Klein Bottle Horizontal Glued Comparison of Level 2 Eigenfunctions and Level 1 Eigenfunctions By Averaging

SPUR 2016

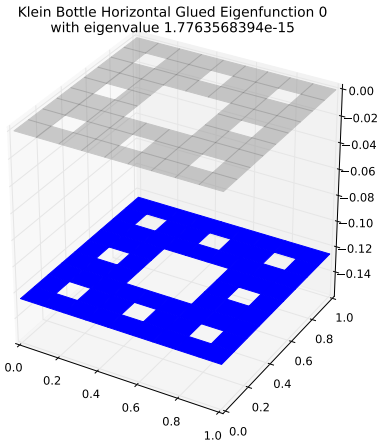
May 23, 2018

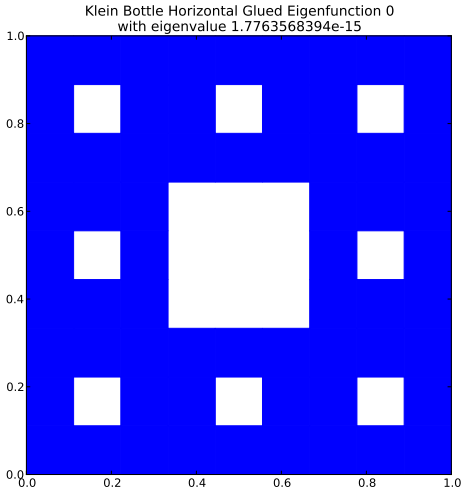
Key to Dot Value

Dot values are in general between 0 and 1; those close to 0 are better matches, while those close to 1 are not good matches. Dot value 2 indicates the eigenvalue averages to the zero function. Dot value 3 indicates the projection onto the closest eigenspace is zero.

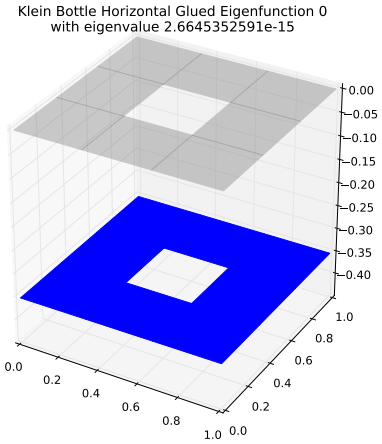
1 $M = 2$ Eigenfunction 0

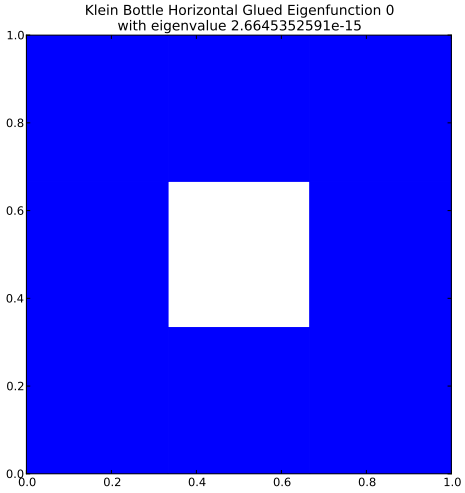
$M = 2$ Eigenfunction 0 has eigenvalue 1.7763568394e-15





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

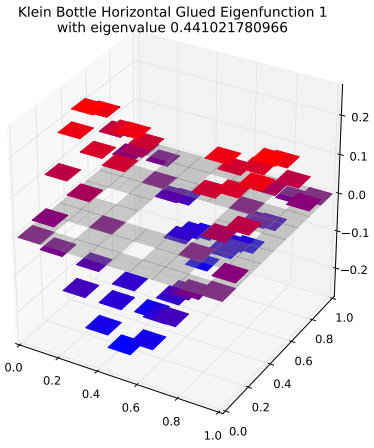


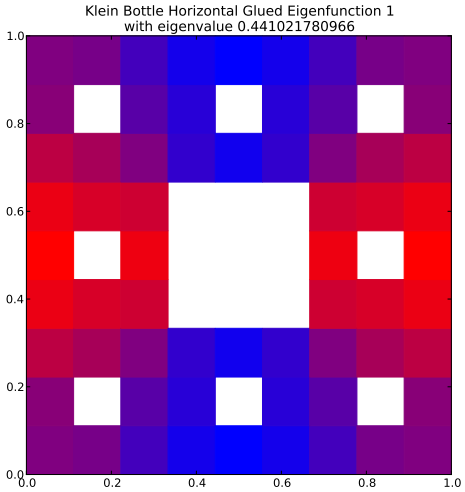


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.666666666667$
Dot Value: 2.220446049250313e-16

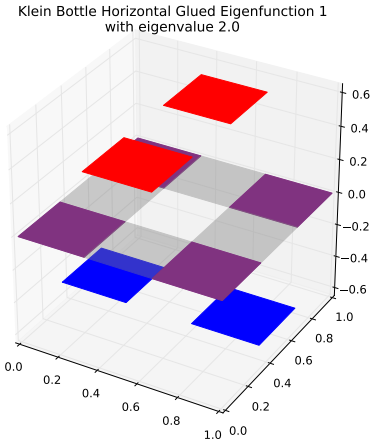
2 $M = 2$ Eigenfunction 1

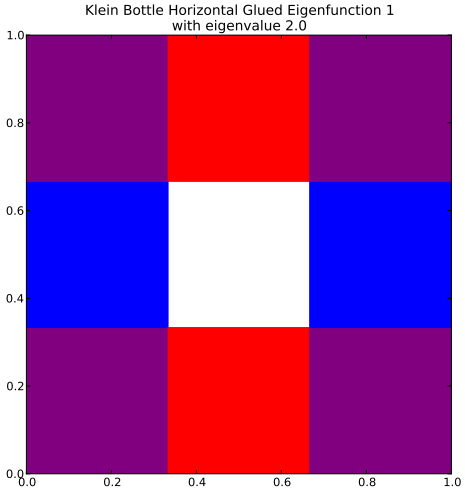
$M = 2$ Eigenfunction 1 has eigenvalue 0.441021780966





Compare to $m = 1$ eigenspace with eigenvalue 2.0

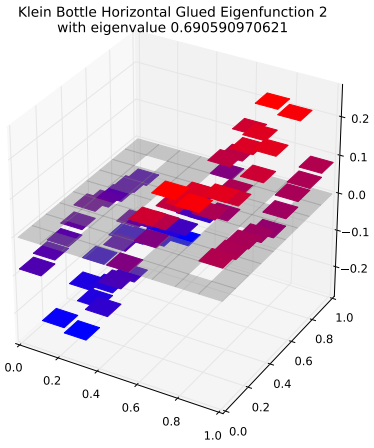


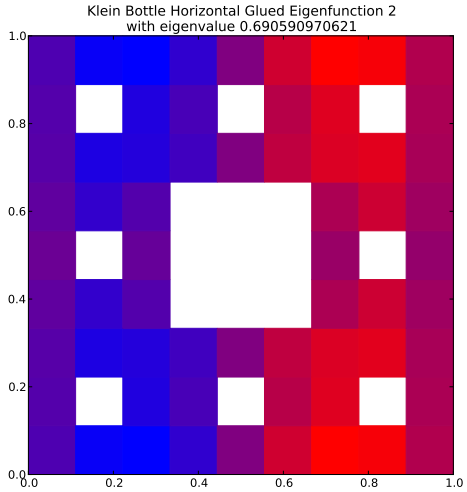


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.220510890483$
Dot Value: 0.0

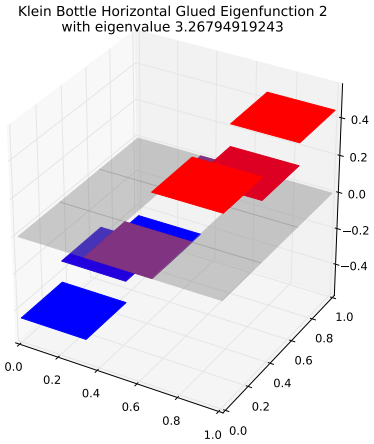
3 $M = 2$ Eigenfunction 2

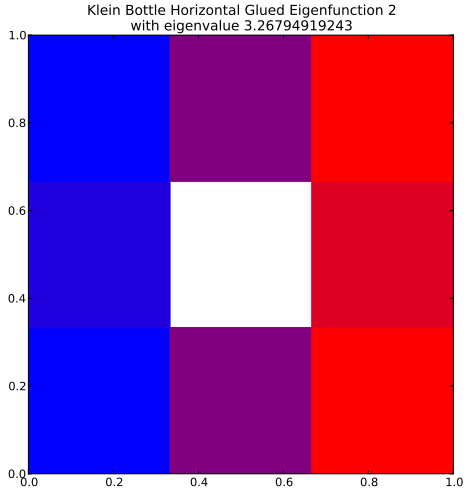
$M = 2$ Eigenfunction 2 has eigenvalue 0.690590970621





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

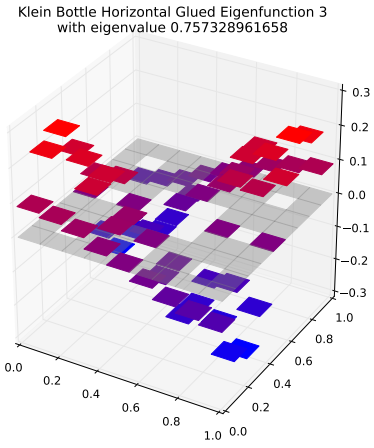


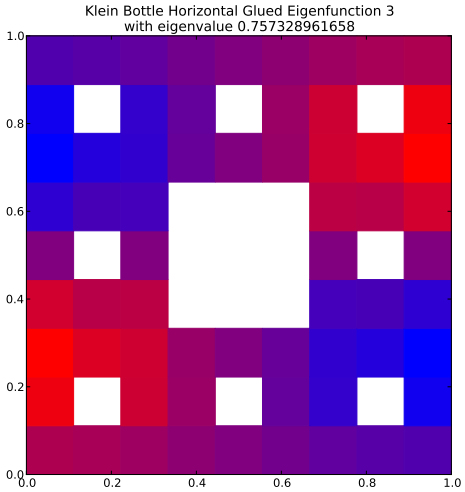


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.211322431885$
Dot Value: 0.007208422818863691

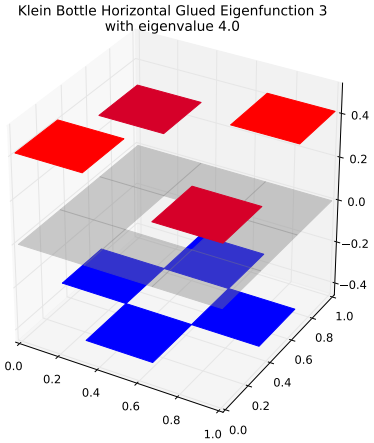
4 $M = 2$ Eigenfunction 3

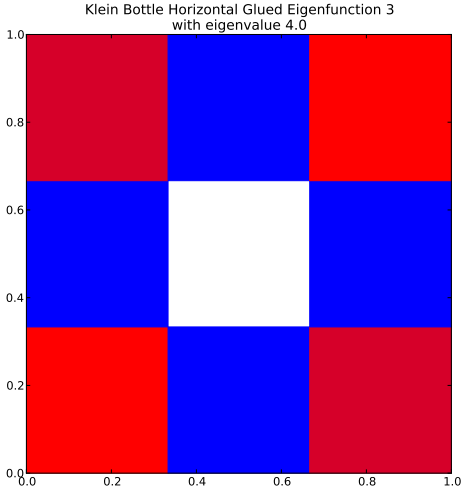
$M = 2$ Eigenfunction 3 has eigenvalue 0.757328961658





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

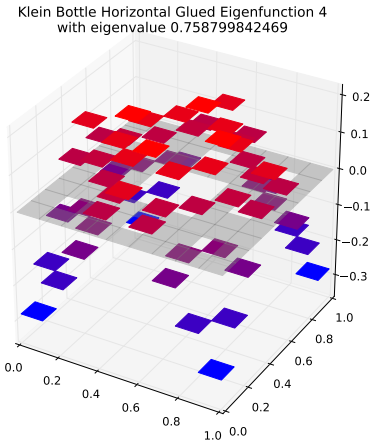


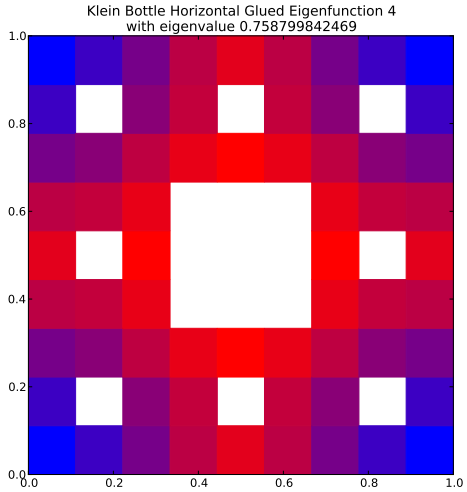


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.189332240415$
Dot Value: 2.220446049250313e-16

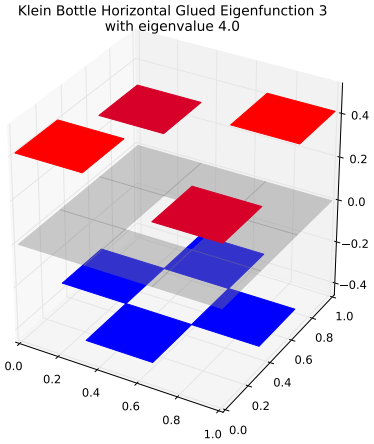
5 $M = 2$ Eigenfunction 4

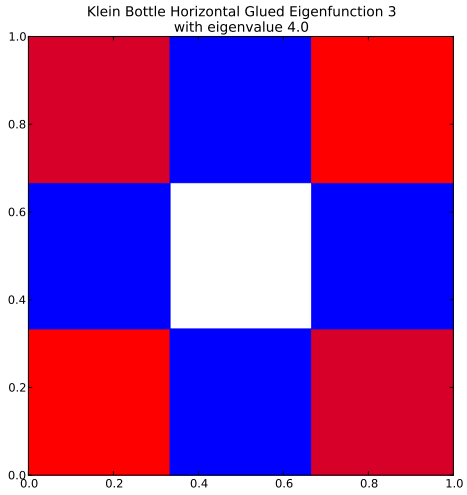
$M = 2$ Eigenfunction 4 has eigenvalue 0.758799842469





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

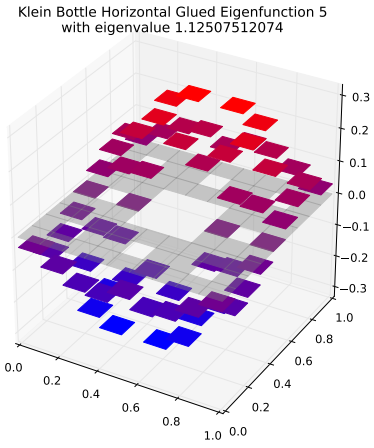


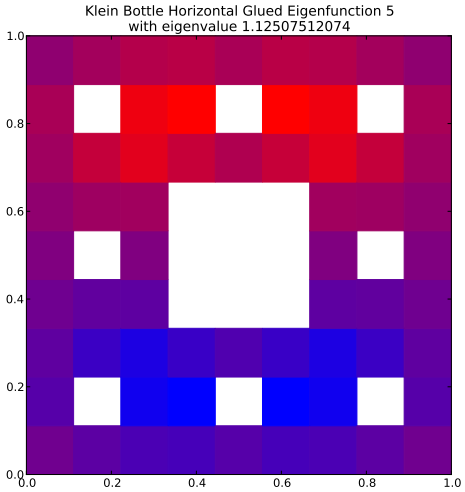


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.189699960617$
Dot Value: 2.220446049250313e-16

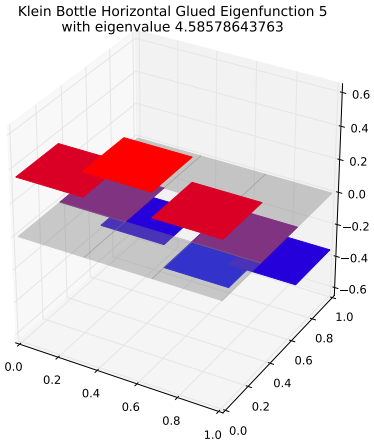
6 $M = 2$ Eigenfunction 5

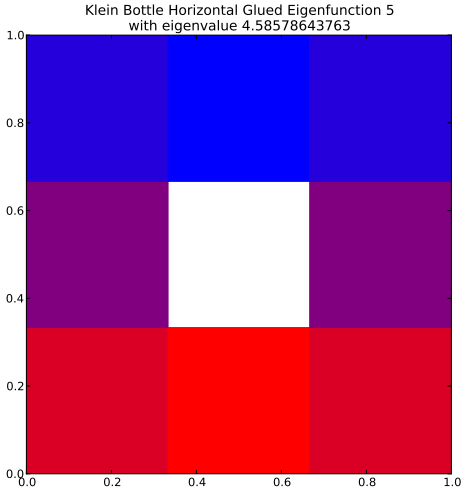
$M = 2$ Eigenfunction 5 has eigenvalue 1.12507512074





