Math 4310 Homework Expectations

Writing mathematics. Communication of ideas is a central part of doing mathematics, and proofs are best communicated by using language and not just symbols or equations! Accordingly, your homework should be written up in complete sentences with proper spelling, grammar, and punctuation. Some guidelines for writing mathematics well are:

- Your proofs should be a coherent argument that can be read linearly down the page.
- You should specify any variables or symbols you use - for instance many proofs may start out like "Let V be a vector space and v be a vector in V..." but should not start out using the symbols V and v as variables without saying what they are. (Of course you don't need to re-specify symbols used in the problem statement. You also don't need to specify symbols like det and $\mathbb{R}$ which represent specific standard things we've used that are not variables).
- Equations or symbolic manipulations you do should be accompanied by an explanation in words of what you're computing and how it contributes to what you're trying to prove.
- You should clearly mention any results from the textbook or lectures you use, and explain how you're using them (e.g. carefully saying what you are applying the theorem to, justifying why the hypotheses are true, and showing how you get from the conclusion of the theorem to the way you are using it).
- To accomplish all of this, you should not be turning in your scratchwork or your "first draft" of solving a problem! You should work out what you need to do to complete the proof of what you're solving, and *then* write up a solution in a linear manner. Your writeup should look like something that you'd be happy to see in a textbook.

Some other resources for writing mathematics well are available on the course website.

Extended Glossaries. In addition to the "standard" homework problems, many weeks will include an *extended glossary*. This is an exercise in writing mathematics clearly and formally - you will be given a mathematical term and be asked to define it formally, provide an example and non-example, and prove a theorem involving that term. You should write a page or two that you could imagine seeing as a page in a textbook (consisting of the four things asked for, plus perhaps some extra commentary between them like you'd see in between such things in a textbook). Some examples of glossaries are provided on the course website.

If you work together on the extended glossaries, please come up with at least *two* examples/non-examples/theorems among your group, and please have each of them appear in at least one group member's glossary.

Collaboration. First of all, I strongly recommend you at least think about all of the homework problems on your own. You may work with other students to solve the problems, but you must **write up your solutions in your own words** even if you come up with the solutions collaboratively, and you should **write the names of your collaborators** on your assignment (this will not affect your grade). Similarly, if you find something in a book, on the internet, or elsewhere that contributes to your solution to a problem, you should again write up the idea in your own words and provide a citation for what you found. Copying solutions from other students or from anywhere else is a violation of the Code of Academic Integrity.

Math 4310 Homework 1 - Due February 1

Problem 1. Prove by induction that the following identity holds for all natural numbers $n \geq 1$:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 2. Let a, b, c be integers, and prove the following facts about divisibility:

(a) If $b|a$ then $b|ac$.

(b) If $b|a$ and $c|b$ then $c|a$.

(c) If $b|a$ and $b|(a+c)$ then $b|c$.

Problem 3. (a) For $2 \leq n \leq 8$, compute the prime factorization of the integer $2^n - 1$.

(b) For the *composite* numbers n above, you should have found that $2^n - 1$ is composite. Prove in general that if $n \geq 2$ is composite, then $2^n - 1$ is composite.

A number of the form $2^p - 1$ that is prime is called a *Mersenne prime* (note that part (b) tells us that if $M_p = 2^p - 1$ is prime then p must be prime itself). In part (a) you should have found that M_2, M_3, M_5, and M_7 were Mersenne primes. Unfortunately most numbers of the form $2^p - 1$ seem *not* to be prime (for instance it fails for $M_{11} = 2047 = 23 \cdot 89$). In fact there are only 49 Mersenne primes known - the most recent one to be found, $2^{74207281} - 1$, was just announced two weeks ago!

Extended Glossary. Give a definition of a **congruence** between two integers. Give an example and a non-example of a congruence. Then state and prove a theorem about congruences.