Math 3040 Discussion questions, Nov. 8, 2019

1. Let (x_n) be a Cauchy sequence. Define

$$b_k = \sup\{x_k, x_{k+1}, \dots\}.$$

   Prove that b_k is a decreasing sequence which is bounded above and bounded below. (You may assume the result of hw problem # 1.)

2. Define a binary relation $\sim$ on $\mathbb{N} \times \mathbb{N}$ by

$$(n_1, n_2) \sim (n_1', n_2')$$

   if and only if $n_1 + n_2' = n_2 + n_1'$.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) Define a binary operation $\oplus$ on the equivalence classes of $\sim$ by

$$[(n_1, n_2)] \oplus [(n_1', n_2')] = [(n_1 + n_1', n_2 + n_2')]$$

   Prove that $\oplus$ is well defined.