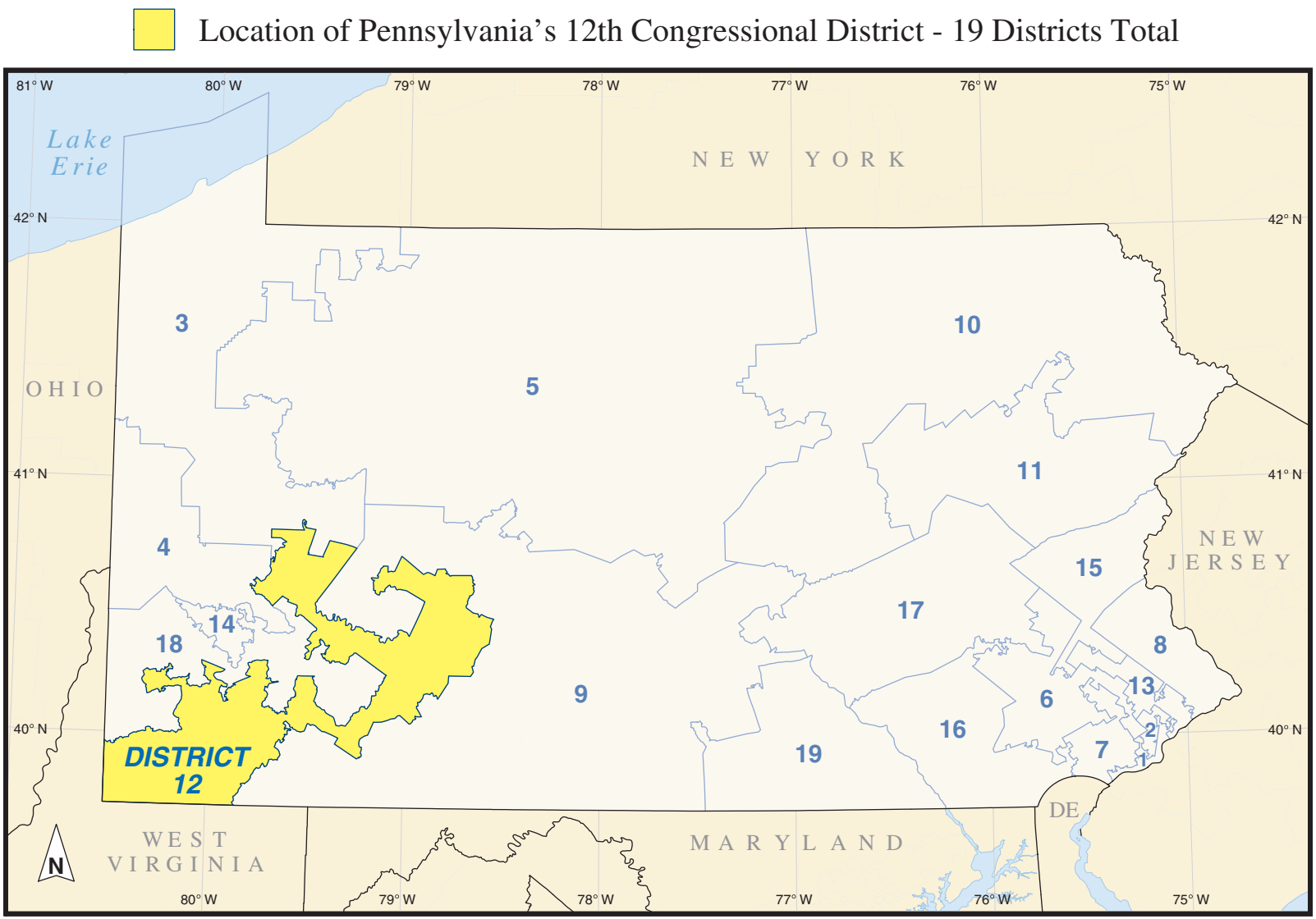
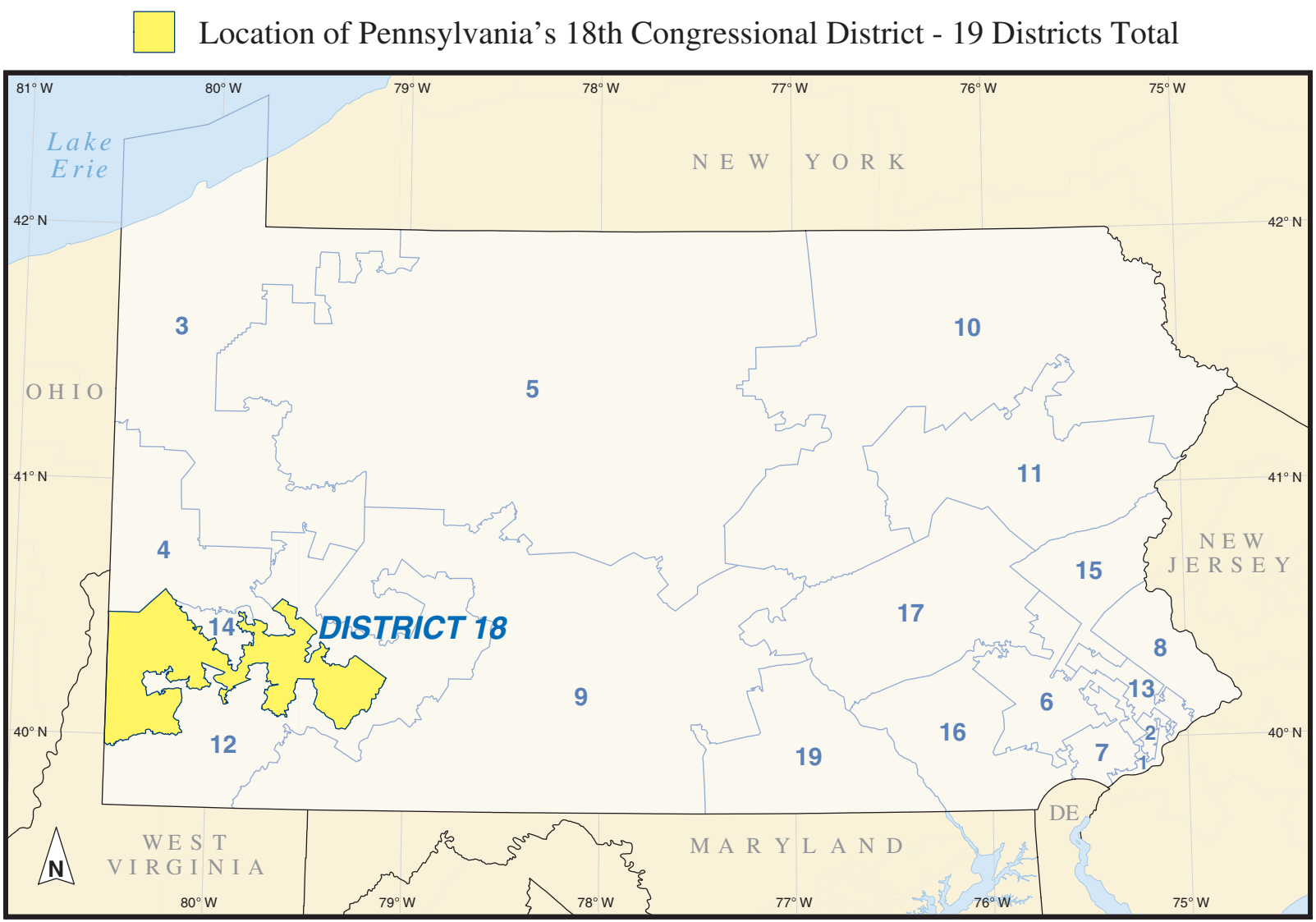