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

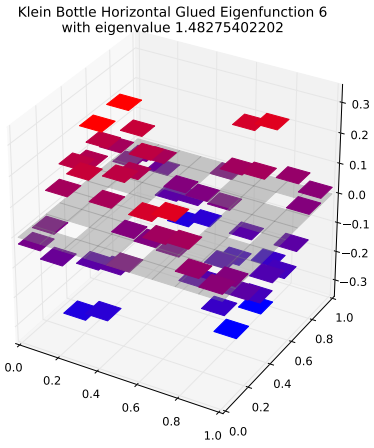


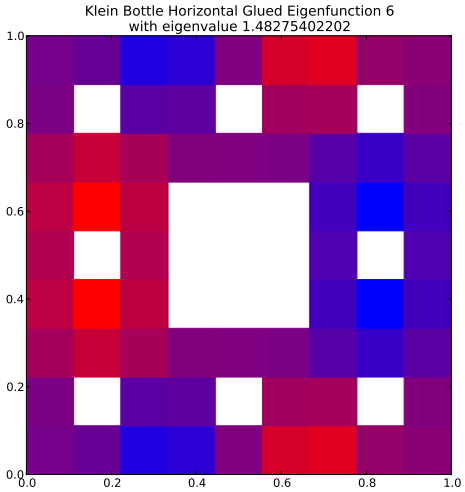


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.245339624085$
Dot Value: 0.0006505301638103367

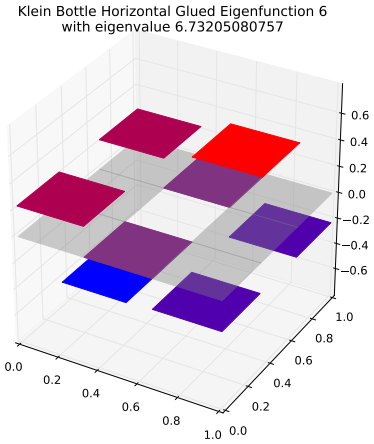
7 $M = 2$ Eigenfunction 6

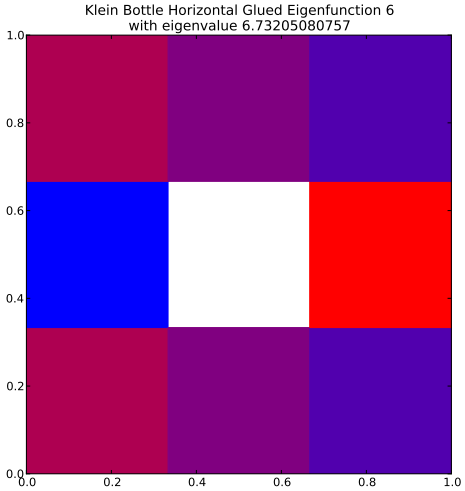
$M = 2$ Eigenfunction 6 has eigenvalue 1.48275402202





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

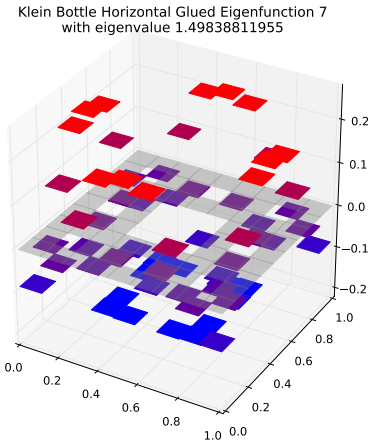


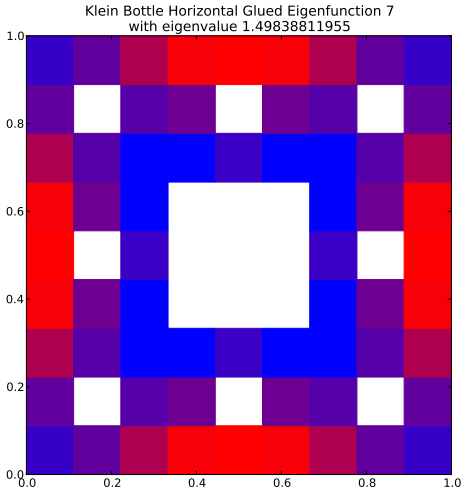


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.220252945856$
Dot Value: 0.08916117818499358

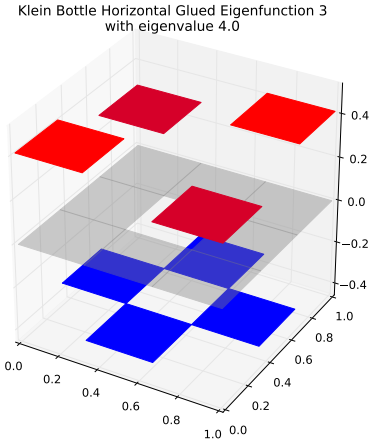
8 $M = 2$ Eigenfunction 7

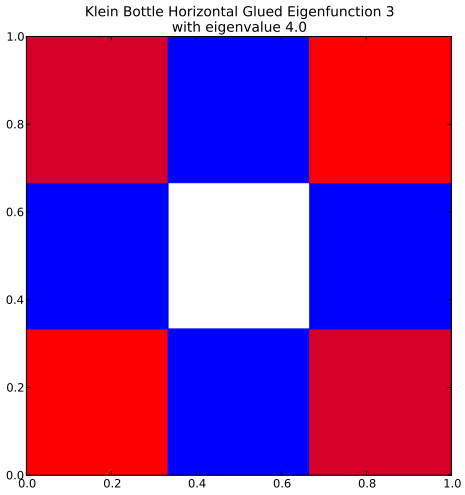
$M = 2$ Eigenfunction 7 has eigenvalue 1.49838811955





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

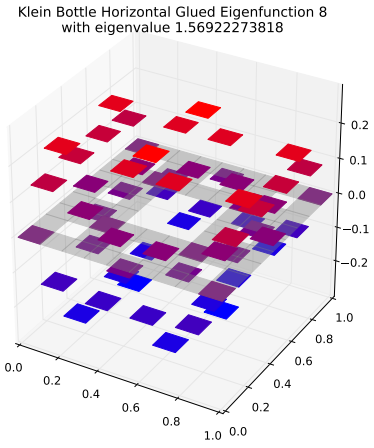


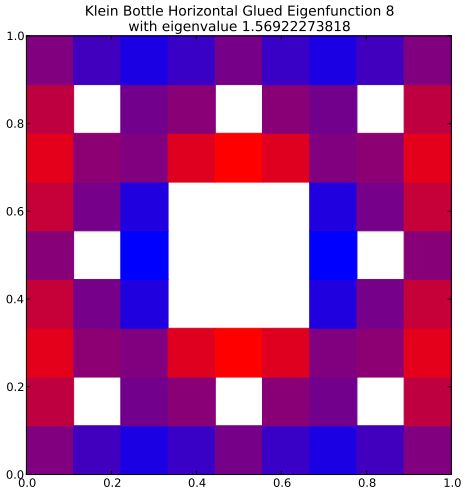


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.374597029888$
Dot Value: 2.220446049250313e-16

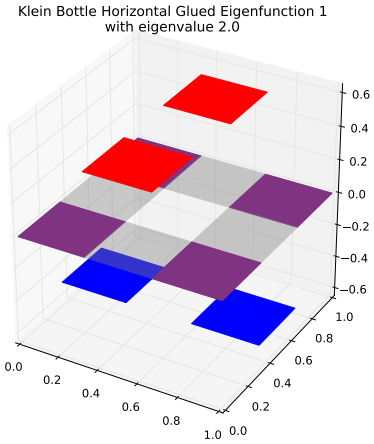
9 $M = 2$ Eigenfunction 8

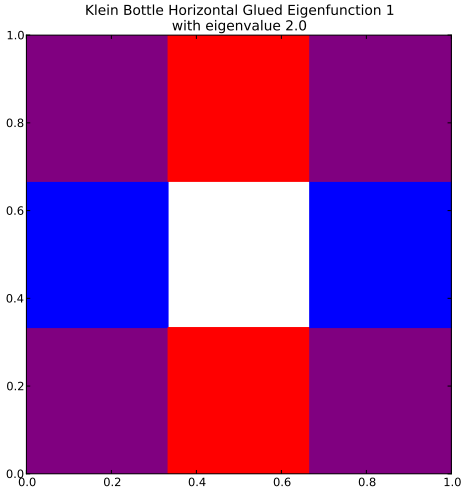
$M = 2$ Eigenfunction 8 has eigenvalue 1.56922273818





Compare to $m = 1$ eigenspace with eigenvalue 2.0

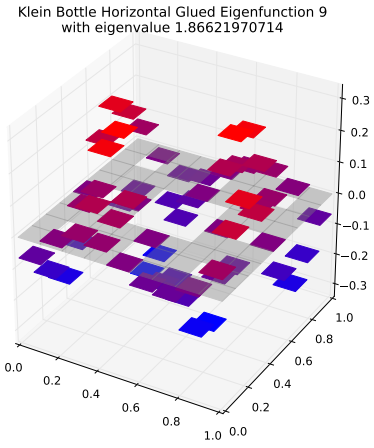


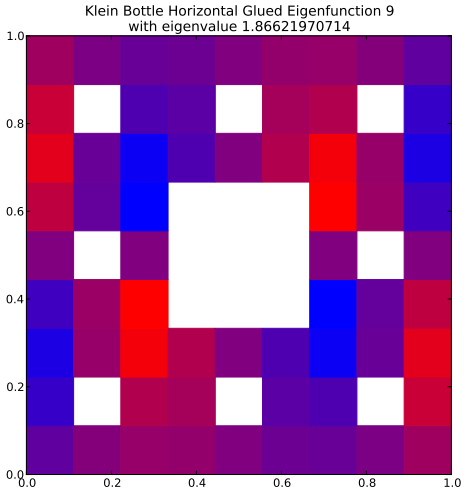


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.78461136909$
Dot Value: 0.0

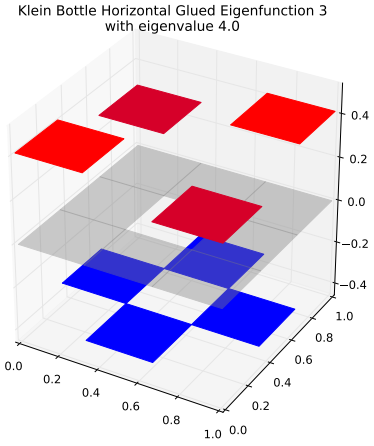
10 $M = 2$ Eigenfunction 9

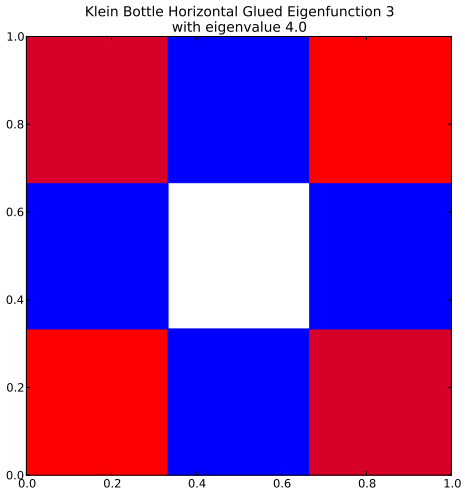
$M = 2$ Eigenfunction 9 has eigenvalue 1.86621970714





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

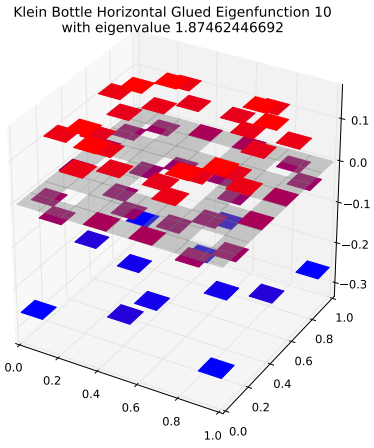


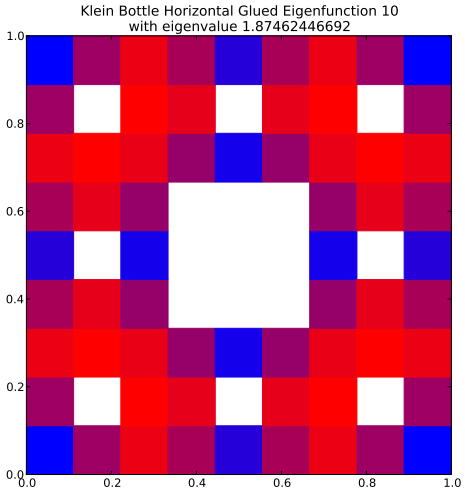


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.466554926786$
Dot Value: 1.1102230246251565e-16

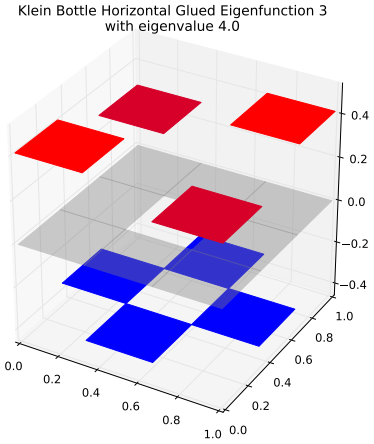
11 $M = 2$ Eigenfunction 10

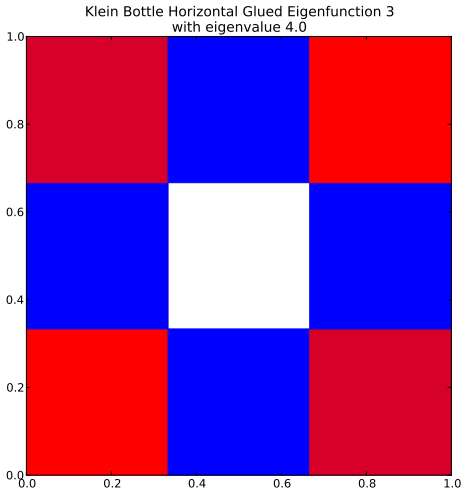
$M = 2$ Eigenfunction 10 has eigenvalue 1.87462446692





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

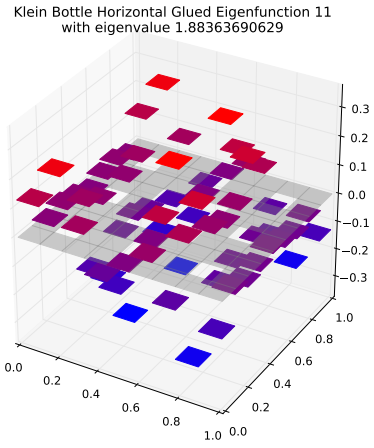


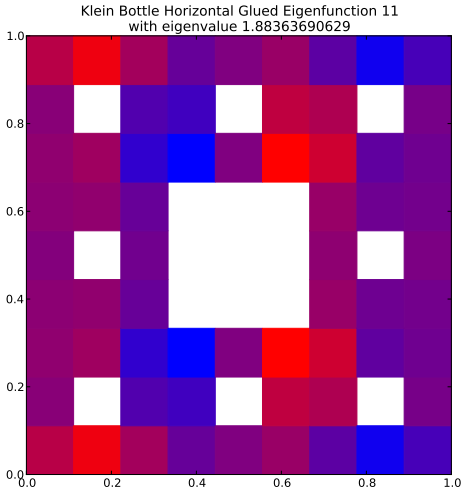


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.468656116731$
Dot Value: 0.0

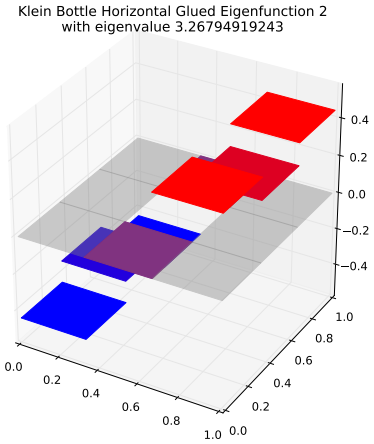
12 $M = 2$ Eigenfunction 11

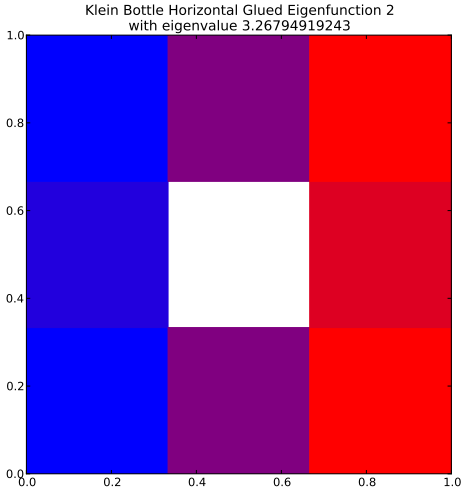
$M = 2$ Eigenfunction 11 has eigenvalue 1.88363690629





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

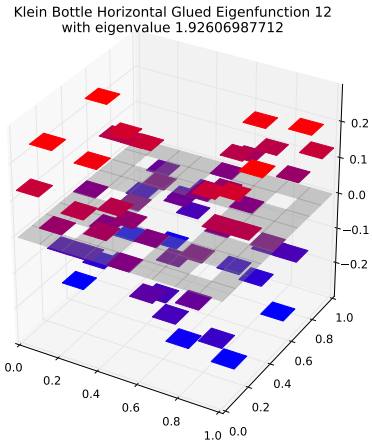


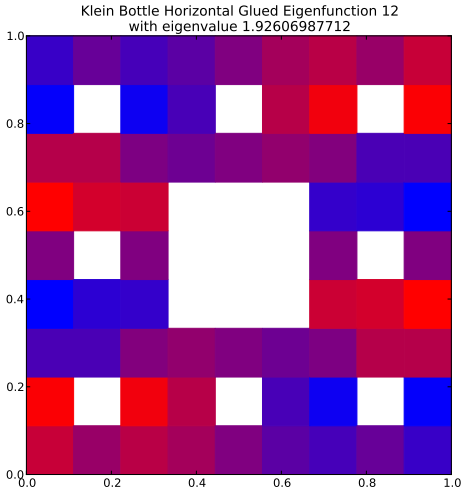


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.576397243462$
Dot Value: 0.11312368070720147

