Math 3040 HW 9 - Due Nov. 18, 2019

1. Let (x_n) be a Cauchy sequence with $x_n \in \mathbb{R}$ for all n. By discussion question 1, we can define an infinite sequence of real numbers,
$$b_k = \sup\{x_k, x_{k+1}, \dots\},$$
and this sequence is bounded below and decreasing. Hence it has a limit L. Prove that
$$\lim_{n\to\infty} x_n = L.$$

2. Let $(x_n)_{n=0}^{\infty}$ be a decreasing sequence of positive real numbers such that $\lim_{n\to\infty} x_n = 0$. Prove that
$$\sum_{n=0}^{\infty} (-1)^n x_n = L$$
for some $L \in \mathbb{R}$.