Furthermore, Galois himself acknowledges that the existence of such functions t was known to Abel before him.

(2) The proof of *Lemme III* is a splendidly controversial matter. Poisson was unable to understand it and made a note on the manuscript to say so, but he accepted that the lemma was true by a result of Lagrange. Galois, incensed, appended "On jugera" to Poisson's note. Edwards feels that Galois was right and he gives a line of argument that undoubtedly completes the proof. But to do this he has to read very much more than is there into what Galois actually wrote, and I find his justification rather far-fetched. On balance I side with Poisson: it was up to Galois to be both clear and correct, whereas what he wrote is far too easily misunderstood.

I have other small criticisms of the book. For example, although it is primarily a contribution to mathematical exposition, not to the history of mathematics, I would have liked to see a paragraph or two about Abel's contributions and his influence (or lack of it) on Galois. Then again, the explanation of what is meant by solubility of cyclotomic equations by radicals is not entirely happy: elsewhere 'solution by radicals' involves using roots of equations $x^p - k = 0$, so why is a root of the equation $x^{p-1} + x^{p-2} + \cdots + x + 1 = 0$ not immediately acceptable as a radical in virtue of the fact that it is a root of $x^p - 1 = 0$? But all that these criticisms prove is that the author is right when he advises his students to 'Read the masters.' The reader must form his own judgment after reading what Galois and Harold Edwards themselves have written. That is one of the many points on which I am in complete agreement with him.

At the end of his famous testamentary letter, written on the night before the duel, Galois commends his manuscripts to Chevalier's care and writes

> il se trouvera, j'espère, des gens qui trouveront leur profit à déchiffrer tout ce gâchis.
>
> [there will, I hope, be people who will find it profitable to decipher all this mess.]

With his latest book Harold Edwards joins the select band of these *gens*. He has added another significant item to the new *genre* of mathematical publication that he created with his two earlier books *The Riemann Zeta Function* and *Fermat's Last Theorem*. Just as Galois' paper '... résolubilité des équations par radicaux' is very aptly named, so *Galois theory* has an unusually accurate title: this is not only a splendid textbook of that subject, but also an excellent contribution to the study of Galois the mathematician.

Winning Ways for Your Mathematical Plays, Volumes I & II. By Elwyn R. Berlekamp, John H. Conway, and Richard K. Guy. Academic Press, New York, 1982. Volume I, xxxi + 426 pp.; Volume II, xxxi + 424 pp.

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"It's only a game, like dying is only death."—Tom Paxton

Imagine the following offhand conversation between two erudite mathematicians, Right and Left, at a prestigious institution of higher learning, as they pick up their mail.

Left: "Wow! Look at this! A book that shows you how to win at Dots-and-Boxes!"

Right (somewhat bored): "Splendid. Now you can win against your seven-year-old."

Left (unperturbed, but defiant): "Dots is a subtle game".

Right (sarcastically): "Umm. I'll bet. You should write a paper on it."

Left: "Look. The book is loaded with fascinating mathematical games."

Right: "I don't like games. I give at the office."

Left: "This *is* the office."

Mathematicians (or perhaps mathematics itself) have a strong love-hate relationship with mathematical games. Witness "game theory". Axioms are the rules of the mathematics game. Theorems are simply statements, "I win."

On the other hand, it is embarrassing to be caught playing a game (or mathematics) that is "trivial" or "too easy". It is almost cheating. And even if your game is "difficult", it helps if you can justify it with something about importance, relevance, etc. Being "just a game", in some eyes, makes it almost automatically "not serious mathematics", "child's play".

Mathematicians can be classified as either games players or not. There seems to be a reasonably large proportion of Chess players, Go players, etc., among mathematicians. For them games and mathematics are too compelling to deny.

On the other hand, many feel as Right above that "serious" games are too much like mathematics itself and "do not provide adequate diversion to their mathematical interests." If you believe that, they can explain away uncounted other onerous character flaws as well.

Winning Ways is not for the games hater or the person who wants a tidy theory neatly worked out in the greatest possible generality. Conway's previous book, *On Numbers and Games*, affectionately called ONAG, is a bit more in this mathematical style. *Winning Ways* is chaotic, rich with more examples than you can digest, full of diagrams, cartoon pictures, tables, in-jokes, silly jokes, silly puns, mnemonic devices, who knows what other devices, and it has more information and game winning strategies than you would ever want to know. In short, it is more fun than the Three Stooges, albeit more intellectual. For example, the following figure copied from WWI (*Winning Ways*, volume I), page 121, gives some of the flavor of the book.