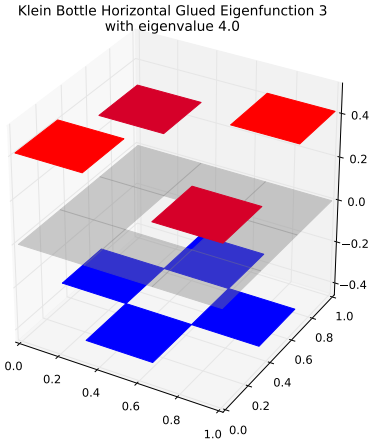
13 $M = 2$ Eigenfunction 12

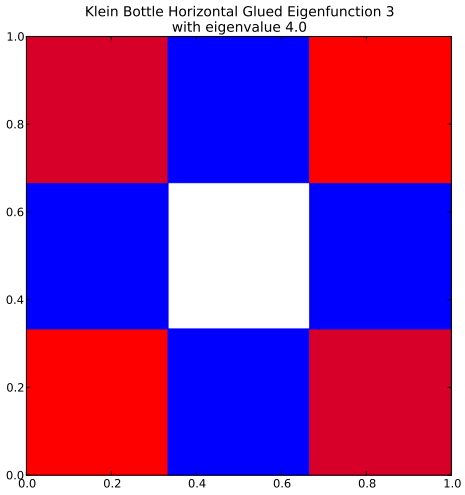
$M = 2$ Eigenfunction 12 has eigenvalue 1.92606987712





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

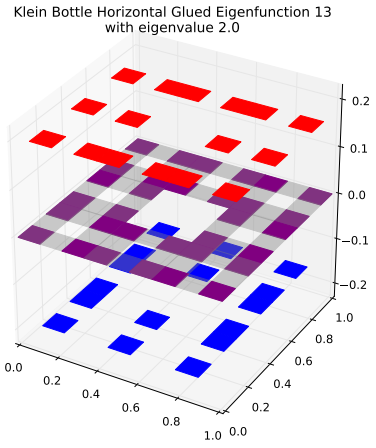


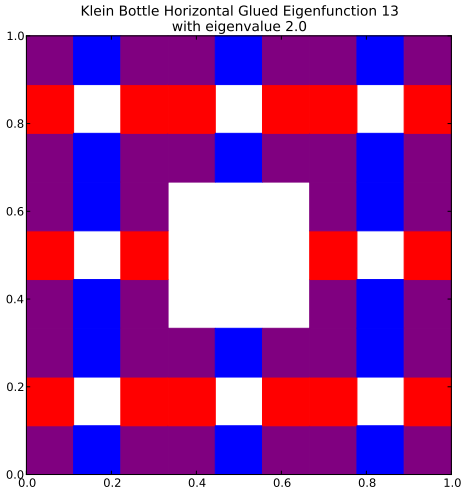


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.481517469281$
Dot Value: 0.0

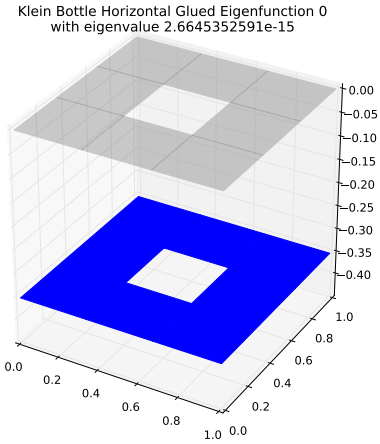
14 $M = 2$ Eigenfunction 13

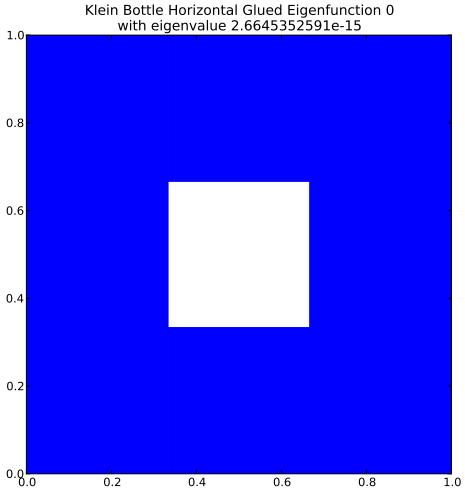
$M = 2$ Eigenfunction 13 has eigenvalue 2.0





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

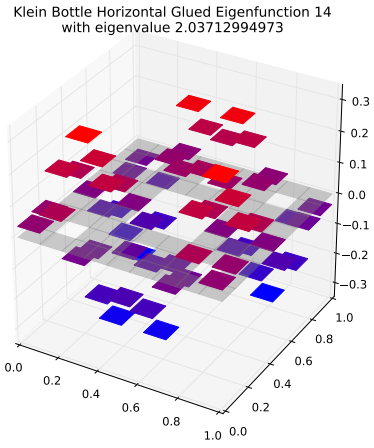


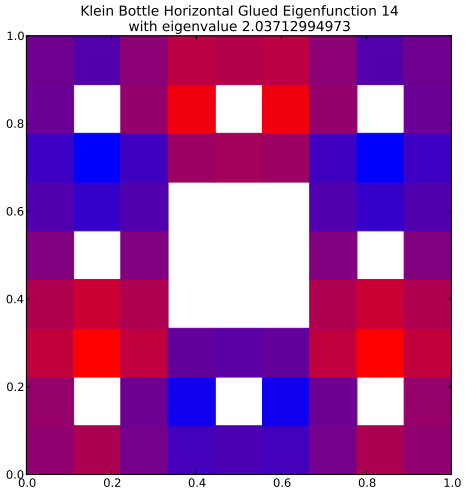


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 7.50599937895e + 14$
Dot Value: 2

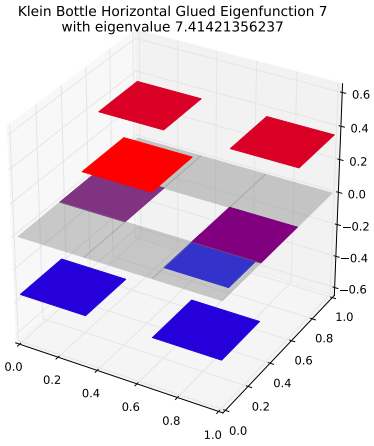
15 $M = 2$ Eigenfunction 14

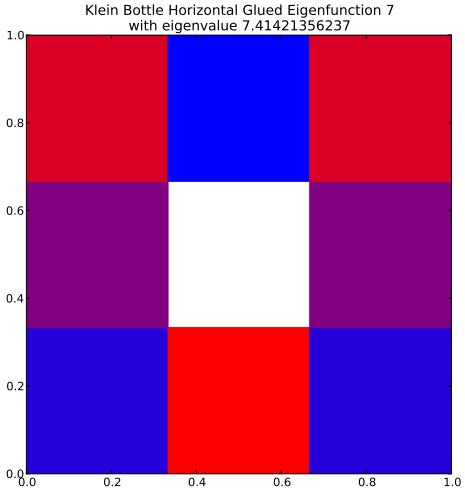
$M = 2$ Eigenfunction 14 has eigenvalue 2.03712994973





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

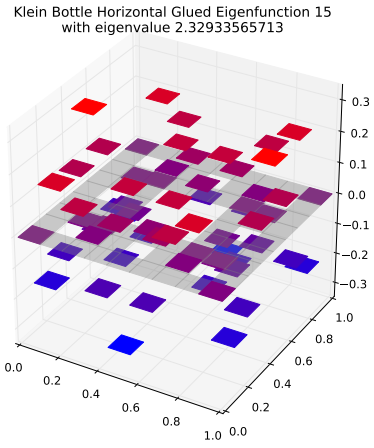


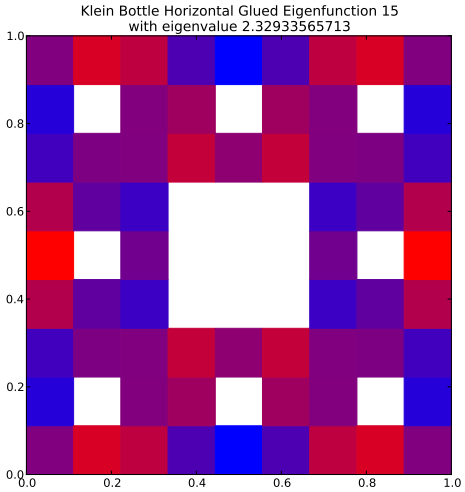


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.274760085152$
Dot Value: 0.0017108198585333856

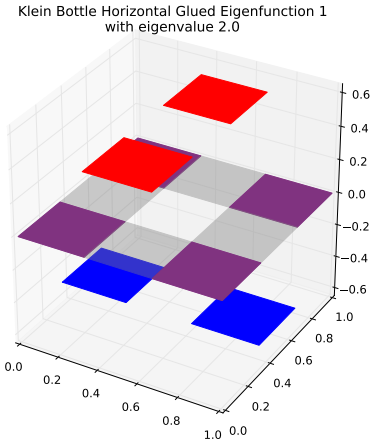
16 $M = 2$ Eigenfunction 15

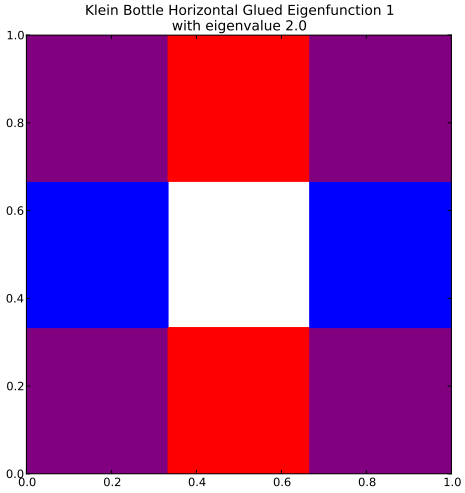
$M = 2$ Eigenfunction 15 has eigenvalue 2.32933565713





Compare to $m = 1$ eigenspace with eigenvalue 2.0

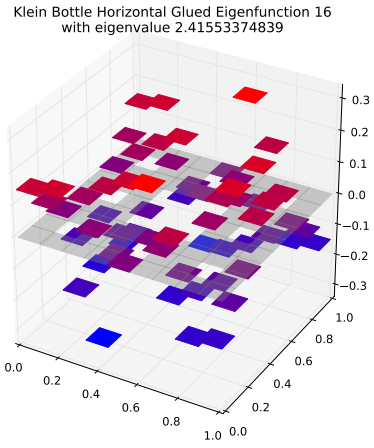


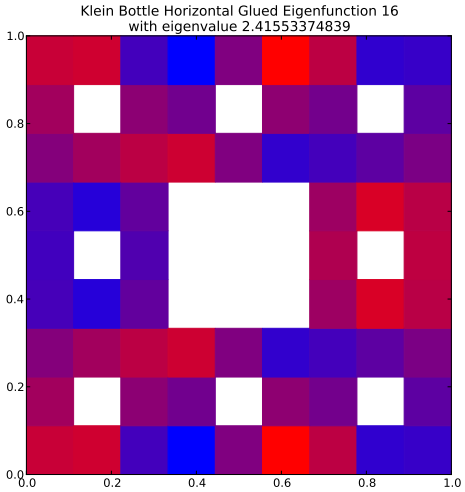


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.16466782857$
Dot Value: 0.0

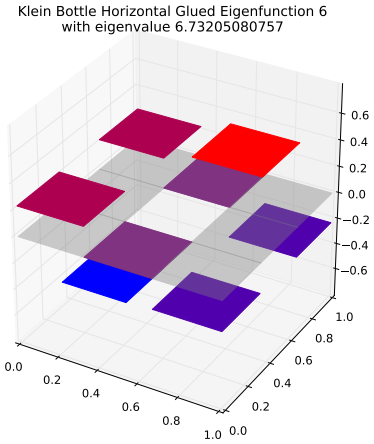
17 $M = 2$ Eigenfunction 16

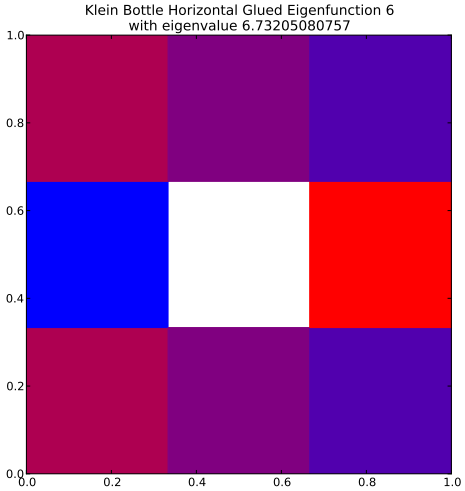
$M = 2$ Eigenfunction 16 has eigenvalue 2.41553374839





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

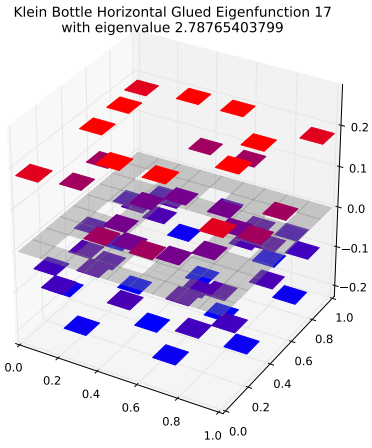


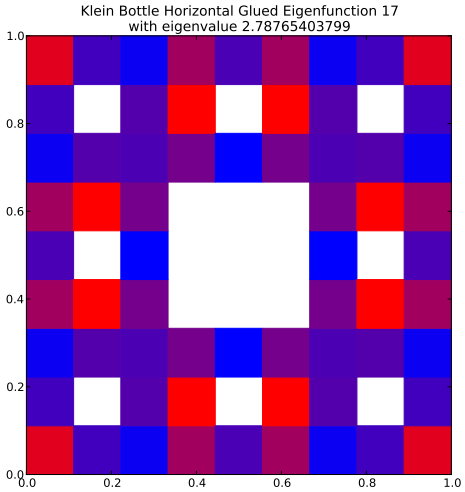


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.358810980106$
Dot Value: 0.011440672902887083

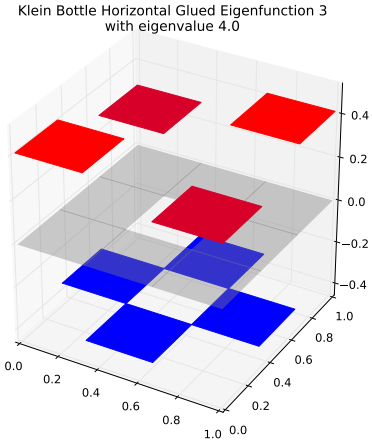
18 $M = 2$ Eigenfunction 17

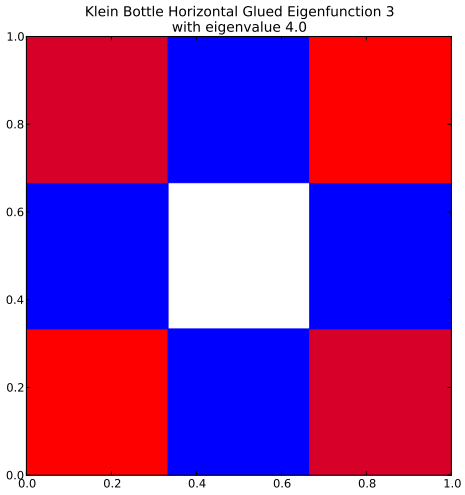
$M = 2$ Eigenfunction 17 has eigenvalue 2.78765403799





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

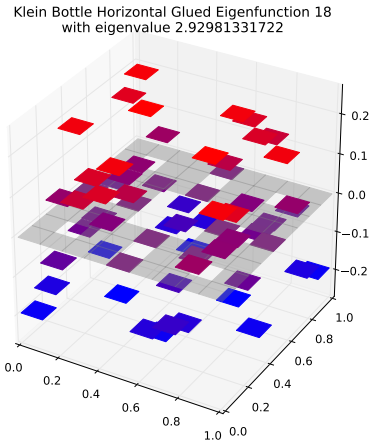


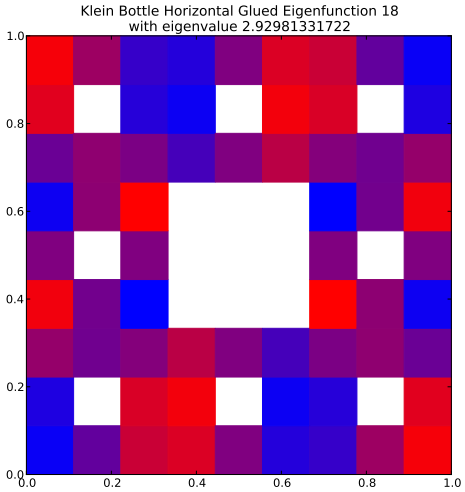


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.696913509497$
Dot Value: 2.220446049250313e-16

