**Math 762 Homework Assignment, Due Thursday, May 3**

**1.** Let $f : K \to \mathbb{E}^2$ be a singular, non-degerate, orientable, combinatorial 2-manifold, $M$, where $K$ is an orientable, combinatorial 2-manifold and $f$ is affine linear on each simplex of $K$. For each 2-simplex $\sigma$ of $K$ choose a vertex $q_\sigma$ in $\mathbb{E}^2$, such that if $\sigma$ and $\tau$ share a common 1-simplex $< p_1, p_2 >$, then $p_2 - p_1$ is perpendicular to $q_\sigma - q_\tau$. The configuration $q$, as above, with edges corresponding to the 1-simplices of $K$, when it exists, is called a *reciprocal* of $M$.

**a.** Show that the set of reciprocals for a fixed $M$ has a natural linear structure as a vector space.

**b.** Show that there is a natural linear isomorphism from the space of reciprocals for $M$ to the space of equilibrium stresses for the 1-skeleton of $M$. (Hint: The stress corresponding to the edge $< p_1, p_2 >$ is $|q_\sigma - q_\tau| / |p_2 - p_1|$, using the notation from above, and with due regard to the orientation of $M$.)

**c.** Suppose that we have a packing of circles in the plane such as the one indicated in the Figure below. (The Figure is taken from a paper of Allan J. Wilks.) Show that the vertices of the centers of the circles are the vertices of a singular non-degenerate combinatorial 2-manifold $M$ such as above, and that it has a reciprocal, and thus a non-zero equilibrium stress.