

## MATH 3560 HOMEWORK 13 DUE APRIL 30

(1) A function  $f : \mathbb{C} \rightarrow \mathbb{C}$  is defined to be an *affine transformation* if it is of the form (in vector notation)

$$f(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$$

where  $A$  is an invertible  $2 \times 2$  matrix with real entries, and  $\mathbf{b}$  is a vector.

- (a) Write down the formula for the affine transformation which fixes the origin, and doubles the distance between the origin and every other point in the plane. Also describe in words and write down the formula for the inverse of this transformation.
- (b) Show that the composition of two affine transformations is an affine transformation.
- (c) Let  $f$  be an affine transformation, with  $f(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$ . Find a formula for the inverse of  $f$ , and show that  $f^{-1}$  is an affine transformation.
- (d) Use parts (b) and (c) to show that the set of affine transformations of the plane forms a group under composition. This group is denoted  $\text{Aff}(\mathbb{C})$ .
- (e) Explain why  $\text{Isom}(\mathbb{C})$  is a subgroup of  $\text{Aff}(\mathbb{C})$ .

(2) For each of the following finite groups  $G$ , draw a wallpaper pattern such that  $G$  is the stabilizer of the origin in the group of symmetries of this pattern. Make sure you clearly identify the origin in your pictures. (Hint: think about tilings of the plane.)

- (a) The group  $\langle M \rangle$ , where  $M$  is reflection in the  $x$ -axis.
- (b) The dihedral group  $D_2$  (this is the group generated by  $M$  and the reflection in the  $y$ -axis).
- (c) The dihedral group  $D_3$ .
- (d) The dihedral group  $D_4$ .
- (e) The dihedral group  $D_6$ .
- (f) The group  $\langle R_\pi \rangle$ , where  $R_\pi$  is rotation about the origin by  $\pi$  radians.
- (g) The group  $\langle R_{2\pi/3} \rangle$ , where  $R_{2\pi/3}$  is rotation about the origin by  $2\pi/3$  radians.
- (h) The group  $\langle R_{\pi/2} \rangle$ , where  $R_{\pi/2}$  is rotation about the origin by  $\pi/2$  radians.
- (i) The group  $\langle R_{\pi/3} \rangle$ , where  $R_{\pi/3}$  is rotation about the origin by  $\pi/3$  radians.