Math 739 – Important Groups (homework 9, due Apr 5)

Exercise 9.1. Show that the sub-monoid generated by x_0, x_1 and x_1^{-1} is the free product of the monoid generated by x_0 and the subgroup generated by x_1.

Exercise 9.2. Show that F is torsion free.

Exercise 9.3. Let $\mathcal{C}$ be a class of groups and let $\overline{\mathcal{C}}$ denote the smallest class of groups containing $\mathcal{C}$ that is closed with respect to extensions. Prove that $\overline{\mathcal{C}}$ is closed with respect to subgroups and quotients provided $\mathcal{C}$ is closed with respect to subgroups and quotients.

Exercise 9.4. Show that Thompson's group F admits a total order $<$ such that, for all f, g, and h,

$$f < g \Longrightarrow hf < hg.$$

Hint: It suffices to figure which elements shall be in the positive cone $P := \{f \mid 1 < f\}$. This cone has to satisfy the following conditions:

1. $F = P^{-1} \uplus \{1\} \uplus P$.
2. P is closed under multiplication.
3. $f^{-1}Pf \subseteq P$ for each $f \in F$.

Exercise 9.5 (extra credit). Prove that $\mathrm{H}_m(F, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}$, for all $m \geq 1$. In particular, $\operatorname{cd} F = \infty$.

It suffices to solve, on the average, half of the problems correctly.