Math 749 – Important Groups
Homework #1

Exercise 1. *Let G be a group acting on the set M. For any sequence (F_i) of finite subsets in M, the set*

$$\left\{ g \in G \;\middle|\; \lim_{i\to\infty} \frac{|gF_i \triangle F_i|}{|F_i|} = 0 \right\}$$

is a subgroup of G.

Exercise 2. *Show that $\mathcal{CF}_{\mathbb{N}}$, the filter of cofinite sets in $\mathbb{N}$, and $\mathcal{CI}_{\mathbb{N}}$, the filter generated by coinitial segments in $\mathbb{N}$ regarded as a directed set, coincide.*

Exercise 3. *Show that every abelian group is amenable:*

1. *Show that the direct product of two amenable groups is amenable.*
2. *Show that a group is amenable if all its finitely generated subgroups are amenable (i.e., locally amenable groups are amenable). Hint: The system of finitely generated subgroups inside G is a directed set. Use an ultralimit construction to obtain a measure on G from the measures on the finitely generated subgroups of G.*

From (1) infer that finitely generated abelian groups are amenable. Then (2) implies that abelian groups are amenable.

Exercise 4. *Add a little twist to what you did on direct products and show that a group is amenable if it has an amenable normal subgroup such that the quotient is also amenable. I.e., amenable-by-amenable groups are amenable. Infer that solvable groups are amenable.*

Exercise 5. *Show that subgroups of amenable groups are amenable.*

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`www.math.cornell.edu/~bux/teaching/2002_S-749/index.html`

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