Math 3040 HW 8- Due Nov. 8, 2019

1. Let (x_n) be a Cauchy sequence in an ordered field. Prove that $\{x_1, x_2, x_3, \dots\}$ is bounded above and below.

2. Let A be a nonempty set in an ordered field and assume that $\inf A = s$ and $t < s$. Prove that there is NO sequence (x_n) such that $x_n \in A$ for all n, and $\lim_{n\to\infty} x_n = t$.

3. Let A be a nonempty set in an ordered field and assume that $\inf A = s$. Prove that there is a sequence (x_n) such that $x_n \in A$ for all n and $\lim_{n\to\infty} x_n = s$.

4. Let x_n be a sequence such that for every $m \in \mathbb{N}$, $m \geq 2$ the sequence $\lim_{n\to\infty} x_{mn} = L$. Prove or provide a counterexample: $\lim_{n\to\infty} x_n = L$.