Math 3040 Discussion questions, Sept. 30, 2019

1. Project 3.7 i) - iii) in TAP.

2. Prove that

$$\sum_{i=1}^{j} f_i = f_{j+2} - 1,$$

   where f_i is the ith Fibonacci number.

3. Consider a modified Fibonacci sequence as follows. Let $(Z, +, \cdot)$ be a commutative ring. Choose a, b in Z and define a sequence of elements of Z as follows:

   - $f_1 = a$.
   - $f_2 = b$.
   - $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

   Find a 'similar' formula to the previous problem for this modified Fibonacci sequence and prove it.

4. Define a_n to be the number of subsets of $[n] = \{1, \ldots, n\}$ which do not contain two consecutive integers. For example, $a_2 = 3$ since the subsets of $[2] = \{1, 2\}$ which do not contain consecutive integers is

$$\{\emptyset, \{1\}, \{2\}\}$$

   and there are three of them. Find a formula for a_n.