Math 762 Homework Assignment, Due Thursday, March 15

1.

a. Let S by a d-by-d matrix. Show that for all vectors v and w in $\mathbb{E}^d$, $v^T S w = -w^T S v$ if and only if S is skew-symmetric. In other words $S^T = -S$.

b. Suppose that S is a linear transformation defined on a linear subspace X of $\mathbb{E}^d$, and let B be a basis for X. Suppose that for all vectors v and w in B, $v^T S w = -w^T S v$. Show that S extends to a linear transformation on all of $\mathbb{E}^d$ given by a skew symmetric matrix.

2. Use the result of Problem 1 to show that any k-simplex, consisting of all bars for members, $k \leq d$, is infinitesimally rigid in $\mathbb{E}^d$.

3. Suppose that $p = (p_0, p_1, \dots, p_d)$ is an affine independent set in $\mathbb{E}^d$. Let G be a bar graph where all the vertices are pinned, except p_0. Show that the bar framework $G(p)$ is infinitesimally rigid in $\mathbb{E}^d$. In this case it means that any infinitesimal flex $p' = 0$, where we assume $p'_1 = p'_2 = \cdots = p'_d = 0$.

4. Consider the bar graph $G(p)$ in $\mathbb{E}^d$, whose affine span is all of $\mathbb{E}^d$ and such that all pairs of vertices of G are connected by a bar and there are no pinned vertices. Show that $G(p)$ is infinitesimally rigid.

5. Suppose that p is a configuration in $\mathbb{E}^d$ such that the affine span of p is all of $\mathbb{E}^d$. Show that an infinitesimal flex p' of p is non-trivial if and only if for some $i \neq j$, $(p_i - p_j) \cdot (p'_i - p'_j) \neq 0$.

6. Find an example of a bar framework in the plane with the smallest number of vertices that you can think of that is globally rigid, not infinitesimally rigid, and no three vertices are collinear. (A prize to the person with the graph with the smallest number of vertices.)