Math 3040 HW 1 - Due Sept. 16, 2019

1. Read Section 1.3 from TAP. Prove Proposition 1.27 parts ii) and iii).

2. Suppose $(Z, +, \cdot)$ is an integral domain. Define 3 as in class. Prove or give a counterexample: If $3 \cdot m = m$, then $m = 0$.

3. For this problem Z is a two-element set, $Z = \{a, b\}$. Define binary operations $+$ and $\cdot$ on Z with the following tables.

$+$	a	b
a	b	a
b	a	b

$\cdot$	a	b
a	a	b
b	b	b

   You may assume that this $(Z, +, \cdot)$ satisfies the five axioms in Axiom 1.1 of TAP. Determine whether or not $(Z, +, \cdot)$ satisfies Axioms 1.2, 1.3, 1.4 and 1.5 in TAP. In each case either prove that it does satisfy the axiom or give an example which shows it does not.

4. Assume that $(Z, +, \cdot)$ is a commutative ring. Prove that $(Z, +, \cdot)$ is an integral domain if and only if whenever $m \cdot n = 0$, then $m = 0$ or $n = 0$.