anah Tisdale's original Gerrymander, as it appear
e, March 26, 1812. (From James Parton, *Caricature*
ew York: Harper and Brothers, 1877, p. 316.)

Pennsylvania's gerrymander: upside-down Chinese dragon?



Pennsylvania's gerrymander: supine seahorse?



The rotten electoral state of the United States

	2002	2004	2006
Incumbent candidates	386	392	394
Incumbent candidates reelected	380	389	371
Incumbent candidates who lost to outsiders	4	3	23
Elected candidates ahead by $\geq$20% of votes	356	361	318
Elected candidates ahead by $\geq$16% of votes	375	384	348
Elected candidates ahead by $\leq$10% of votes	36	22	56
Elected candidates ahead by $\leq$6% of votes	24	10	39
Candidates elected without opposition	81	66	59
Republicans elected	228	232	202
Democrats elected	207	203	233

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Entirely possible for a majority party in the House to be elected by a minority!

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Connecticut:

- 2004: Democratic votes exceeded Republican by 156,000, yet only elected 2 to the Republican's 3 Representatives,
- 2006: 44% of votes gave Republicans only 1 seat (20%) to Democrat's 4.

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And yet, every one of Texas's 32 districts had a census population of **651,619** or **651,620**: a perfectly carved state!

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Note in passing: Political gerrymandering is perfectly ecumenical.

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*Caliper Corp.'s *Maptitude for Redestricting* does this for $6,000.

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In 2002, 7 Democrats (1 unopposed) and 12 Republicans (4 unopposed) were elected.

Recourse in the Courts

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A few more clicks of the mouse: each district's population either 646, 371 or 646,372. But

- supine seahorses and upside down Chinese dragons,
- 21 counties and 81 municipalities fractured.

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Justice Antonin Scalia announced the judgement (joined by only 3 other justices), concluding:

> *Eighteen years of essentially pointless litigation have persuaded us that Bandemer [1986] is incapable of principled application. We would therefore overrule that case, and decline to adjudicate these political gerrymandering claims. The judgement of the District Court is affirmed.*

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Only one criterion is accepted as capable of application, the numbers. *Kirkpatrick v. Preisler 394 U.S. 526 (1969)*:

> *[The] "nearly as practicable" standard requires that the State make a good-faith effort to achieve precise mathematical equality. Unless population variances among congressional districts are shown to have resulted despite such effort, the State must justify each variance, no matter how small.*

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The Supreme Court's decisions and dissenting opinions, taken together, have left a legacy of utter confusion.

