Math 739 – Important Groups (homework 7, due Mar 15)

Exercise 7.1. Show that the following presentations define isomorphic groups:

$$\begin{aligned} F_{\text{finite,a}} &:= \left\langle a, b \,\middle|\, b^{aa} = b^{ab},\, b^{abb} = b^{abb^a} \right\rangle \\ F_{\text{finite,b}} &:= \left\langle c, d \,\middle|\, d^{cc} = d^{cd},\, d^{cdc} = d^{cdd} \right\rangle \\ F_{\text{infinite}} &:= \left\langle x_0, x_1, x_2, \ldots \,\middle|\, x_i^{x_j} = x_{i+1} \text{ for } j < i \right\rangle. \end{aligned}$$

Here, we use the convention $g^h := h^{-1}gh$.

Fact 7.2. *Let $H \leq G = \langle \Sigma \mid R \rangle$, and fix a set T of words $w_1, \ldots, w_r$ in the free group F_Σ representing the right cosets of H in G. (Such a set is called a* <u>*Schreier transversal*</u>*.) For each $g \in G$, let $\overline{g}$ be the word w_i representing the coset Hg.*

For each $w \in T$ and $x \in \Sigma$, define

$$y_{w,x} := wx\overline{(wx)}^{-1}$$

Then H is generated by the $y_{w,x}$ with defining relations wrw^{-1} where $w \in T$ and $r \in R$.

Exercise 7.3. Prove (7.2).

Exercise 7.4. Prove that a virtually finitely presented group is finitely presented.

Exercise 7.5. Let G be finitely generated. Show that every generating set for G contains a finite subset that already generates G.

Exercise 7.6. Let G be finitely presented. Show that any finite generating for G set can be extended to a finite presentation for G.

Exercise 7.7. Let

$$G = \langle x_1, \ldots x_r \mid r_1, r_2, \ldots \rangle$$

be a finitely presented group. Show that in the above presentation all but finitely many relations are redundant.

Exercise 7.8. Prove that finitely presented by finitely presented groups are finitely presented.

It suffices to solve, on the average, half of the problems correctly.