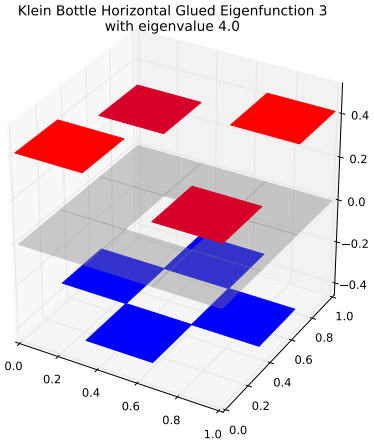
19 $M = 2$ Eigenfunction 18

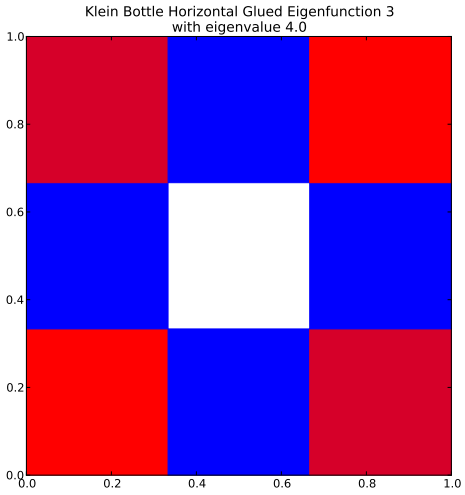
$M = 2$ Eigenfunction 18 has eigenvalue 2.92981331722





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

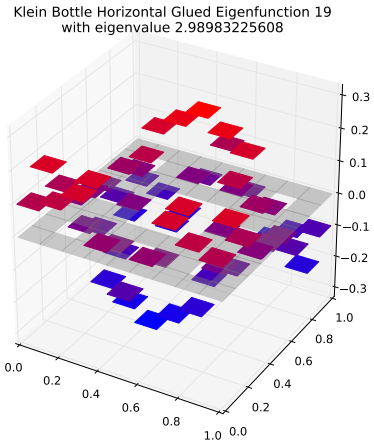


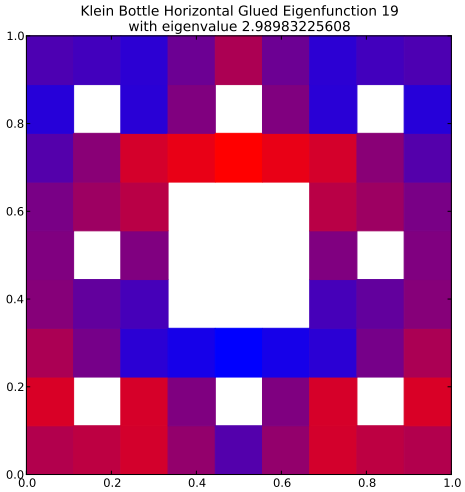


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.732453329304$
Dot Value: 0.0

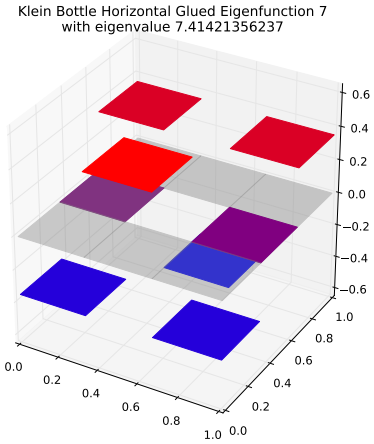
20 $M = 2$ Eigenfunction 19

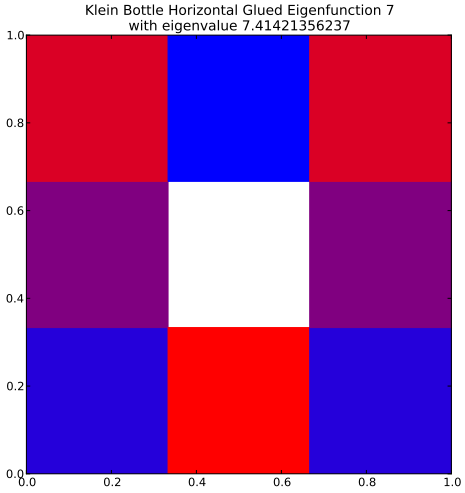
$M = 2$ Eigenfunction 19 has eigenvalue 2.98983225608





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

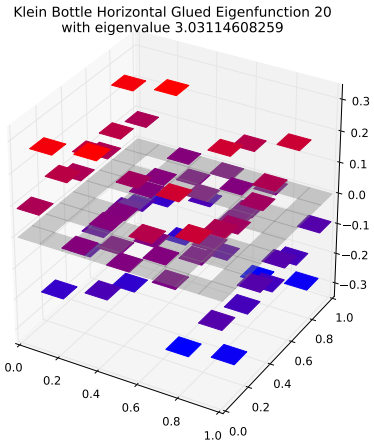


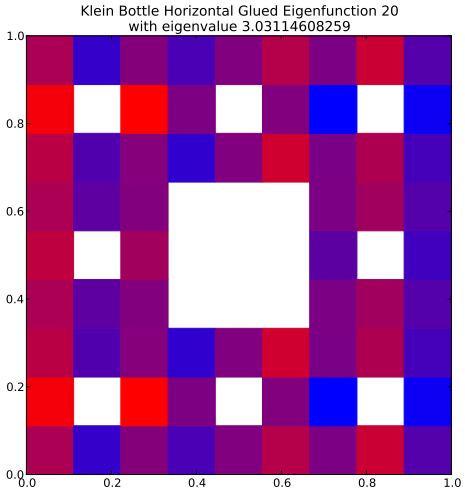


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.403256829727$
Dot Value: 0.008815596088504352

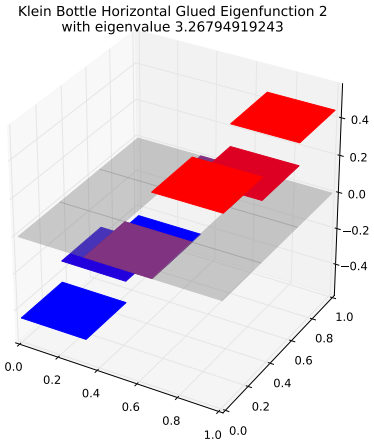
21 $M = 2$ Eigenfunction 20

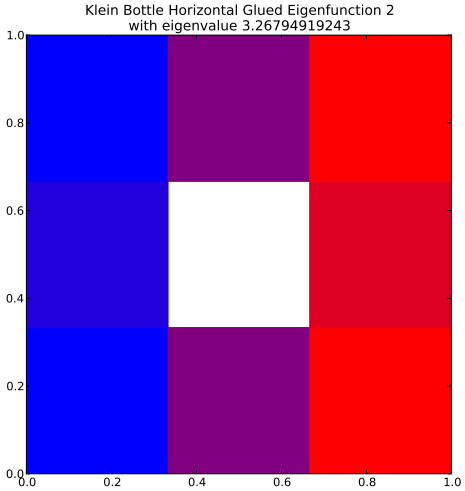
$M = 2$ Eigenfunction 20 has eigenvalue 3.03114608259





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

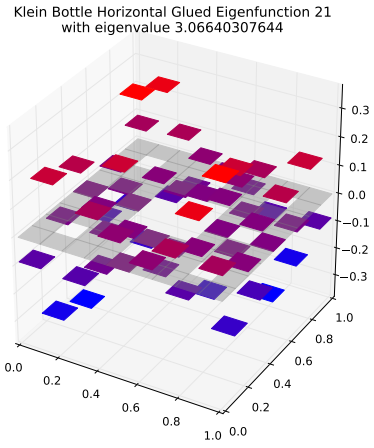


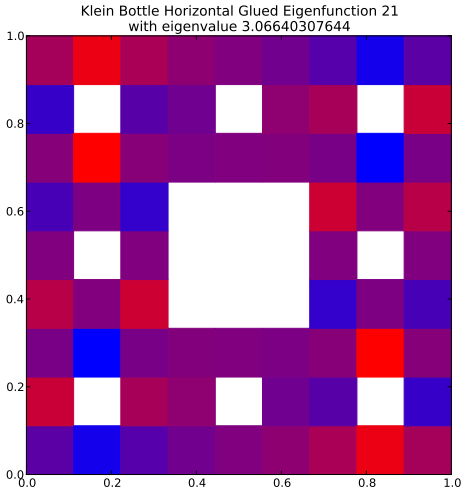


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.927537701507$
Dot Value: 0.007550097877229822

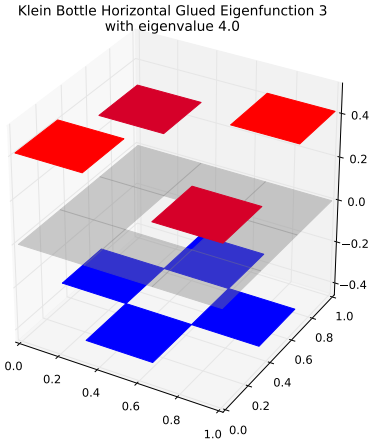
22 $M = 2$ Eigenfunction 21

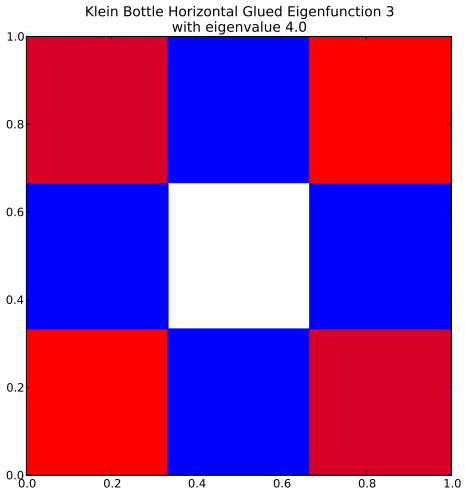
$M = 2$ Eigenfunction 21 has eigenvalue 3.06640307644





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

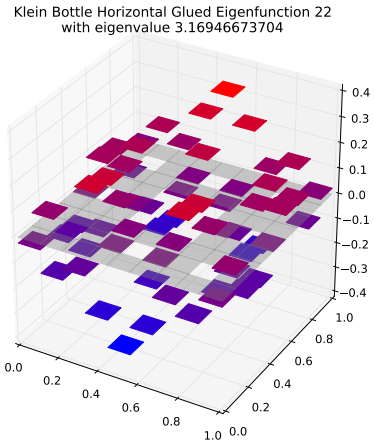


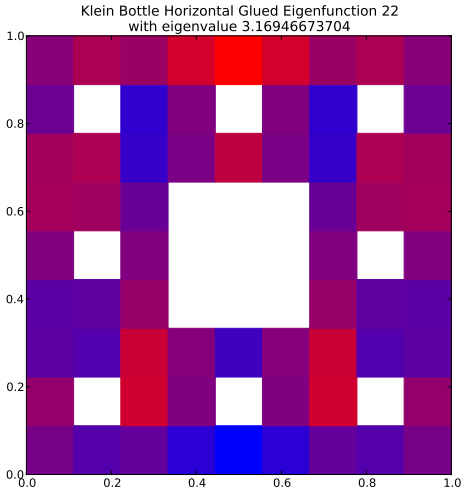


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.766600769111$
Dot Value: 2.220446049250313e-16

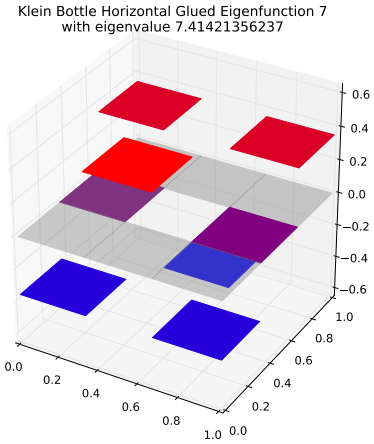
23 $M = 2$ Eigenfunction 22

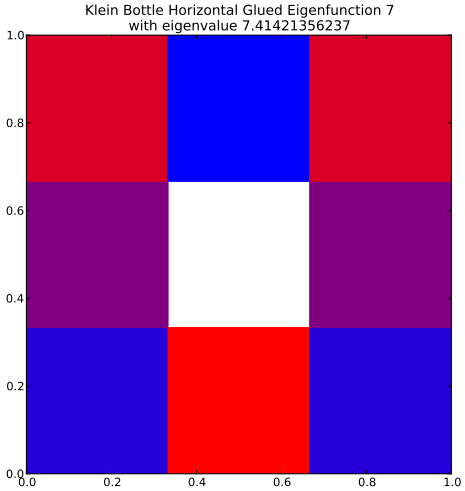
$M = 2$ Eigenfunction 22 has eigenvalue 3.16946673704





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

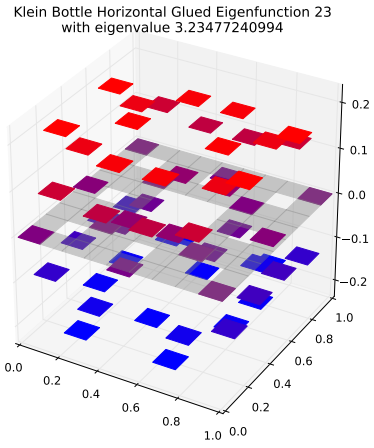


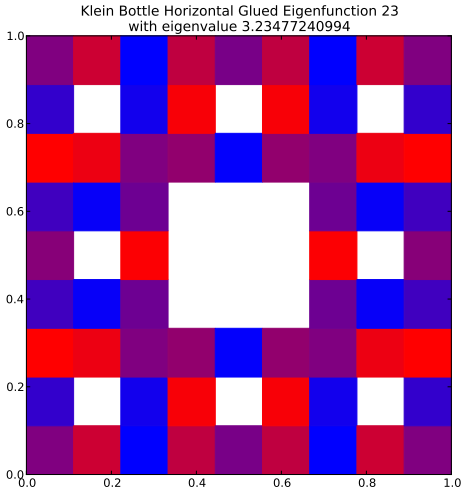


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.42748522286$
Dot Value: 0.2602622797711073

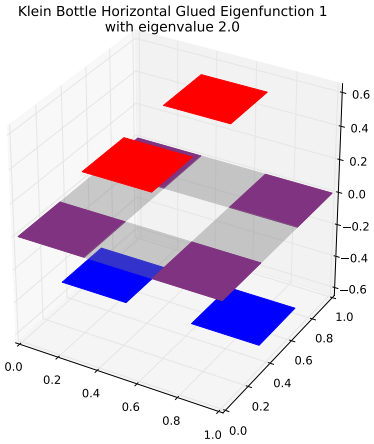
24 $M = 2$ Eigenfunction 23

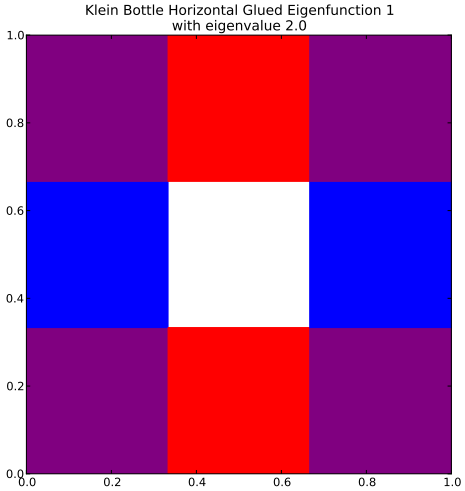
$M = 2$ Eigenfunction 23 has eigenvalue 3.23477240994





Compare to $m = 1$ eigenspace with eigenvalue 2.0

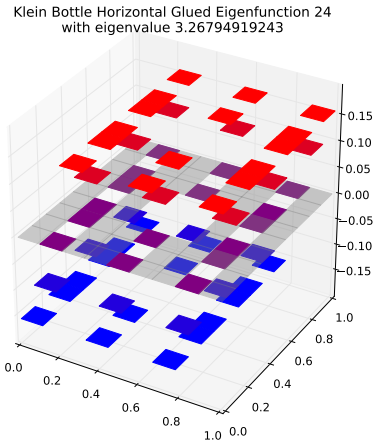


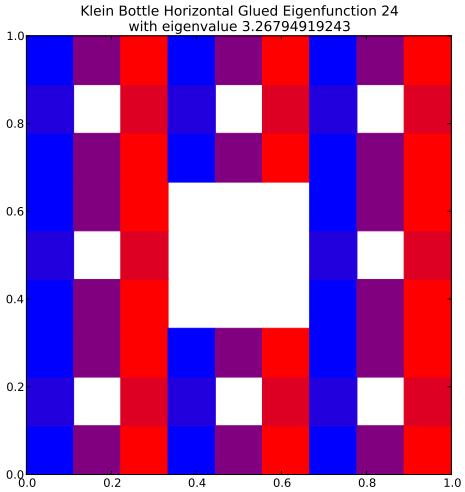


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.61738620497$
Dot Value: 0.0

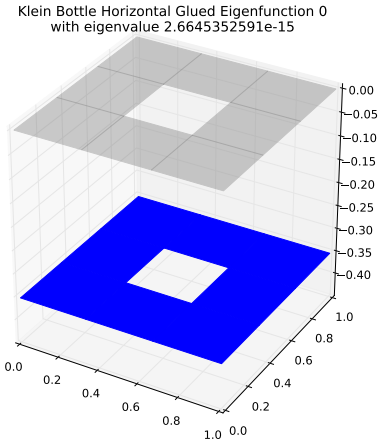
25 $M = 2$ Eigenfunction 24

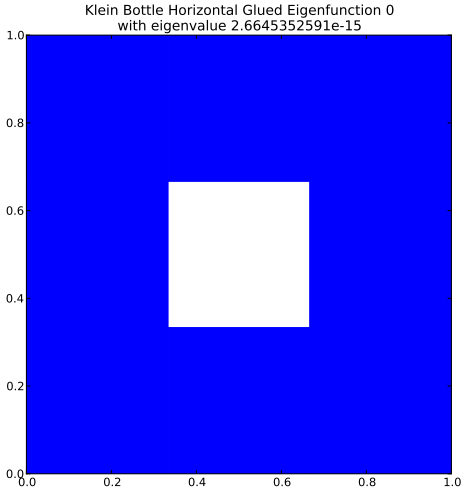
$M = 2$ Eigenfunction 24 has eigenvalue 3.26794919243





