HOMEWORK MATH 6110

1. Homework (due Wednesday, September 18)

Pages 37-40: 1, 2, 3, 4, 8, 10.

2. Homework (due Wednesday, October 2)

Pages 40-45: 13, 16, 19, 20, 28, 29, 32, 33, 34, 35, 36.

3. Homework (due Friday, October 16)

Page 89: 6, 9, 10, 11, 12, 13, 19. Page 94: 21, 22, 23, 24.

4. Homework (due Thursday, October 31)

Pages 145-146: 1, 2, 4, 5. In addition, solve also the following problem.

Problem 4.1. *Prove the following "triangle inequality" for the quasi-Banach space* $L^{1,\infty}(\mathbb{R})$*:*

$$\|f_1 + \dots + f_N\|_{1,\infty} \leq C \log N (\|f_1\|_{1,\infty} + \dots + \|f_N\|_{1,\infty}).$$

Then show that the inequality is sharp in the sense that the $\log N$ *factor cannot be removed or replaced by a smaller one (such as* $(\log N)^{1/2}$*, for example).*

(Hint: For the inequality, show first that if f *is a function in* $L^{1,\infty}(\mathbb{R})$ *so that* $0 < \alpha < f(x) \leq 1$ *for* $0 < \alpha < 1/2$ *and every* x *in the support of* f*, then one has* $\|f\|_1 \leq C(\log(1/\alpha))\|f\|_{1,\infty}$*. For the counterexample, consider the function* $\sum_{j=1}^{N} \frac{1}{j}\chi_{[j,j+1]}$*.)*

5. Homework (due Tuesday, November 12)

Page 147: 8, 9 , 10, 11, 12, 17. Page 150: 24. Page 151: 29.

6. Homework (due Friday, December 6)

Page 152: Problem 2. Page 154: Problem 7. Page 312: 3. Page 381: 10. In addition, solve also the following problem.

Problem 6.1. *Consider the linear map defined on functions of* d *variables with values in functions of* $d+1$ *variables, and given by the formula*

$$Tf(x,t) := \sum_{k,l: k \leq 0; k+l \geq 0} \frac{1}{2^{(k+l)d}} \langle f, 1_{[0,2^k]^d} \rangle 1_{[0,2^l]^d}(x) 1_{[0,4^l]}(t).$$

Prove that T *is a bounded linear map from* $L^p(\mathbb{R}^d)$ *into* $L^p(\mathbb{R}^{d+1})$*, as long as* $p > \frac{2(d+1)}{d}$[1]*.*

[1] Here, by $\langle F, G \rangle$ one denotes the scalar product between the functions F and G. Also, if J is an interval, one denotes by J^d the cross product $J \times \dots \times J$ of the interval J with itself, taken d times. In addition, $x \in \mathbb{R}^d$ while $t \in \mathbb{R}$. Hint: One may want to use the duality between the spaces L^p and $L^{p'}$, where $1/p + 1/p' = 1$, as you learned during the Discussion Section !