§11.4: Absolute/Conditional Convergence
§11.5 Ratio and Root Tests

Name: ____________________

Math 1910 November 21, 2017

Rapid Review

(1) **Absolute Convergence:** A series $\sum_{n=1}^{\infty} a_n$ **converges absolutely** if $\boxed{}^{(1)}$ converges.

(2) **Absolute Convergence Theorem:** If $\boxed{}^{(2)}$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

(3) **Conditional Convergence:** A series $\sum_{n=1}^{\infty} a_n$ **converges conditionally** if $\boxed{}^{(3)}$ converges but $\boxed{}^{(4)}$ diverges.

(4) **Alternating Series Test:** If the sequence $\{b_n\}$ is positive and decreasing, and $\lim_{n\to\infty} b_n = 0$, then $\boxed{}^{(5)}$ converges.

(5) **Ratio Test:** Assume that $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ exists. Then the series $\sum_{n=1}^{\infty} a_n$

(a) converges absolutely when $\boxed{}^{(6)}$

(b) diverges when $\boxed{}^{(7)}$

(c) inconclusive if $\boxed{}^{(8)}$

(6) **Root Test:** Assume that $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$ exists. Then the series $\sum_{n=1}^{\infty} a_n$

(a) converges absolutely if $\boxed{}^{(9)}$

(b) diverges if $\boxed{}^{(10)}$

(c) inconclusive if $\boxed{}^{(11)}$

PROBLEMS

(1) True or false?

(a) If $\sum_{n=0}^{\infty} |b_n|$ diverges, then $\sum_{n=0}^{\infty} b_n$ also diverges.

(b) If $\sum_{n=0}^{\infty} a_n$ diverges, then $\sum_{n=0}^{\infty} |a_n|$ also diverges.

(c) If $\sum_{n=0}^{\infty} c_n$ converges, then $\sum_{n=0}^{\infty} |c_n|$ also converges.

(2) Show that $\sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{n}{n^2+1}$ converges conditionally.

(3) Does $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$ converges absolutely, conditionally, or not at all?

(4) Apply the ratio test or the root test to determine the convergence or divergence of the following series, or state that the test is inconclusive. If the test is inconclusive, apply another test to determine convergence or divergence, if possible.

(a) $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1} n}{5^n}$

(b) $\sum_{n=1}^{\infty} \dfrac{3n+2}{5n^3+1}$

(c) $\sum_{n=1}^{\infty} \dfrac{2^n}{n}$

(d) $\sum_{n=0}^{\infty} \dfrac{1}{10^n}$

(e) $\sum_{k=0}^{\infty} \left(\dfrac{k}{k+10}\right)^k$