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

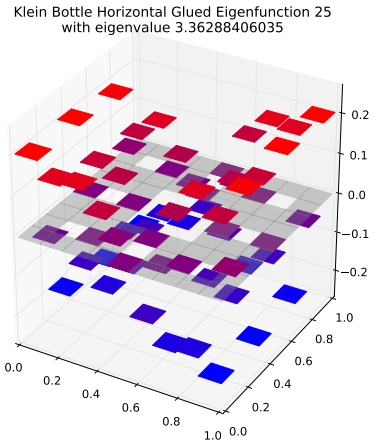


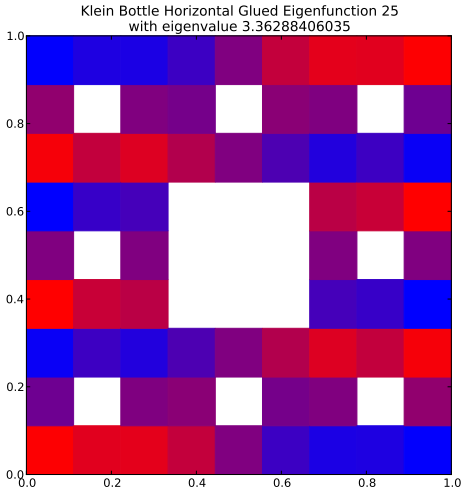


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.22646123044e + 15$
Dot Value: 2

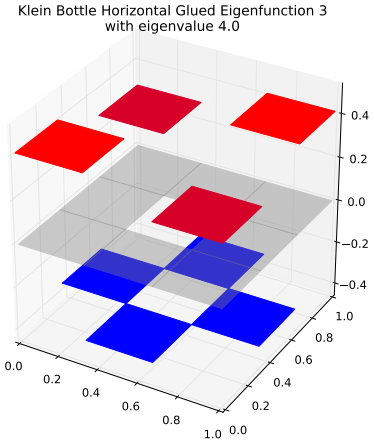
26 $M = 2$ Eigenfunction 25

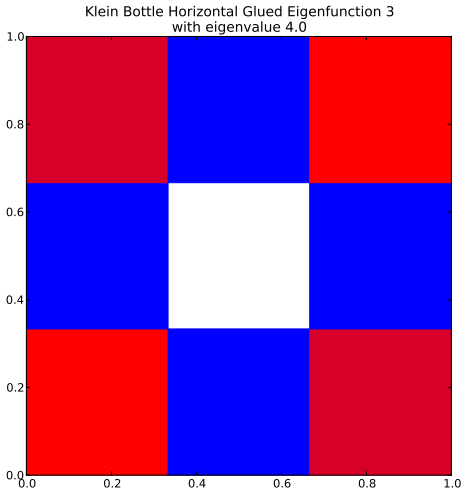
$M = 2$ Eigenfunction 25 has eigenvalue 3.36288406035





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

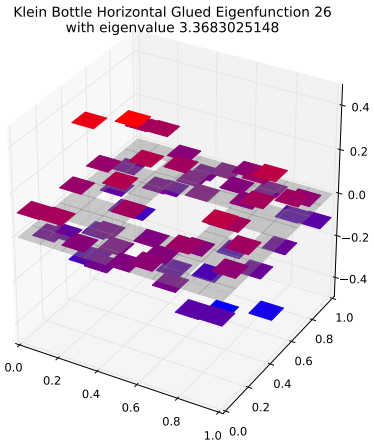


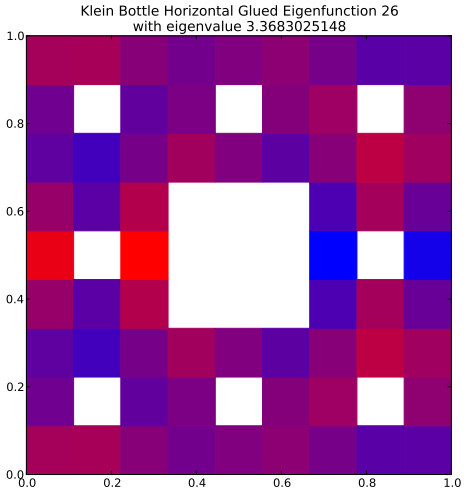


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.840721015087$
Dot Value: 0.0

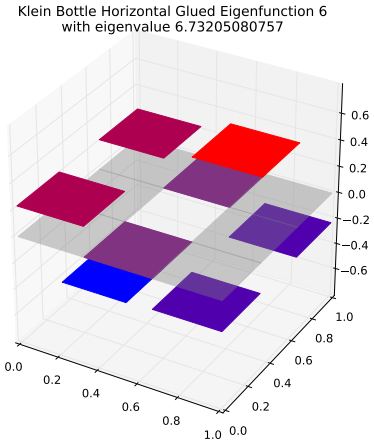
27 $M = 2$ Eigenfunction 26

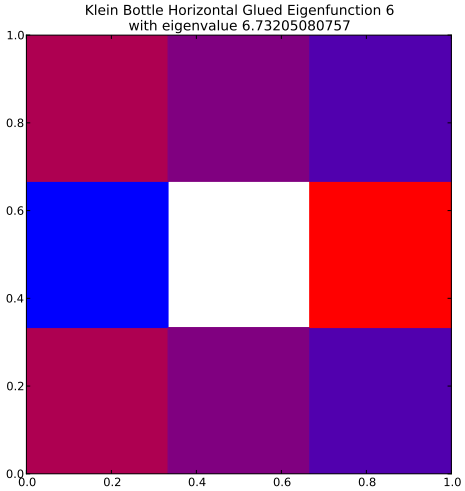
$M = 2$ Eigenfunction 26 has eigenvalue 3.3683025148





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

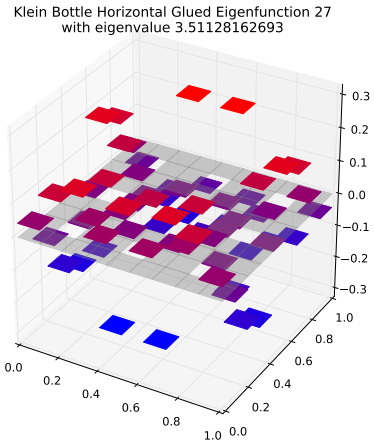


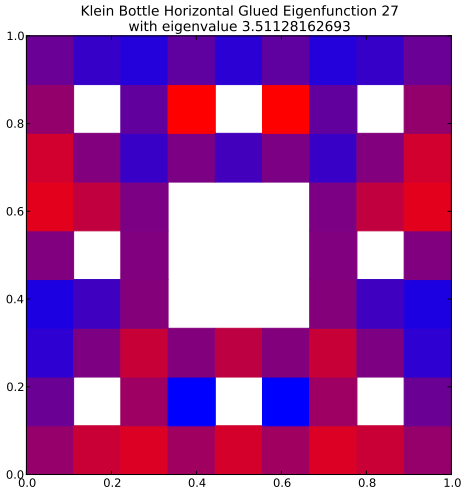


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.500338249232$
Dot Value: 0.021125970899166124

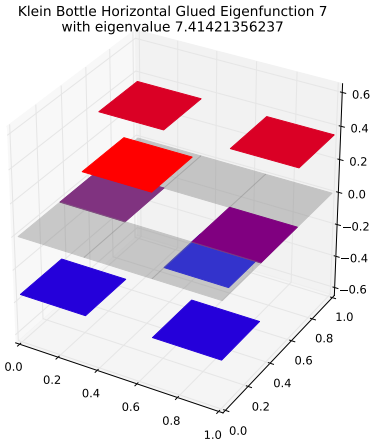
28 $M = 2$ Eigenfunction 27

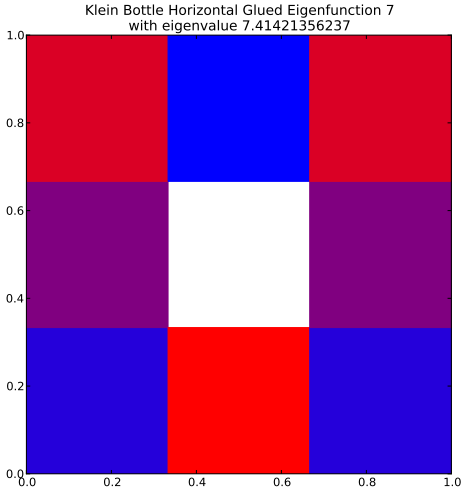
$M = 2$ Eigenfunction 27 has eigenvalue 3.51128162693





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

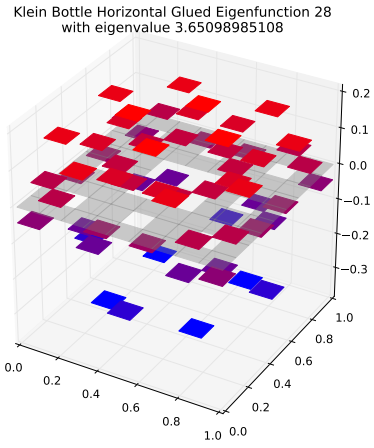


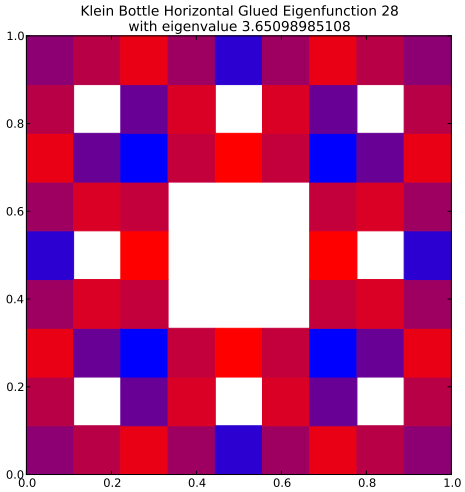


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.473587872454$
Dot Value: 0.19617600407097247

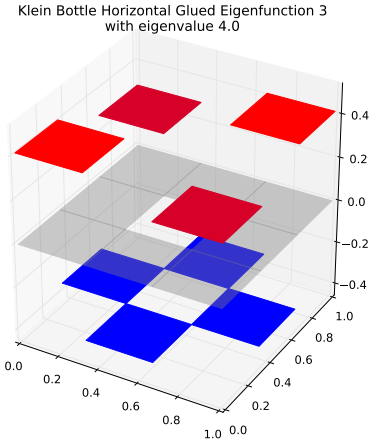
29 $M = 2$ Eigenfunction 28

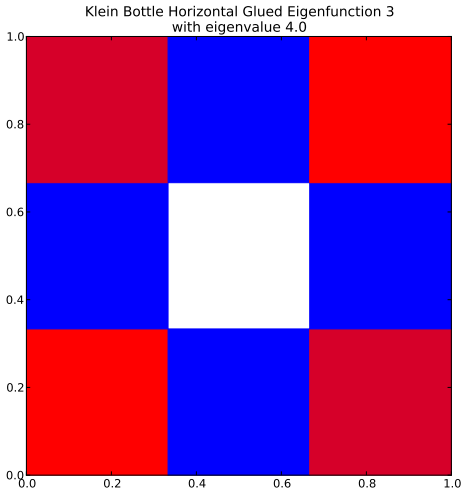
$M = 2$ Eigenfunction 28 has eigenvalue 3.65098985108





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

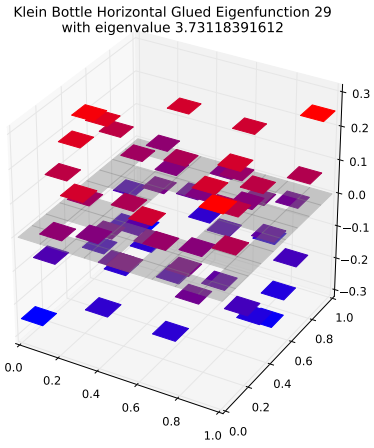


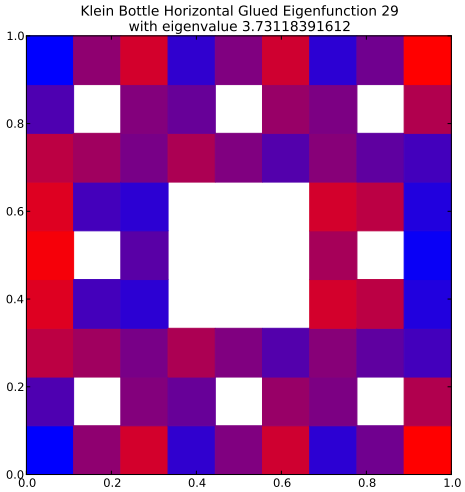


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.912747462769$
Dot Value: 0.0

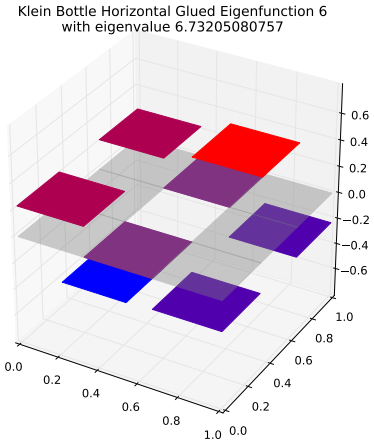
30 $M = 2$ Eigenfunction 29

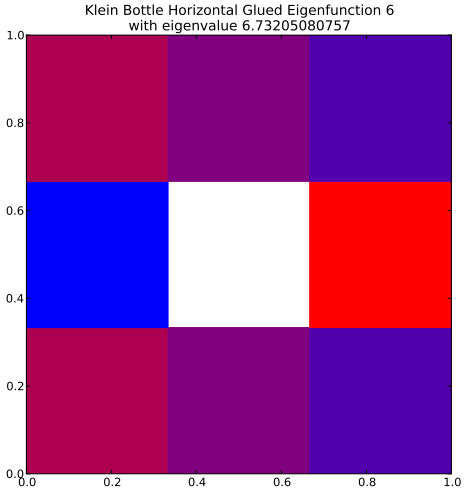
$M = 2$ Eigenfunction 29 has eigenvalue 3.73118391612





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

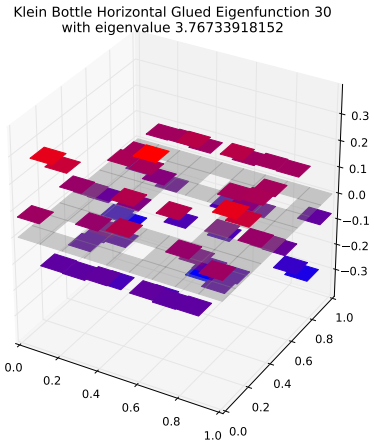


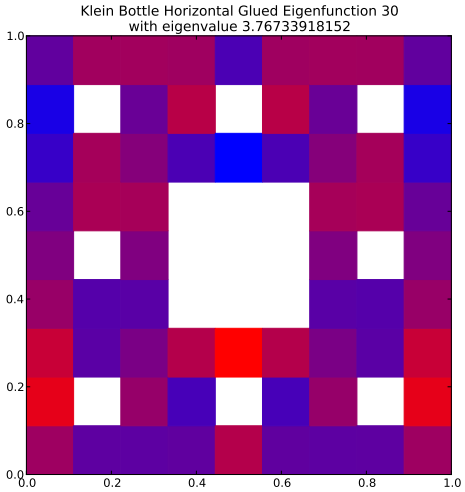


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.554241793886$
Dot Value: 0.02873650859488841

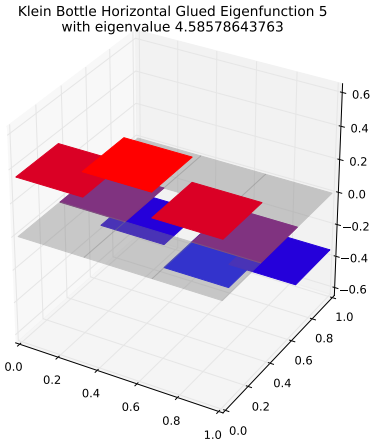
31 $M = 2$ Eigenfunction 30

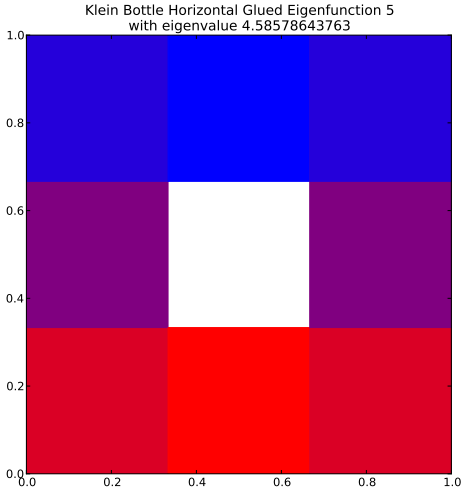
$M = 2$ Eigenfunction 30 has eigenvalue 3.76733918152





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

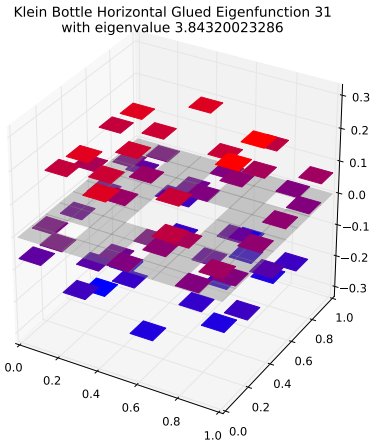


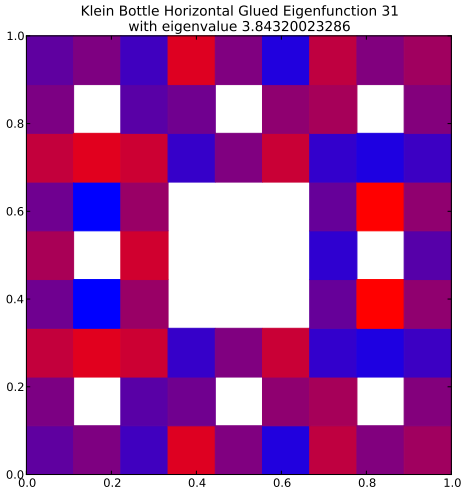


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.821525213344$
Dot Value: 0.021663047229855348

