Publications List with Abstracts – Christopher A. Francisco

Below are abstracts of my two completed papers and of my Ph.D. thesis, which I expect to finish in May 2004. Preprints of the two papers are available from my website at http://www.math.cornell.edu/~ chris.

1. C. A. Francisco, Almost complete intersections and the Lex-Plus-Powers Conjecture, to appear, *Journal of Algebra*, 2004.

We prove the almost complete intersection case of the Lex-Plus-Powers Conjecture on graded Betti numbers. We show that the resolution of a lex-plus-powers almost complete intersection provides an upper bound for the graded Betti numbers of any other ideal with regular sequence in the same degrees and the same Hilbert function. A key ingredient is finding an explicit comparison map between two Koszul complexes. Additionally, we obtain bounds on the Hilbert function of an almost complete intersection. These results include a special case of a conjecture of Eisenbud-Green-Harris on the growth of Hilbert functions of Artinian ideals with a regular sequence in prescribed degrees. The Eisenbud-Green-Harris conjecture on Hilbert function growth implies their generalized Cayley-Bacharach Conjecture and is thus important in algebraic geometry.

2. C. A. Francisco, Minimal graded Betti numbers and stable ideals, *Communications in Algebra* **31** (2003), 4971-4987.

Let k be a field, and let $R = k[x_1, x_2, x_3]$. We study the graded Betti numbers that are possible for stable ideals, motivated by work of Deery when he was a student of Geramita. Stable ideals are particularly important since they arise naturally in commutative algebra as generic initial ideals. Given a Hilbert function H for a cyclic module over R, we give an algorithm to produce a stable ideal I such that R/I has Hilbert function H and uniquely minimal graded Betti numbers among all R/J with the same Hilbert function, where Jis another stable ideal in R. We also show that such an algorithm is impossible in more variables, finding incomparably minimal sets of graded Betti numbers for stable ideals with the same Hilbert function, and we disprove a related conjecture of Deery.

3. C. A. Francisco, Hilbert functions and graded free resolutions, Ph.D. Thesis, Cornell University, in progress.

My Ph.D. thesis will contain results on four topics relating to the connections between Hilbert functions and graded free resolutions, including the two papers above. Below are descriptions of the other material.

- We find Hilbert functions for which all modules with that Hilbert function have the same graded Betti numbers, producing several infinite families of examples. Our tools include Peeva's work on consecutive cancellations of Betti numbers and a theorem of Evans and Richert on order ideals that rules out certain sets of graded Betti numbers.
- We investigate a conjecture of Huneke-Srinivasan, generalized by Herzog-Srinivasan, that bounds the multiplicity of an ideal in terms of the shifts in its graded free resolution. In this work, we describe a new attack that investigates the resolutions that occur for a given Hilbert function, and we discuss some computational evidence.