Justice Harlan's charge

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Justice John Harlan was unusually prescient in a 1969 dissenting opinion:

> *[The] rule of absolute equality is perfectly compatible with "gerrymandering" of the worst sort. A computer may grind out district lines which can totally frustrate the popular will . . . The legislature must do more than satisfy one man, one vote; it must create a structure which will in fact as well as theory be responsive to the sentiments of the community. . . Even more than in the past, district lines are likely to be drawn to maximize the political advantage of the party temporarily dominant in public affairs.*

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From this perspective, many electors are very badly represented!

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Two rules decide which candidates are elected:

- The *requisite number* of representatives elected by each party is determined by Jefferson's method on the basis of total party votes.
- The candidates elected—exactly one in each district and the requisite number of each party—are determined by the procedure that is about to be described.

The problem

The 2004 Connecticut congressional elections: votes.

District	1st	2nd	3d	4th	5th	Total
Republican	73,273	165,558	68,810	149,891	165,440	622,972
Democratic	197,964	139,987	199,652	136,481	105,505	779,589

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The district-winners of the justified-votes are 3 Democrats and 2 Republicans: FMV elects them.

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If not, why? The vote was "unbalanced": 2nd district's vote counts for too much (or the 4th's for too little), so votes among the candidates of districts are rescaled ($136,480/139,987 \approx 0.9749$).

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If not, why? The vote was "unbalanced": 2nd district's vote counts for too much (or the 4th's for too little), so votes among the candidates of districts are rescaled ($136,480/139,987 \approx 0.9749$).

Connecticut's justified-votes:

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Fair majority voting: focus on parties (not districts)

If the requisite number of candidates with the most votes in each party—the party-winners—give each district 1 Representative, that is the solution. Connecticut's votes:

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The exact same solution: **it always is**.

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Such rescalings can always be found (for any number of parties and districts). They always yield the identical set of elected candidates. No other set of feasible candidates is coherent with majority decision for every contested pair.

Fair majority voting: Pros and cons

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- Every candidate incited to seek as many votes as possible (as vs. "proportional representation").

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This is unavoidable. The evidence shows electorates are prepared to accept it.

Contents

1 Electoral Realities

2 Fair majority voting

3 Biproportionality

The Zürich story

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Mr. Schmidt was the resident of a district with 3 representatives; he regularly cast his votes for a party that never received enough votes in his district to elect one of its candidates. The Court ruled he was right!

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2^{nd}	2846	1379	653	1082	541	176	464	198	16
3^{rd}	2052	629	349	786	315	79	699	108	13
4^{th}	2409	968	1092	842	440	342	230	111	10
5^{th}	3632	1642	3015	1499	837	618	323	144	17
6^{th}	2628	1972	754	572	708	615	154	333	16
7^{th}	2938	1630	1272	807	696	391	212	124	12
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Multipliers can always be found to rescale rows (or votes in districts) and/or columns (or votes for parties) so that rounding the results to the nearest integers yields an apportionment that gives to each district and each party the seats it deserves.

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1^{st}	3.92	2.12	3.00	1.72	1.02	0.45	0.42	0.23
2^{nd}	6.52	3.19	1.49	2.51	1.25	0.46	1.34	0.46
3^{rd}	5.08	1.57	0.86	1.96	0.79	0.22	2.18	0.27
4^{th}	3.61	1.47	1.64	1.28	0.67	0.59	0.44	0.17
5^{th}	5.45	2.49	4.52	2.27	1.27	1.06	0.61	0.22
6^{th}	5.51	4.17	1.58	1.21	1.49	1.48	0.41	0.70
7^{th}	4.48	2.51	1.94	1.24	1.07	0.68	0.41	0.19
8^{th}	6.27	4.49	2.19	1.41	1.65	1.53	0.51	0.70
9^{th}	3.27	2.56	0.76	0.55	1.23		0.13	0.52

Biproportional apportionment

The solution:

	A	B	C	D	E	F	G	H	
1^{st}	4	2	3	2	1	0	0	0	12
2^{nd}	7	3	1	3	1	0	1	0	16
3^{rd}	5	2	1	2	1	0	2	0	13
4^{th}	4	1	2	1	1	1	0	0	10
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Theorem

Such multipliers can always be found (for any number of parties and districts with any number of seats). They always yield the identical set of elected candidates. No other set of feasible candidates is coherent with the simple rounding rule ***for every pair of party-district lists****.*

Messenger's charge and Tocqueville's remark

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Alexis de Tocqueville's remark in a January 5, 1851 letter to his cousin and close friend Gustave de Beaumont, puts this charge in perspective:

> *"How sad it is that everywhere on earth governments are always precisely as roguish as the morals of their subjects permit them to be!*
>
> *Their vices have found but that one limit."*