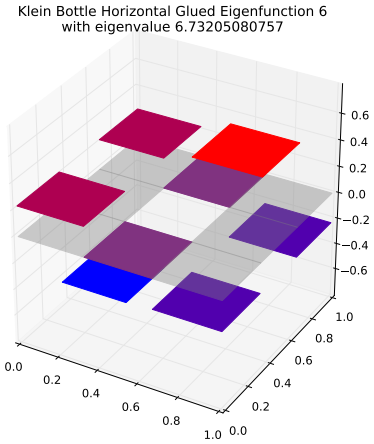
32 $M = 2$ Eigenfunction 31

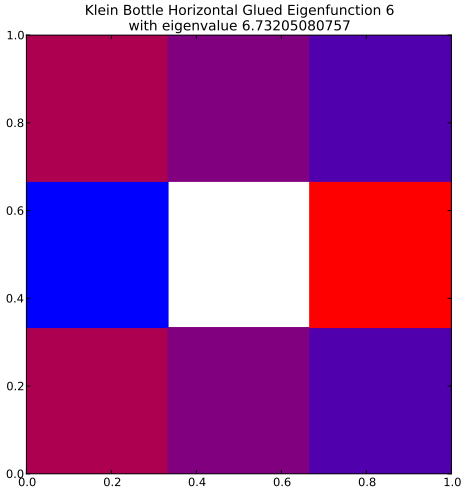
$M = 2$ Eigenfunction 31 has eigenvalue 3.84320023286





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

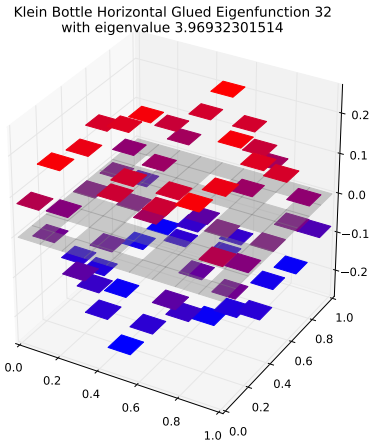


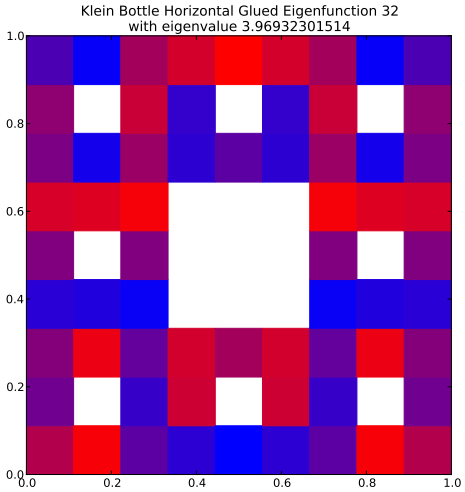


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.570881049878$
Dot Value: 0.08754815732229282

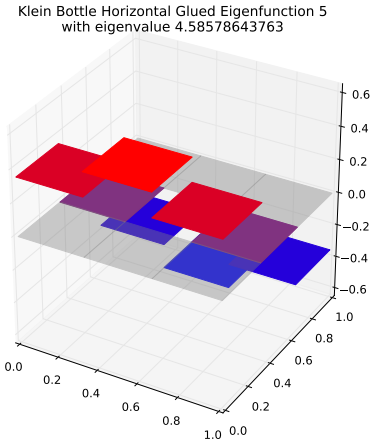
33 $M = 2$ Eigenfunction 32

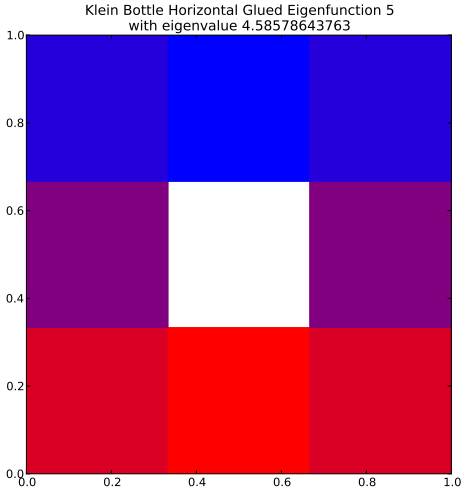
$M = 2$ Eigenfunction 32 has eigenvalue 3.96932301514





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

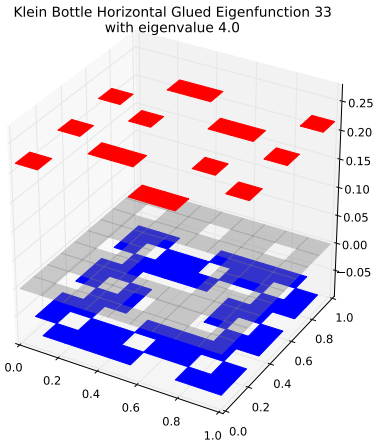


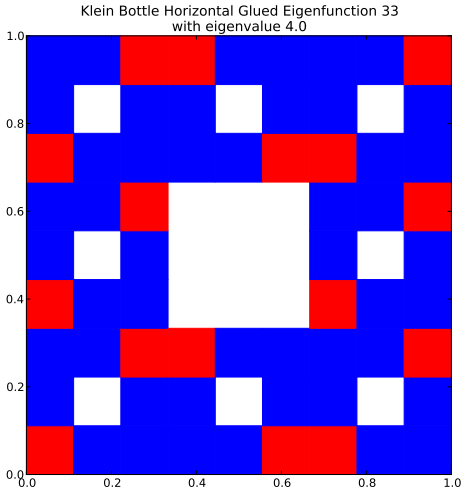


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.865570839185$
Dot Value: 0.11467013435711004

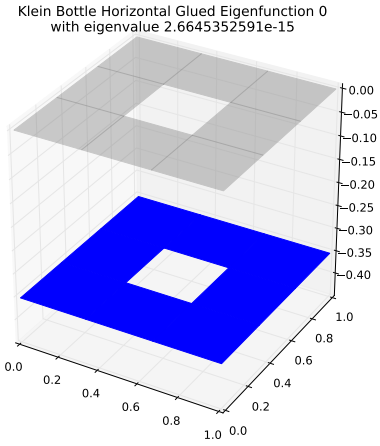
34 $M = 2$ Eigenfunction 33

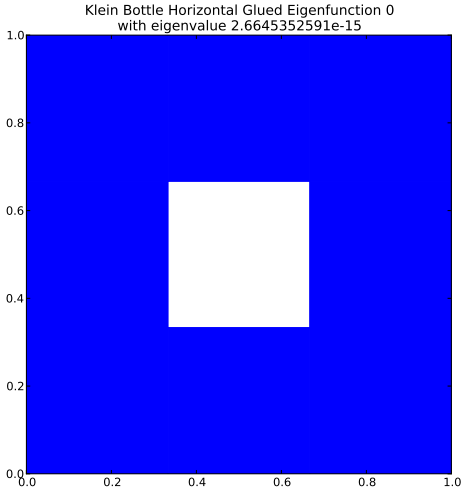
$M = 2$ Eigenfunction 33 has eigenvalue 4.0





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

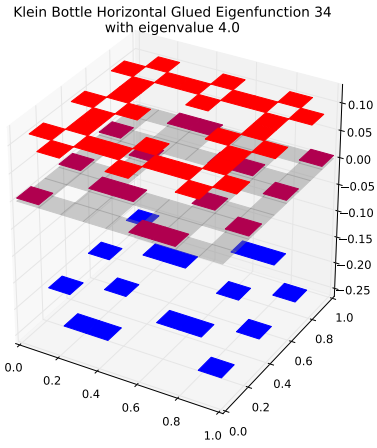


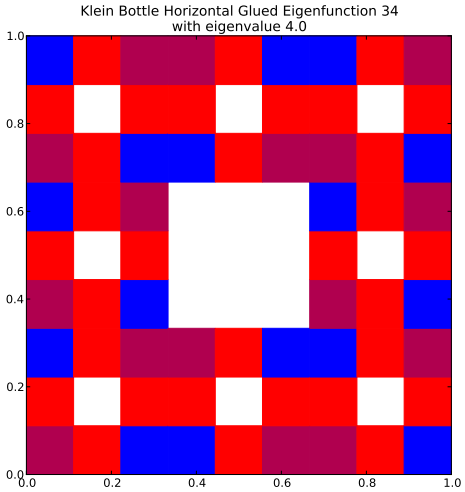


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.50119987579e + 15$
Dot Value: 2

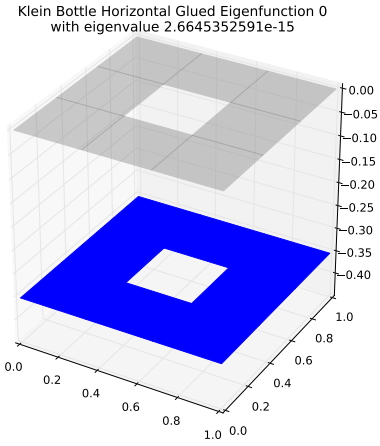
35 $M = 2$ Eigenfunction 34

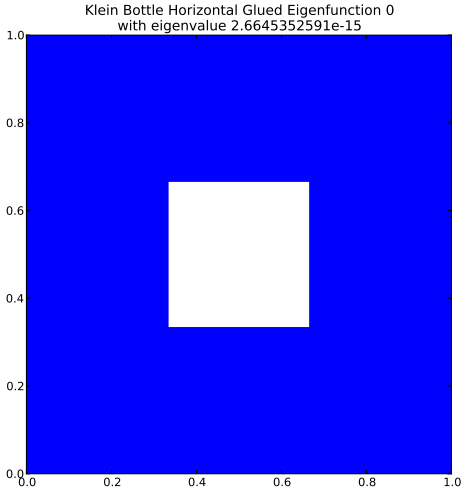
$M = 2$ Eigenfunction 34 has eigenvalue 4.0





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

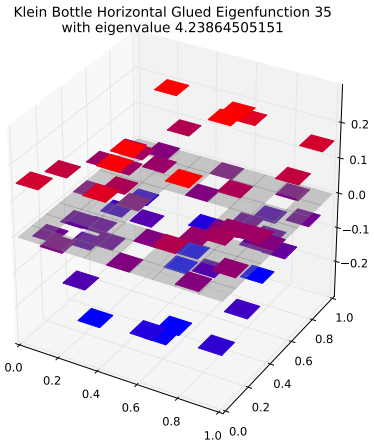


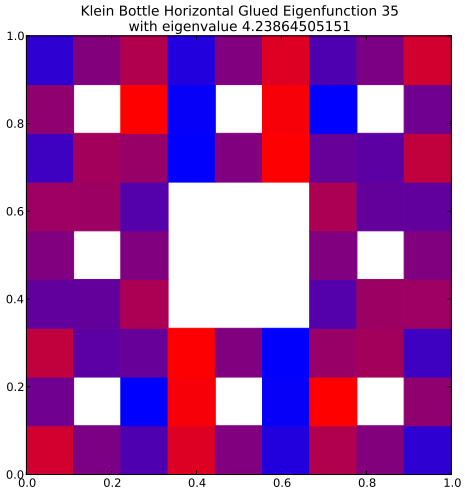


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.50119987579e + 15$
Dot Value: 2

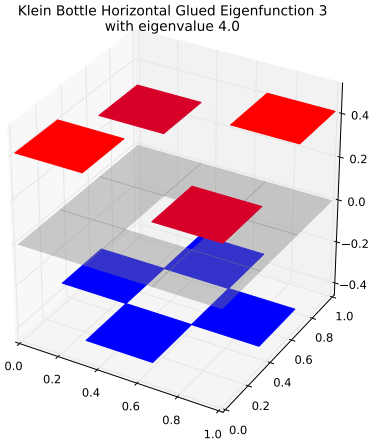
36 $M = 2$ Eigenfunction 35

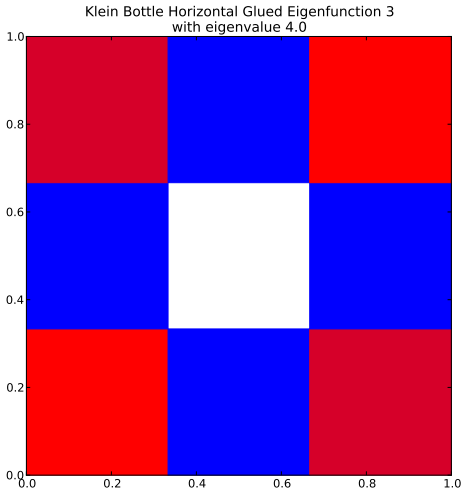
$M = 2$ Eigenfunction 35 has eigenvalue 4.23864505151





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

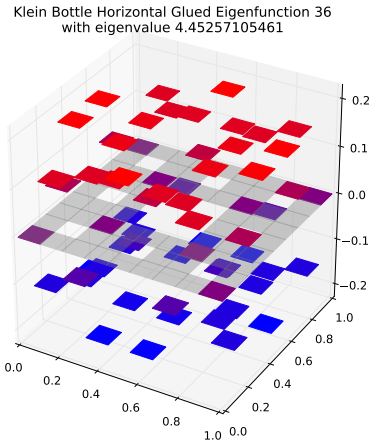


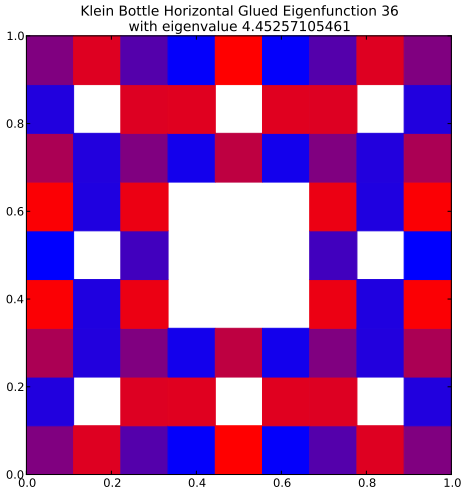


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.05966126288$
Dot Value: 2.220446049250313e-16

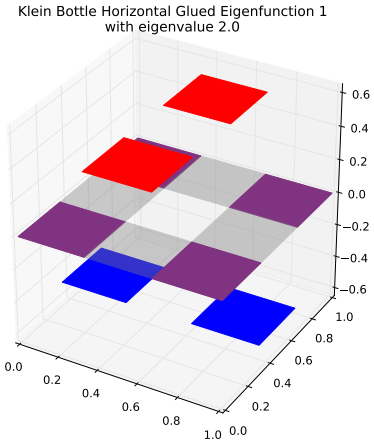
37 $M = 2$ Eigenfunction 36

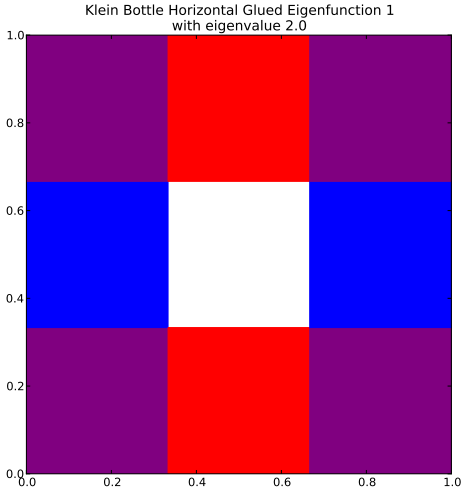
$M = 2$ Eigenfunction 36 has eigenvalue 4.45257105461





Compare to $m = 1$ eigenspace with eigenvalue 2.0

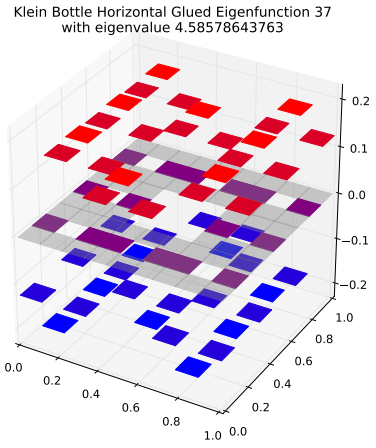


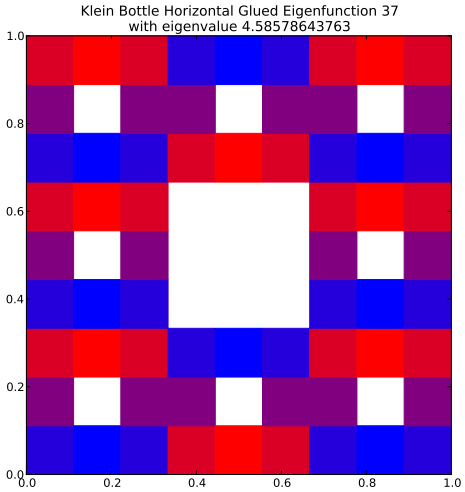


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 2.2262855273$
Dot Value: 0.0

