## Math 3040 HW 6 - Due Oct. 25, 2019

- 1. Let  $f : A \to B$  and  $g : B \to C$ . For each statement below either prove it or construct f, g, A, B, C which show that the statement is false.
  - (a) If  $g \circ f$  is surjective, then g is surjective.
  - (b) If  $g \circ f$  is surjective, then f is surjective.
  - (c) If  $g \circ f$  is injective, then f and g are injective.
- 2. Let  $n \in \mathbb{N}$  and  $f: [n] \to [n]$  a function. Prove that f is a surjection if and only if f is an injection.
- 3. Let  $(Z, +, \cdot)$  and  $(W, \oplus, \otimes)$  be commutative rings. Let  $f : Z \to W$  be a function with the following property: For all  $x, y \in Z$ ,  $f(x + y) = f(x) \oplus f(y)$ . Prove the following.
  - (a)  $f(0_Z) = 0_W$ .
  - (b) For all  $x \in Z$ , f(-x) = -f(x).
  - (c) If  $f(x) = 0_W$  implies that  $x = 0_Z$ , then f is injective.