

## RAPID REVIEW

- (1) Given a function  $f$ , the **derivative** of  $f$  at the point  $a$  is defined by

$$f'(a) := \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

- (2) The line tangent to  $(a, f(a))$  is  $y - f(a) = f'(a)(x - a)$ .

- (3) Differentiation rules:

(a)  $(cf)' = cf'$  if  $c$  is a constant.

(b)  $(f + g)' = f' + g'$

(c) **Product rule:**  $(fg)' = f'g + fg'$ .

(d) **Quotient rule:**  $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$ .

(e) **Chain rule:**  $(f(g(x)))' = f'(g(x))g'(x)$ .

- (4) **Implicit differentiation** is used to compute  $\frac{dy}{dx}$  when the variables  $x$  and  $y$  are related by an equation, such as  $x^3 - y^3 = 4$ . This is a special instance of the chain rule. To perform implicit differentiation, take the derivative of both sides. Remember that  $y$  is a function of  $x$ , so  $\frac{d}{dx}f(y) = f'(y)y'$ .

- (5) **The first derivative test:** If  $f$  is differentiable and  $c$  is a critical point, then the type of critical point can be found in the table.

Sign Change of $f'(x)$	Type of Critical Point
From + to -	Local max
From - to +	Local min

- (6) A function  $f$  is **concave up** on  $(a, b)$  if  $f'$  is increasing, and **concave down** if  $f'$  is decreasing. A **point of inflection** is a point  $(c, f(c))$  where the concavity changes. We can use the first derivative test on the derivative  $f'$  to find the inflection points of  $f$ .

## PROBLEMS

(1) Compute  $\frac{dy}{dx}$ .

(a)  $y = 3x^5 - 7x^2 + 4$

(b)  $y = \frac{x}{x^2+1}$

(c)  $y = (x^4 - 9x)^6$

(d)  $y = \sqrt{x + \sqrt{x}}$

(e)  $y = \tan(x)$

(f)  $y = \sin(2x) \cos^2(x)$

(g)  $y = \tan(\sqrt{1 + \csc x})$

(h)  $x^3 - y^3 = 4$

(i)  $y = xy^2 + 2x^2$

(j)  $y = \sin(x + y)$

(2) Find the points on the graph of  $f(x) = x^3 - 3x^2 + x + 4$  where the tangent line has slope 10.

(3) Find the critical points of  $f$  and determine if they are minima or maxima.

(a)  $f(x) = x^3 - 4x^2 + 4x$

(b)  $f(x) = x^2(x+2)^3$

(c)  $f(x) = x^{2/3}(1-x)$

(4) Find the points of inflection of the function  $f$

(a)  $f(x) = x^3 - 4x^2 + 4x$

(b)  $f(x) = x - 2 \cos x$

(c)  $f(x) = \frac{x^2}{x^2+4}$

(5) Find conditions on  $a$  and  $b$  that ensure  $f(x) = x^3 + ax + b$  is increasing on  $(-\infty, \infty)$ .