

Math 4180 Homework

- Homework 1 (Due Thursday, February 7)
 - Page 8: 12, 13, 14, 15, 16, 17
 - Page 23: 7, 8
 - Page 39: 1
 - Page 40: 6, 7, 8
- Homework 2 (Due Thursday, February 14)
 - Page 42: 2.1, 5.3
 - Page 52: 4, 8, 10
- Homework 3 (Due Thursday, February 21)
 - Page 61: 10, 15
 - Page 69: 2, 4
 - Page 76: 1
- Homework 4 (Due Thursday, March 7)
 - Page 95: 1.5, 5.1, 5.2, 5.3, 5.4
 - Page 105: 1
 - Page 116: 1, 2, 3
- Homework 5 (Due Thursday, March 14)
 - Page 125: 1, 3, 4, 5
 - Page 132: 2, 3
 - Page 138: 1, 2, 3, 6
- Homework 6 (Due Thursday, March 21)
 - Page 132: 2, 3
 - Page 138: 8
 - Page 145: 2, 4, 6, 12
 - Page 152: 1, 2, 3

In addition, solve also the following problem:

Denote by $P_1(x)$ the function defined by $P_1(x) := \frac{1}{\pi} \frac{1}{1+x^2}$ (and notice that $\int_{\mathbb{R}} P_1(x) dx = 1$). Define also for $y > 0$ and x a real number $P_y(x) := \frac{1}{y} P_1(\frac{x}{y})$.

(a) Show that the function $(x, y) \rightarrow P_y(x)$ is harmonic on the upper half space $R_+^2 := \{(x, y) : x \in \mathbb{R}, y > 0\}$.

(b) Let f be a continuous and absolutely integrable function on the real line and define $u(x, y) := f * P_y(x) := \int_{\mathbb{R}} f(t) P_y(x - t) dt$. Prove that u is harmonic on R_+^2 and that $u(x, y)$ converges to $f(x)$ for every x , as $y \rightarrow 0$, $y > 0$.

(c) Conversely, consider now a smooth and positive function Φ on the real line whose integral is 1 and whose first and second derivatives are bounded functions, and define as before $\Phi_y(x) := \frac{1}{y} \Phi(\frac{x}{y})$. Assume that all the functions of the form $(x, y) \rightarrow f * \Phi_y(x)$ are harmonic for any function f as above. Show that in this case Φ must be equal to P_1 .

- Homework 7 (Due Thursday, April 18)
 - Page 152: 5, 6, 7, 8, 10, 11
 - Page 162: 10
- Homework 8 (Due Thursday, April 25)
 - Page 170: 8, 9
 - Page 171: 1.1, 2.1, 2.2., 6.3., 7.1.
- Homework 9 (Due Thursday, April 5)
 - Page 171: 4.2, 5.1, 5.2, 6.2, 6.3, 7.1, 7.4, 8.4, 8.5, 8.8, 8.9.
- Homework 10 (Due Thursday, April 12)
 - Page 184: 1, 2, 5, 8, 9
- Homework 11 (Due Thursday, April 19)
 - Page 195: 1, 2, 3, 4, 5
- Homework 12 (Due Thursday, April 26)
 - Page 202: 1, 2, 3
 - Page 223: 1, 2, 3, 5, 8