

Solutions to HW Set 3

Math 1350

Problem Set 2

1. (a) $25^{-1} \equiv 7 \pmod{29}$, so $x \equiv 18$.
(b) $5^{-1} \equiv 7 \pmod{17}$, so $x \equiv 14$.
(c) We first seek a general solution: $2x \equiv 5 \pmod{7} \implies x \equiv 6$, so this solution works for (c), as do sums of 6 with multiples of 7 (since the factor of 3 annihilates multiples of 7, $\pmod{21}$), giving $x = 6, 13, 20$.
(d) As was done in (c), we would first solve $18x \equiv 4 \pmod{51}$, but this has no solutions, since $18x$ is always a multiple of 3, mod 51 (since 3 divides 51).
2. $5\ell \equiv 1 \pmod{6}$, implies $\ell \equiv 5 \pmod{6}$, which solves $11\ell \equiv 1+6k \equiv x-4 \pmod{66}$, so $x \equiv 59 \pmod{66}$ solves the first two equivalences. Then, solve $59 + 66k \equiv 3 + 17j \pmod{1122}$ by solving $15k \equiv 12 \pmod{17}$, which gives $k \equiv 11 \pmod{17}$, then $x \equiv 785 \pmod{1122}$. Indeed we can check this answer, and because of the coprimality of the three modular numbers, 6, 11, 17, these are precisely the solutions, i.e., $x = 785 + 1122n$ for each integer $n \in \mathbb{Z}$.
3. We have the simultaneous equivalences

$$N \equiv 3 \pmod{17},$$

$$N \equiv 10 \pmod{16},$$

$$N \equiv 0 \pmod{15}.$$

Solving the first two by the same method as in 2 above gives $k = 7 \pmod{16}$ as a solution to $3 + 17k \equiv 10 + 16j$. Then $N \equiv 122 \pmod{17 * 16}$. Then $122 + 17 * 16\ell \equiv 0 \pmod{15}$ is rewritten as $2\ell \equiv 13 \pmod{15}$, which is solved with $\ell \equiv 14 \pmod{15}$. The pirates thus found $122 + 17 * 16 * 14 = 3930$. Checking, 5 and 3 are readily seen to go into 3930, so 15 does. and 122 solved the first two equivalences so adding multiples of $17 * 16$ doesn't change that fact. Finally, $3930 < 17 * 16 * 15 = 4080$, and as these three numbers are co-prime, we know solutions to the three equivalences will be 4080 apart, thus 3930 is the smallest number of coins possible.

2.3

1. (a) (in the back of the book)
 (b) WE WILL NOW DISCUSS IN A LITTLE MORE DETAIL THE STRUGGLE FOR EXISTENCE.

3. (a) (in the back of the book)
 (b) Education has become a prisoner of contemporaneity. It is the past, not the dizzy present, that is the best door to the future.
 (d) I'd rather be a lightning rod than a seismograph.
 (f) The truth is always something that is told, not something that is

known. If there were no speaking or writing, there would be no truth about anything There would only be what is.

4. Four score and seven years ago our fathers brought forth on this continent a new nation conceived in liberty and dedicated to the proposition that all men are created equal. Keyword: Lincoln (yes there's a typo in proposition)

2.4

2. 50 letters, 9 columns, means the first five columns have 6 letters, the last four columns have 5 letters. This gives
WELL IF I CALLED THE WRONG NUMBER WHY DID YOU ANSWER THE PHONE

4.

<i>C</i>	<i>O</i>	<i>M</i>	<i>M</i>	<i>O</i>	<i>N</i>	<i>S</i>
<i>E</i>	<i>N</i>	<i>S</i>	<i>E</i>	<i>I</i>	<i>S</i>	<i>T</i>
<i>H</i>	<i>E</i>	<i>B</i>	<i>E</i>	<i>S</i>	<i>T</i>	<i>D</i>
<i>I</i>	<i>S</i>	<i>T</i>	<i>R</i>	<i>I</i>	<i>B</i>	<i>U</i>
<i>T</i>	<i>E</i>	<i>D</i>	<i>C</i>	<i>O</i>	<i>M</i>	<i>M</i>
<i>O</i>	<i>D</i>	<i>I</i>	<i>T</i>	<i>Y</i>	<i>I</i>	<i>N</i>
<i>T</i>	<i>H</i>	<i>E</i>	<i>W</i>	<i>O</i>	<i>R</i>	<i>L</i>
<i>D</i>	<i>F</i>	<i>O</i>	<i>R</i>	<i>E</i>	<i>V</i>	<i>E</i>
<i>R</i>	<i>Y</i>	<i>M</i>	<i>A</i>	<i>N</i>	<i>I</i>	<i>S</i>
<i>C</i>	<i>O</i>	<i>N</i>	<i>V</i>	<i>I</i>	<i>N</i>	<i>C</i>
<i>E</i>	<i>D</i>	<i>T</i>	<i>H</i>	<i>A</i>	<i>T</i>	<i>H</i>
<i>E</i>	<i>I</i>	<i>S</i>	<i>W</i>	<i>E</i>	<i>L</i>	<i>L</i>
<i>S</i>	<i>U</i>	<i>P</i>	<i>P</i>	<i>L</i>	<i>I</i>	<i>E</i>
<i>D</i>	<i>W</i>	<i>I</i>	<i>T</i>	<i>H</i>	<i>I</i>	<i>T</i>

Next, we arrange the columns according to the keyword, i.e., AABEGLR,
1754326, and read down the columns, which gives

CEHIT OTDRC EESDS TDUMN LESCH LETOI SIOYO ENIAE LH-
MEE RCTWR AVHWP TMSBT DIEOM NTSPI ONESE DHFYO
DIUWN STBMI RVINT LII

7. (in the back of the book)