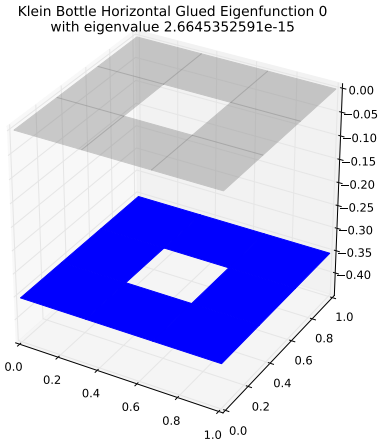
38 $M = 2$ Eigenfunction 37

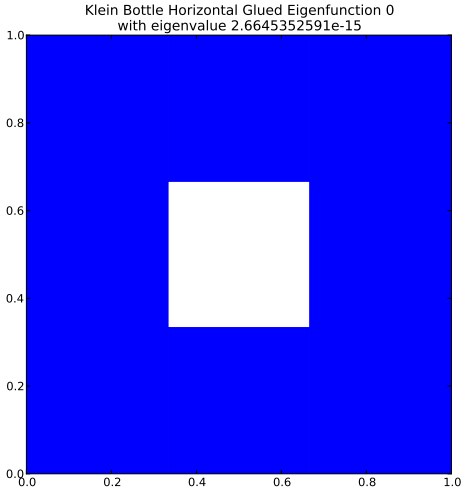
$M = 2$ Eigenfunction 37 has eigenvalue 4.58578643763





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

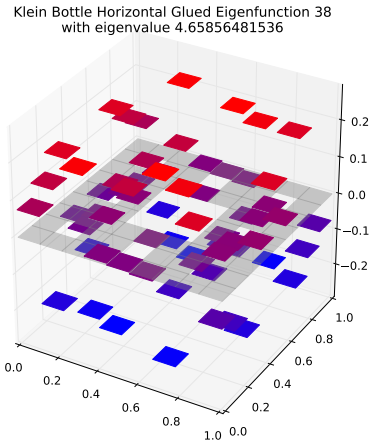


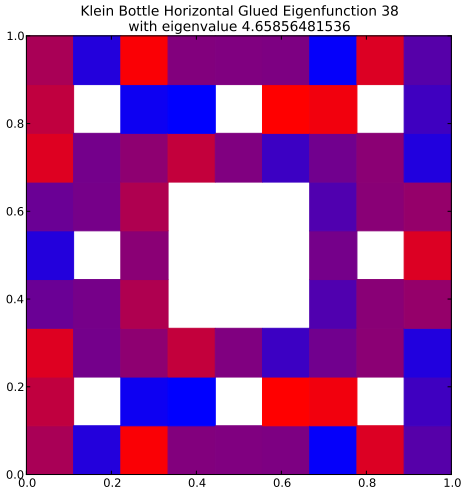


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.72104550764e + 15$
Dot Value: 2

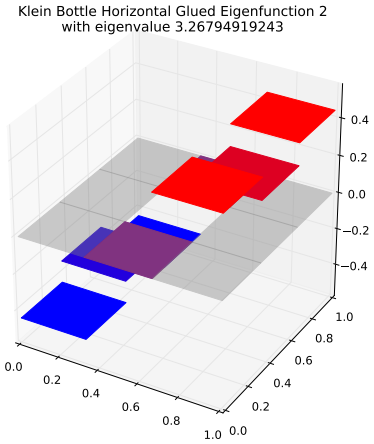
39 $M = 2$ Eigenfunction 38

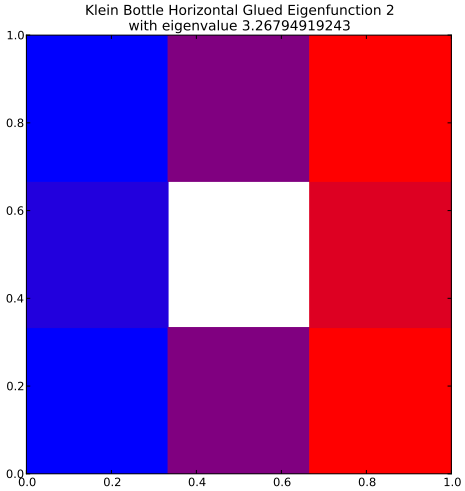
$M = 2$ Eigenfunction 38 has eigenvalue 4.65856481536





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

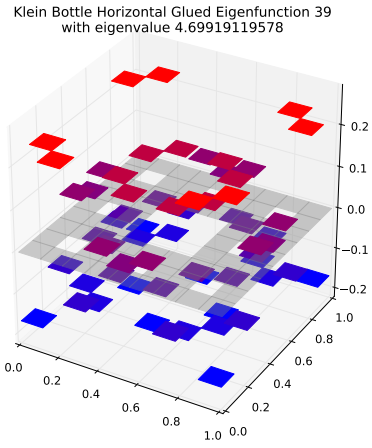


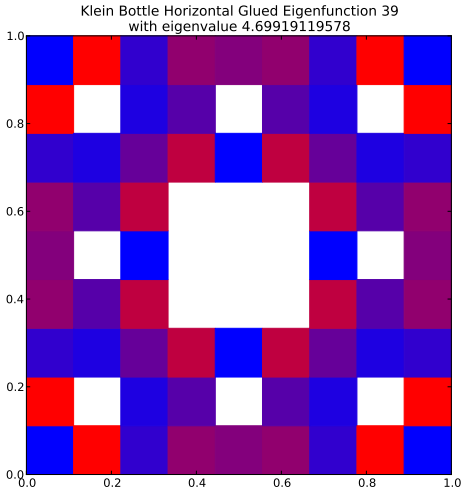


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.42553159215$
Dot Value: 0.26332463973073117

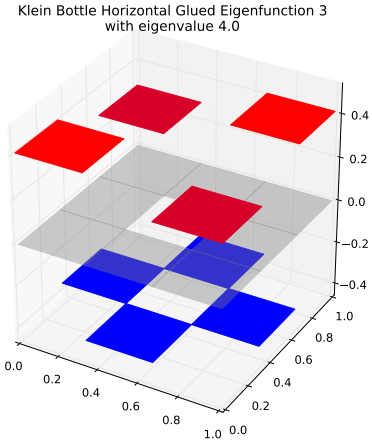
40 $M = 2$ Eigenfunction 39

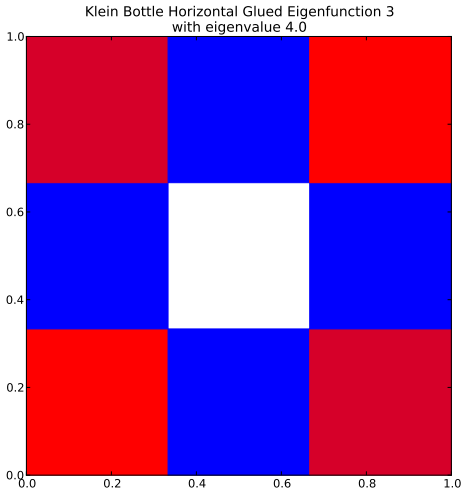
$M = 2$ Eigenfunction 39 has eigenvalue 4.69919119578





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

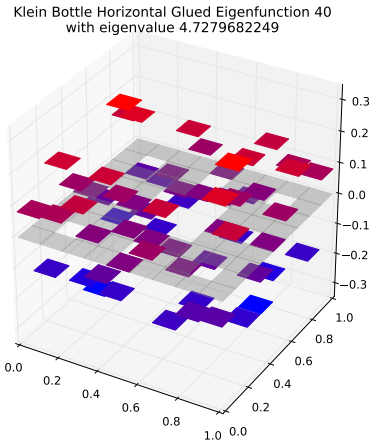


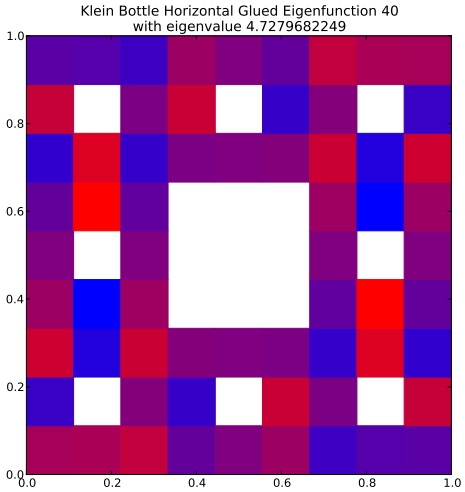


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.17479779895$
Dot Value: 2.220446049250313e-16

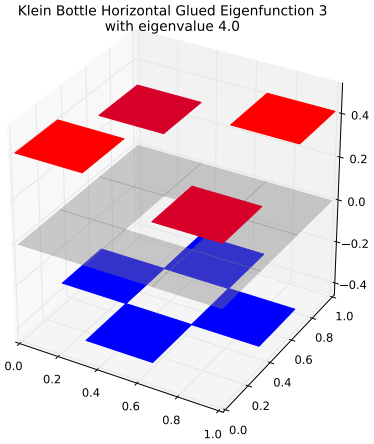
41 $M = 2$ Eigenfunction 40

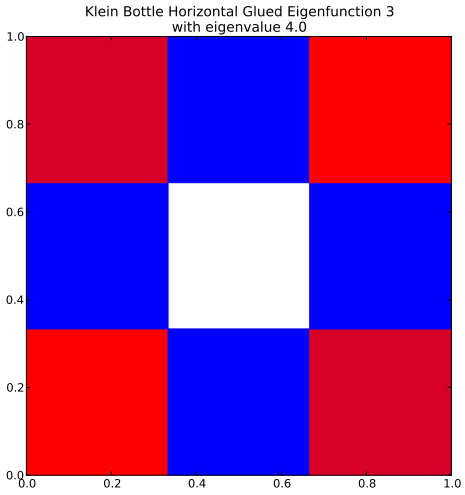
$M = 2$ Eigenfunction 40 has eigenvalue 4.7279682249





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

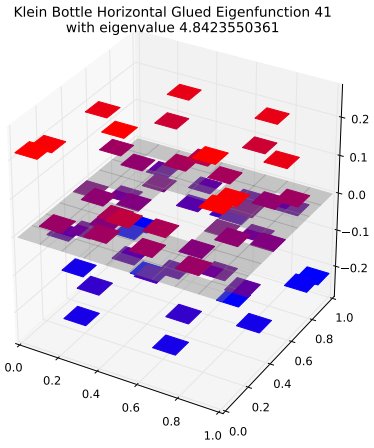


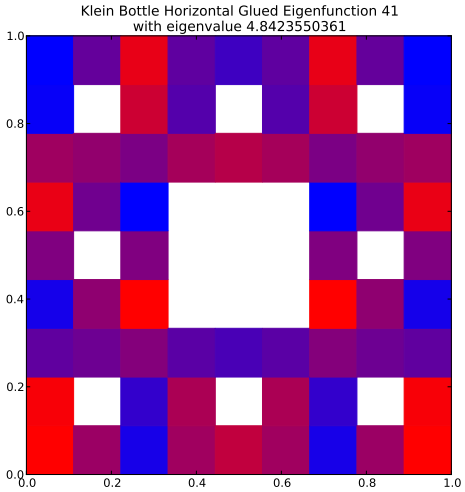


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.18199205622$
Dot Value: 0.0

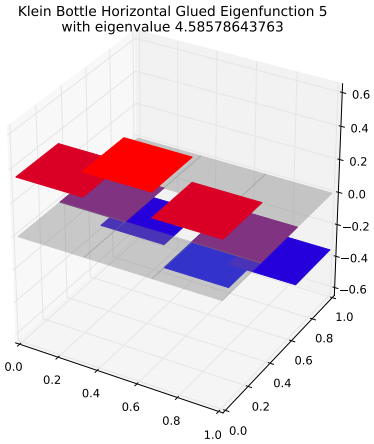
42 $M = 2$ Eigenfunction 41

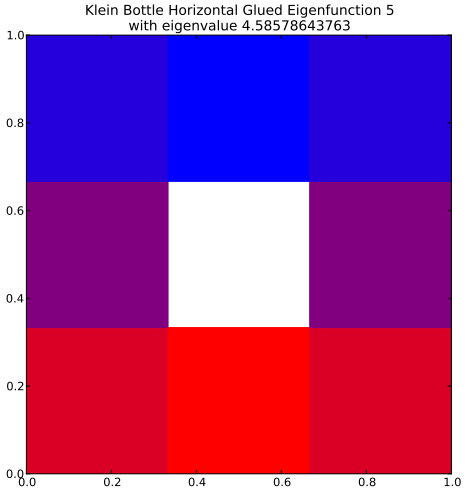
$M = 2$ Eigenfunction 41 has eigenvalue 4.8423550361





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

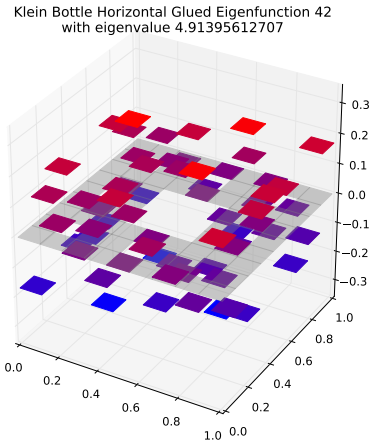


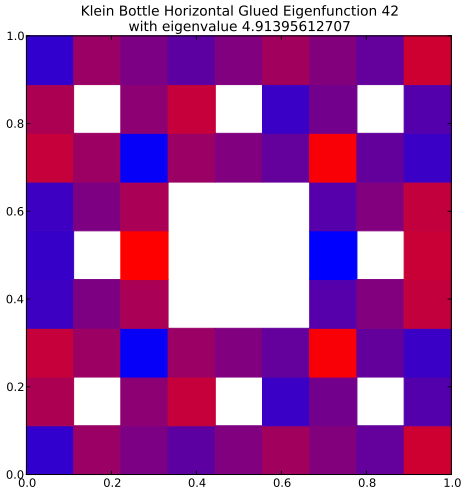


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.05594865831$
Dot Value: 0.009074624468260462

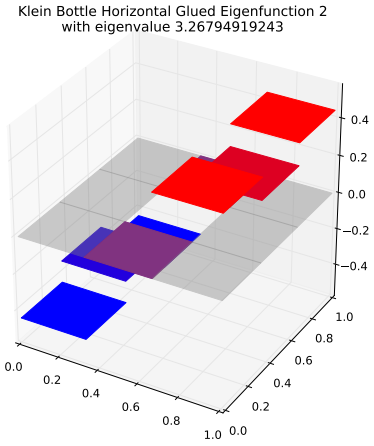
43 $M = 2$ Eigenfunction 42

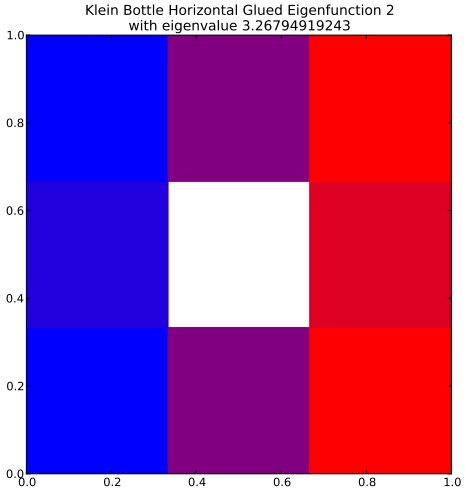
$M = 2$ Eigenfunction 42 has eigenvalue 4.91395612707





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

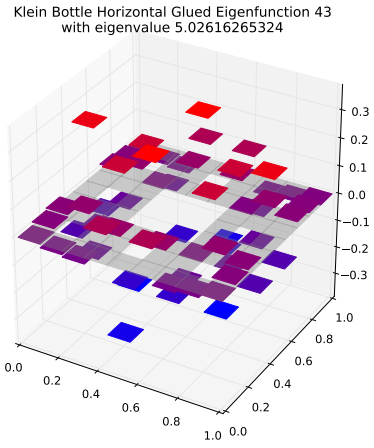


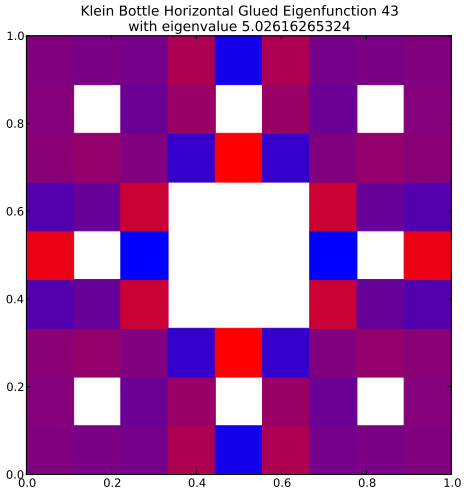


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.50368192335$
Dot Value: 0.11919793065129702

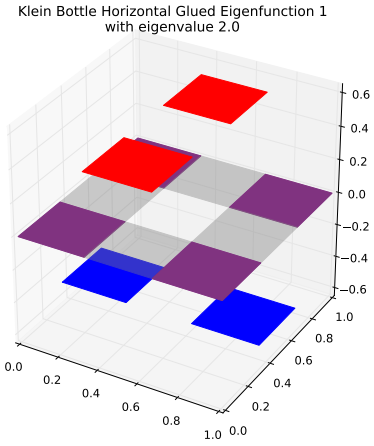
44 $M = 2$ Eigenfunction 43

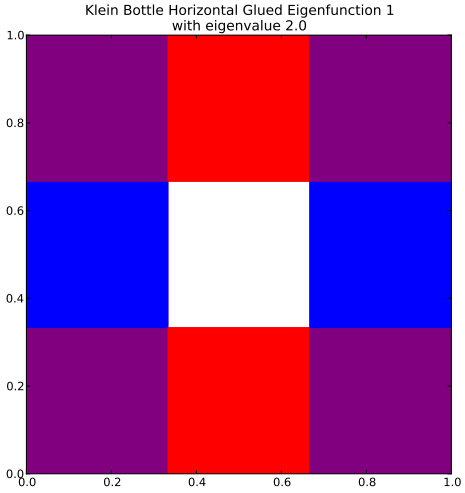
$M = 2$ Eigenfunction 43 has eigenvalue 5.02616265324





