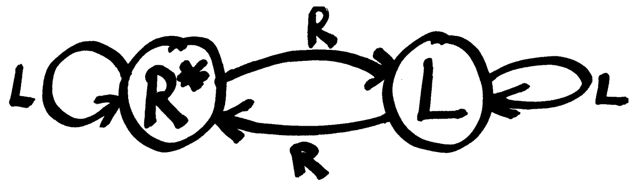
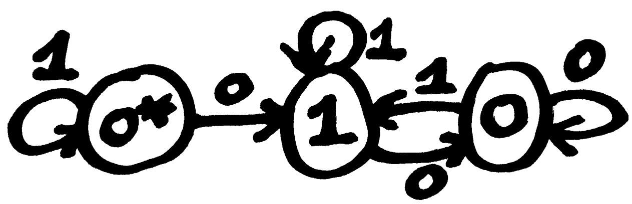
Math 739 – Important Groups (homework 6, due Mar 8)

Exercise 6.1. Prove or disprove: The twist has infinite order. Recall that the twist is defined by the automaton



Exercise 6.2. Is there a recursive system of equations defining a tree automorphism that cannot be realized by a finite state automaton?

Exercise 6.3. This is an automaton over $\{0, 1\}$. What does it do? Is the transformation invertible? Is it of infinite order? If the transformation is invertible, find a recursive definition as small as possible.



Exercise 6.4 (extra credit). Is there an algorithm that takes a finite state automaton as input and decides whether it describes a transformation of finite order?

Exercise 6.5. Prove: A finite state automaton takes ultimately periodic inputs to ultimately periodic outputs.

Exercise 6.6. The decision procedure for equivalence of finite state automata based on checking inputs of length $\leq |A| \times |B|$ is exponential. Find an *effective* algorithm that decides if two automata are equivalent.

Exercise 6.7. Put $D := \langle (\sigma, \delta), (\delta, \sigma) \rangle \leq \mathcal{G}_1 \times \mathcal{G}_1$. Show that the image of $\mathcal{G}_1(1)$ in $\mathcal{G}_1 \times \mathcal{G}_1$ is

$$(B \times B) \rtimes D$$

where the action is conjugation in each component. Infer that $\mathcal{G}_1(1)$ has index 8 in $\mathcal{G}_1 \times \mathcal{G}_1$.

It suffices to solve, on the average, half of the problems correctly.