Math 3040 Discussion questions, Oct. 9, 2019

- 1. Prove there exists an infinite sequence A_i of subsets of \mathbb{N} such that:
 - Each A_i is an infinite set.
 - $\bigcap_{i=1}^{\infty} A_i = \emptyset.$
 - For every $n \in \mathbb{N}$ the intersection $\bigcap_{i=1}^{n} A_i$ is an infinite set.
- 2. Let A be a set and suppose we count the elements of the set ending with m. There are two functions we can associate to this operation, $f : A \to [m]$ where f(x) is the number we said when pointing at x, or $g : [m] \to A$, where g(p) is the element of A we were pointing to when we said p. For each of the four counting mistakes below explain why f and g are not bijections.
 - Skipping a number.
 - Repeating a number.
 - Skipping an element of A.
 - Counting an element of A twice.
- 3. Prove the pigeon hole principle. Precisely Prove that if m < n are in \mathbb{N} and $f : [n] \to [m]$, then f is not an injection.