Math 762 Homework Assignment, Due Thursday, March 29

1. Let $p = (p_1, p_2, p_3, p_4, p_5, p_6)$ be a configuration of points in the plane that all lie on a circle, so $|p_i| = 1$, say. Let $G(p)$ be the bar framework where the the edges of the hexagon are bars and the three antipodal pairs of vertices are bars. So the graph G is the complete bipartite graph $K_{3,3}$. Show that there is a non-trivial infinitesimal flex of $G(p)$. Hint: The infinitesimal flex can be such that each $p_i' = p_i$ or $-p_i$.

2. A fair die is a convex polytope P in three-space, where all its faces are equivalent by means of a congruence of P to itself.

a. Suppose that a convex polytope P has the combinatorial type of a regular octahedron, but the faces are isoscales, instead of equilateral. Find all the combinatorial types of such polytopes. In other words, up to natural equivalence find all the ways to label the edges as either short or long. Cauchy's combinatorial lemma could be a help here.

b. Show that each labeling from part a.) can be realized by an actual convex polytope, and that such polytopes are fair dice. (An extra point for making a cardboard model.)

3. Construct two non-congruent polytopes P and Q that have an incidence preserving correspondence between the faces of P and Q such that corresponding faces are congruent. (Hint: Start with two parallelopipeds, where one is tilted one way and the other is tilted the other. These are congruent, but think of decorating them.)