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(1) Find the volume of a regular tetrahedron whose faces are equilateral triangles of side length $s$.

(2) A frustum of a pyramid is a pyramid with its top cut off. Let $V$ be the volume of a frustum of height $h$ whose base is a square of side length $a$ and whose top is a square of side length $b$ with $a>b \geq 0$.

(a) Show that if the frustum were continued to a full pyramid, it would have height $\frac{h a}{a-b}$.
(b) Calculate the side length of a cross-section of the frustum at height $x$ from the base.
(c) Calculate the volume of the frustum.
(3) A plane inclined at an angle of $45^{\circ}$ passes through a diameter of the base of a cylinder of radius r. Find the volume of the region within the cylinder and below the plane.

(4) The solid $S$ below is the intersection of two cylinders of radius $r$ whose axes are perpendicular.

(a) The horizontal cross-section of each cylinder at a distance $y$ from the central axis is a rectangular strip. Find the area of the horizontal cross-section of $S$ at distance $y$ from the central axis.
(b) Find the volume of $S$ as a function of $r$.