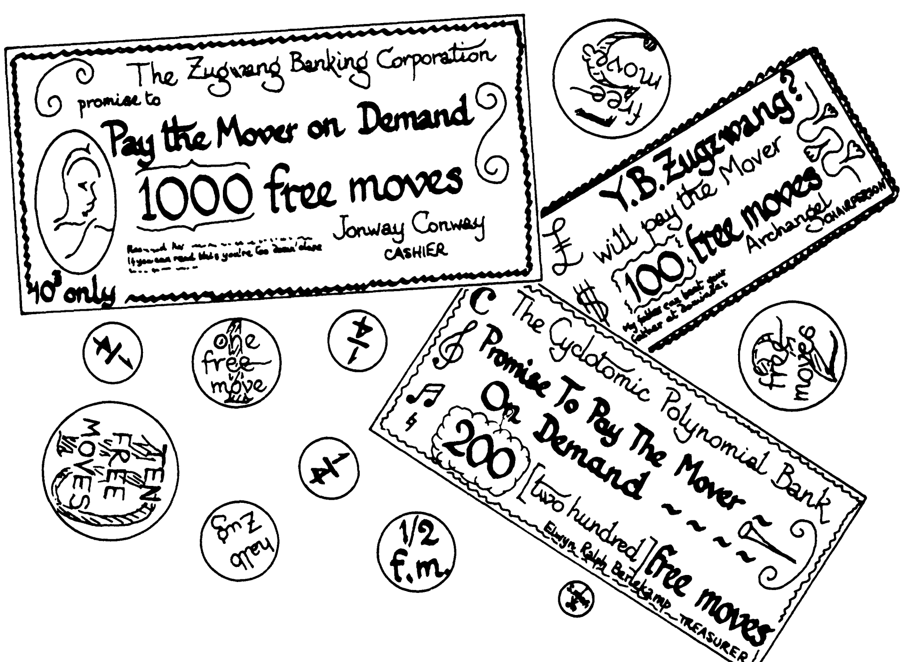


Figure 5. Some Cheques Ready for Cashing.

Zugzwang is a term, often used in Chess, for a position where a player does not want to move. The small print displays some bumper sticker mentality. $\uparrow$ is a game called up.

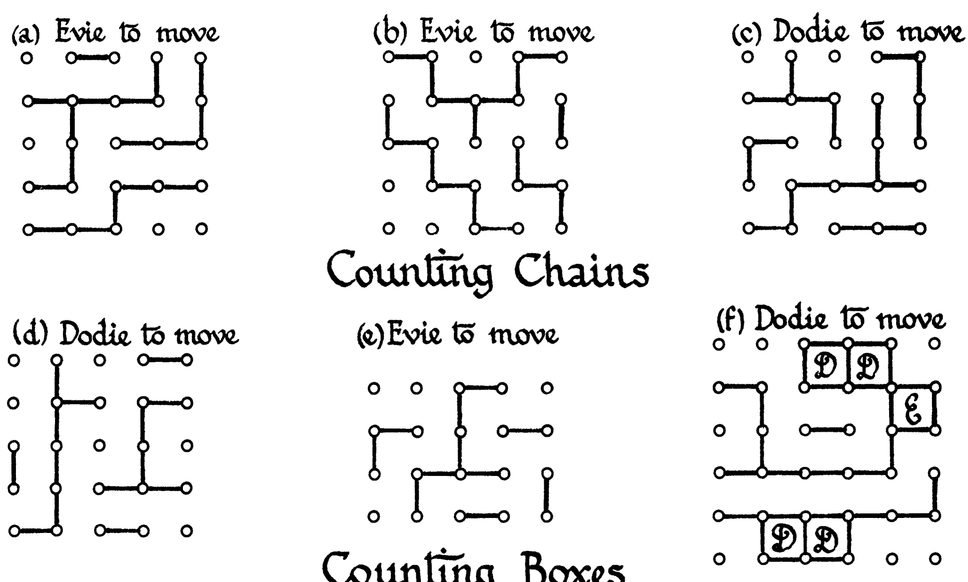
The book is unique. It can be read as a source of interesting combinatorial games (never mind a winning strategy), especially the second volume; one can just look at the funny cartoons; or it can provide much more. In the preface we have the following gratefully appreciated self-review:

