Math 3040 Discussion questions, Sept. 9, 2019

1. Suppose $(Z, +, \cdot)$ is a ring. Prove that if there exists $m \in Z$ such that $m + x = m$, then $x = 0$.
2. Suppose $(Z, +, \cdot)$ is a commutative ring. Prove that if $3|m$ and $3|n$, then $3|mn$ and $3|(m+n)$.
   Remark: What is 3?
3. Suppose $(Z, +, \cdot)$ is an integral domain. Prove that if there exists $m \in Z, m \neq 0$ such that $m \cdot x = m$, then $x = 1$.
4. Let $(Z, +, \cdot)$ be an integral domain. Prove that if $x \in Z$ and $x \cdot x = x$, then $x = 0$ or $x = 1$.

5. Fix n a positive integer. Instead of Z being the integers suppose Z is all $n \times n$ matrices with real coefficients. Let addition and multiplication be addition and multiplication of $n \times n$ matrices. Is Z a ring? commutative ring? integral domain?