Math 1350 Quiz 3 - July 21, 2010

(5 points) Name: Solutions

1. Suppose I have 50 colored marbles - 2 blue ones, 38 red ones, and 10 yellow ones.

   (a) (5 points) In how many ways can I pick 1 blue marble and 3 red ones?

   You have 2 possibilities for picking one blue marble and $C(38, 3)$ ways to pick 3 red ones. The total number of possibilities is:

$$2 \cdot \begin{pmatrix} 38 \\ 3 \end{pmatrix} = \frac{38 \cdot 37 \cdot 36 \cdot 2}{3 \cdot 2} = 38 \cdot 37 \cdot 12$$

   (b) (5 points) If I pull 1 marble out of the bag randomly, what is the probability of getting a yellow one?

$$\text{Prob(Yellow Marble)} = \frac{\begin{pmatrix} 10 \\ 1 \end{pmatrix}}{\begin{pmatrix} 50 \\ 1 \end{pmatrix}} = \frac{10}{50} = \frac{1}{5}$$

2. (5 points) What is the index of coincidence, and what information does it give us?

   The index of coincidence is the probability of choosing two of the same letters from a selection of text. The index of coincidence of encrypted text can help us determine if it has been encrypted with a monoalphabetic cipher (if the I.C. is closer to 0.065) or with a polyalphabetic cipher like the Vigenere Square (if the I.C. is closer to 0.0385). Also, we can use the index of coincidence to determine the length of the keyword for a Vigenere Square.

3. (5 points) Suppose I have given you some encrypted text with I.C. ≈ 0.071 and so $k \approx 0.8$. What kind of cipher was used to encrypt the text? Support your answer with an explanation.

   Because the index of coincidence is closer to 0.065, then we suppose that the text was encrypted with a monoalphabetic cipher, by the explanation in number 2. Alternatively, we can think of it as a Vigenere encrypted text with keyword length 1, using the information given about k. This means that the entire text has been encrypted with a single shift cipher and so is monoalphabetically encrypted.