> "We can supply the reviewer, faced with the task of ploughing through nearly a thousand information-packed pages, with some pithy criticisms by indicating the horns of the polylemma the book finds itself on. It is not an encyclopedia. It is encyclopedic, but there are still too many games missing for it to claim to be complete. It is not a book on recreational mathematics because there's too much serious mathematics in it. On the other hand, for us, as for our predecessors Rouse Ball, Dudeney, Martin Gardner, Kraitchik, Sam Loyd, Lucas, Tom O'Beirne and Fred. Schuh, mathematics itself is a recreation. It is not an undergraduate text, since the exercises are not set out in an orderly fashion, with the easy ones at the beginning. They are there though, and with the hundred and sixty-three mistakes we've left in, provide plenty of opportunity for reader participation. So don't just stand back and admire it, work of art though it is. It is not a graduate text, since it's too expensive and contains far more than any graduate student can be expected to learn. But it does carry you to the frontiers of research in combinatorial game theory and the many unsolved problems will stimulate further discoveries."

Winning Ways is, however, far weightier than its predecessors, both by mass and mathematics. The following are two of the many games discussed:

Nim: Two players start with three piles of beans, the piles having three beans, four beans, and five beans respectively. Each player in his turn must take some number of beans, but at least one bean, from one of the remaining piles. The player to remove the last bean wins. The idea behind the winning strategy is simple, and considerable generalizations of it are at the heart of much of this kind of game theory.

Dot-and-Boxes: This is the child's game where two players start with a rectangular array of dots and take turns joining a pair of horizontal or vertical dots. When a player completes the fourth side of a unit square, he puts his initial in that box and then must draw another line. The player at the end with the greater number of boxes wins.



Apparently this game can be played at about seven levels of sophistication and expert play involves a knowledge of much of the contents of WWI, which involves, among many other things, Sprague-Grundy Theory, which relates all impartial games to Nim.

The figure on page 413 shows some Dots-and-Boxes problems for advanced players, taken from WWII, page 536.

In addition to Nim-type games, much of WWI involves a development of Conway's idea of regarding number systems as particularized forms of games, as described in ONAG, but continued and expanded. Suppose two players Left and Right (no relation to those above) take turns playing a game, where at a position P the options to Left are $a, b, c, \dots,$ and the options to Right are $d, e, f, \dots$. Conway denotes P as $\{a, b, c, \dots / d, e, f, \dots\}$, reminiscent of Dedekind cuts. In fact, some games can be regarded as the real numbers. It is surprising how this simple idea leads to so many useful insights in this theory.

In WWII there is a discussion of Rubik's Cube, Conway's Life (neither a game nor a puzzle, but never mind), sliding block puzzles, Fox and Geese, Spots and Sprouts, String and Wire puzzles, and much, much more. Mercifully, Chess and Go are not discussed. (Nevertheless, the end-game in Go might benefit from some analysis as is in WWI.)

Needless to say, this book is winningly overwhelming.

LETTERS TO THE EDITOR

For instructions about submitting letters for publication in this department see the inside front cover.

Editor:

I am sure that by now you will have received several letters on the subject of Arthur Richert's note, "A Non-Simpsonian use of parabolas in numerical integration" (this MONTHLY, 92 (1985) 425–426). The statement that "the error bound is an improvement over Simpson's rule by a factor of almost eleven" is quite misleading. In fact, as the author himself points out, the Taylor polynomial method requires twice as many function evaluations for a given n as does Simpson's rule. Thus, it would be fairer to poor old Simpson to use $2n$ subdivisions in the error estimate, and this will reduce the Simpson rule error by a factor of approximately 16. Viewed in this light, Simpson's rule is the better rule, a fact which is easily borne out by experiment; e.g., taking $n = 20$ in Richert's formula and $n = 40$ in Simpson's rule, one gets approximations to π of

$$3.14159\;26536\;07233 \text{ and } 3.14159\;26535\;80105$$

using the usual integral for arctan(1) and (IEEE) 64 bit floating point arithmetic ($\pi = 3.14159\;26535\;89793\dots$). Here, the error of the Richert method is almost twice the error of Simpson's rule. The author's method is even more costly in this case, since the second derivative $f''(x)$ of $1/(1 + x^2)$ is computationally more complex than $f(x)$.

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