Math 3040 HW 11 - Will not be collected

1. Let $(G, \cdot)$ be a group such that for all $g \in G,\ g^2 = 1$. Prove that for all g and h in $G,\ g \cdot h = h \cdot g$.

2. Let $(G, \cdot)$ and $(H, \circ)$ be groups. A function $f : G \to H$ is a *homomorphism* if for all $g, g' \in G,\ f(g \cdot g') = f(g) \circ f(g')$. The *kernel* of a homomorphism f is

$$\ker f = \{g \in G : f(g) = 1_H.\}$$

   Prove that $\ker f$ is a subgroup of G.

3. Let $f : G \to H$ be a homomorphism from the group $(G, \cdot)$ to the group $(H, \circ)$. Let $K = \ker f$. Prove that for all $g \in G$,

$$\{g \cdot k : k \in K\} = \{k \cdot g : k \in K\}.$$