Compare to $m = 1$ eigenspace with eigenvalue 2.0

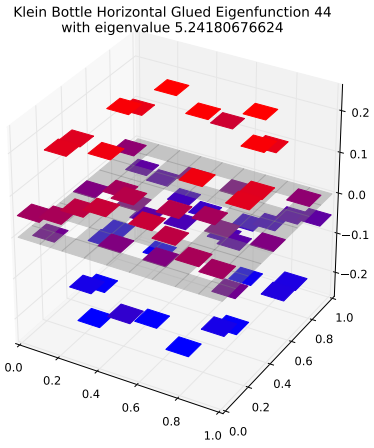


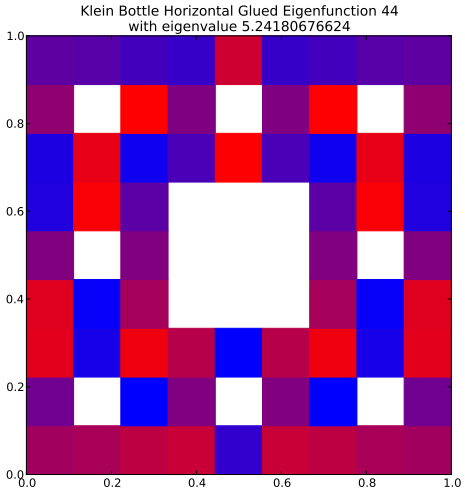


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 2.51308132662$
Dot Value: 2.220446049250313e-16

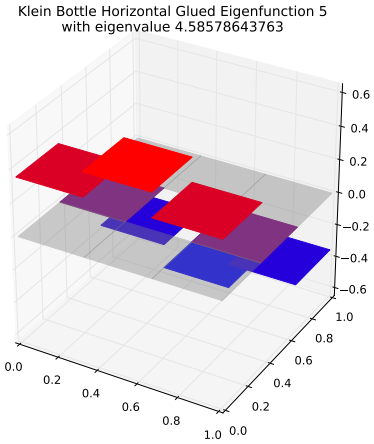
45 $M = 2$ Eigenfunction 44

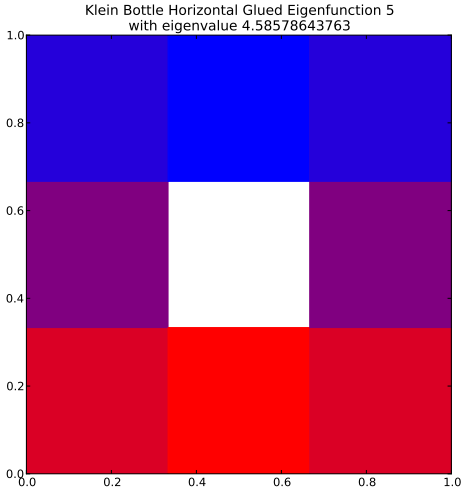
$M = 2$ Eigenfunction 44 has eigenvalue 5.24180676624





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

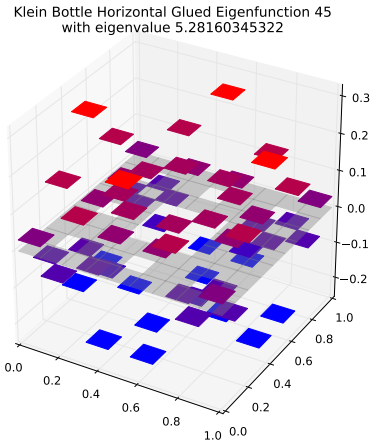


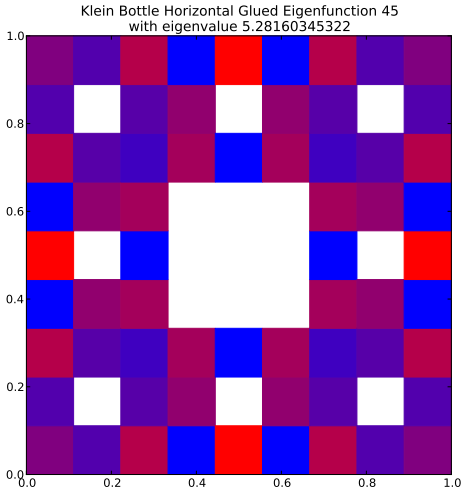


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.14305514169$
Dot Value: 0.09516721095526859

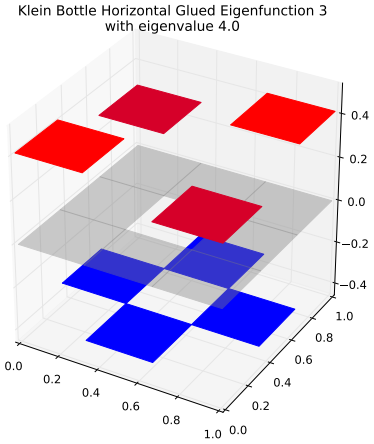
46 $M = 2$ Eigenfunction 45

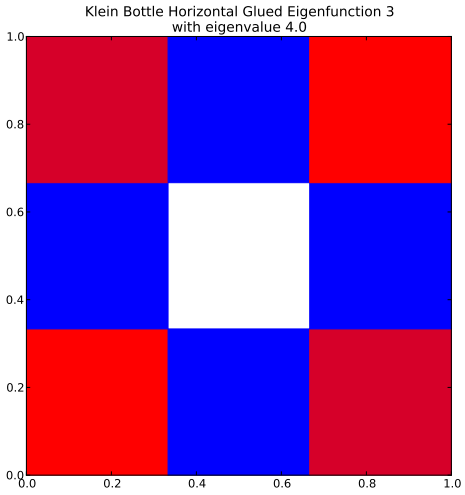
$M = 2$ Eigenfunction 45 has eigenvalue 5.28160345322





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

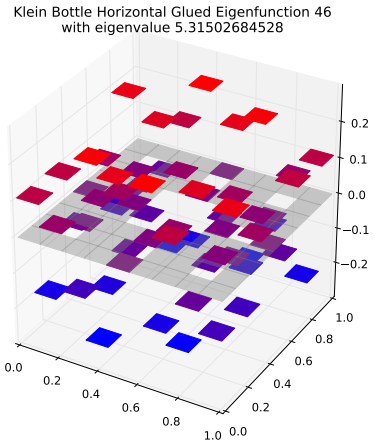


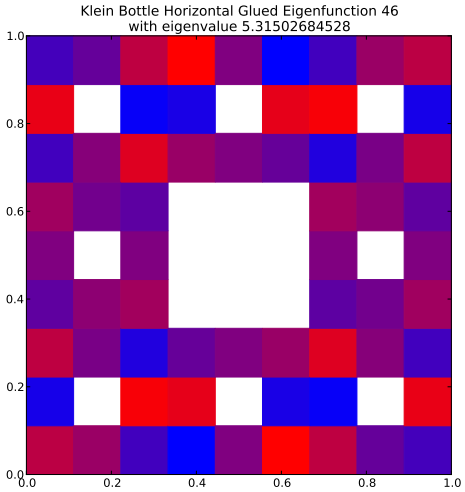


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.3204008633$
Dot Value: 1.1102230246251565e-16

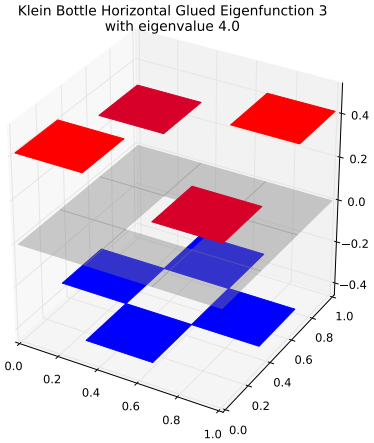
47 $M = 2$ Eigenfunction 46

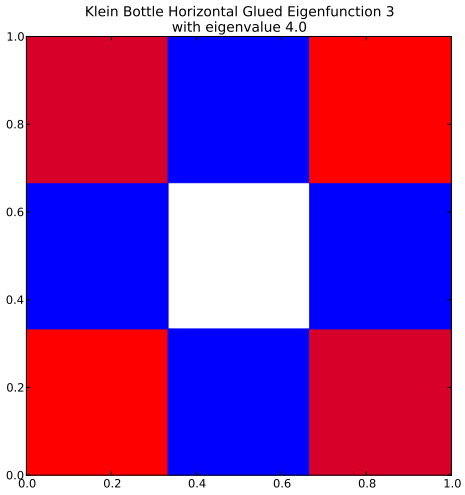
$M = 2$ Eigenfunction 46 has eigenvalue 5.31502684528





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

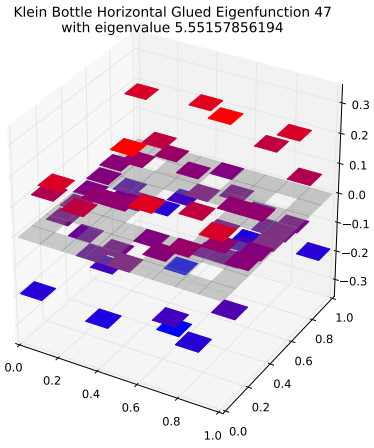


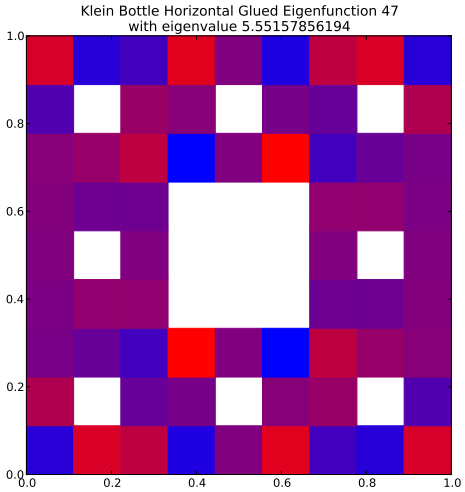


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.32875671132$
Dot Value: 0.0

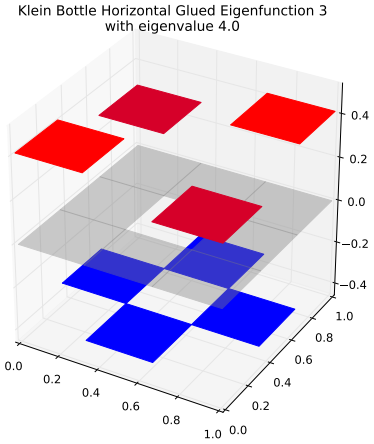
48 $M = 2$ Eigenfunction 47

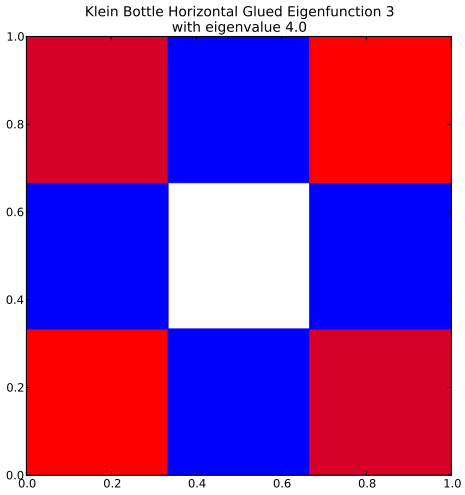
$M = 2$ Eigenfunction 47 has eigenvalue 5.55157856194





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

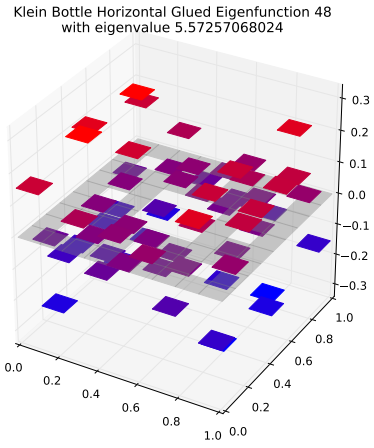


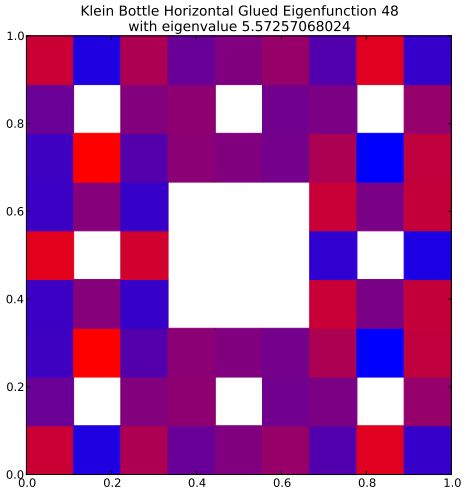


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.38789464049$
Dot Value: 2.220446049250313e-16

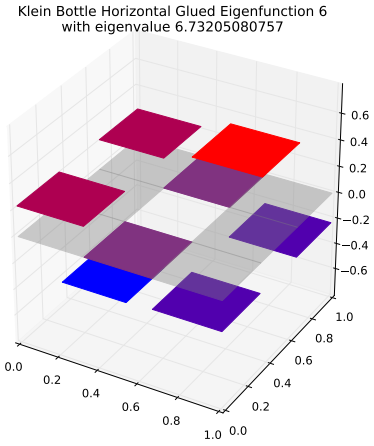
49 $M = 2$ Eigenfunction 48

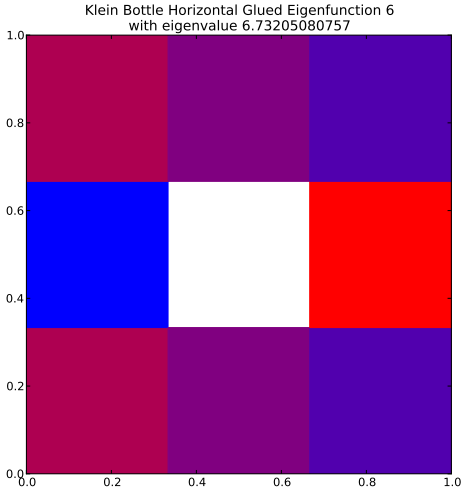
$M = 2$ Eigenfunction 48 has eigenvalue 5.57257068024





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

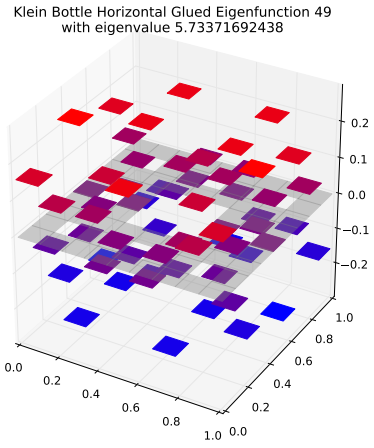


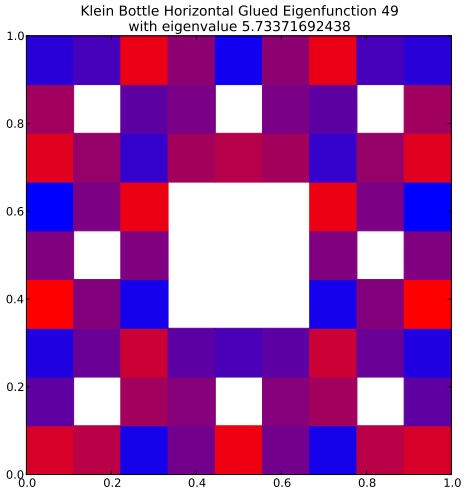


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.827767175194$
Dot Value: 0.008885431644714115

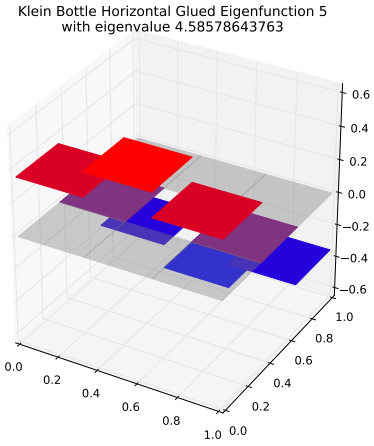
50 $M = 2$ Eigenfunction 49

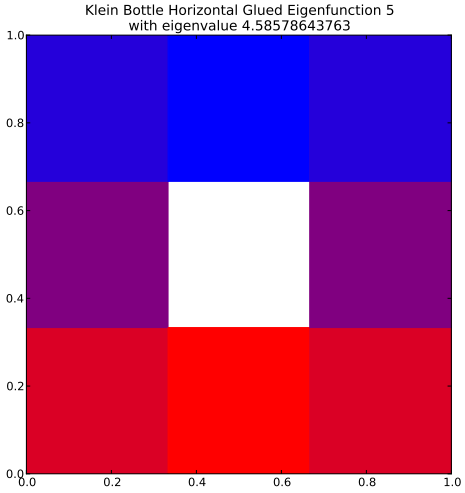
$M = 2$ Eigenfunction 49 has eigenvalue 5.73371692438





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

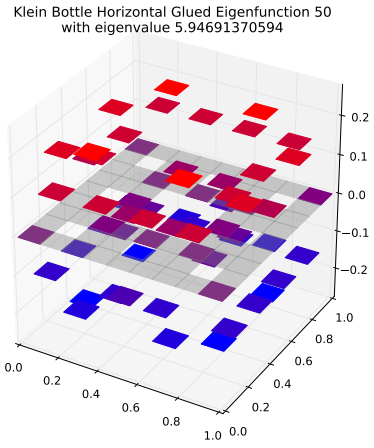


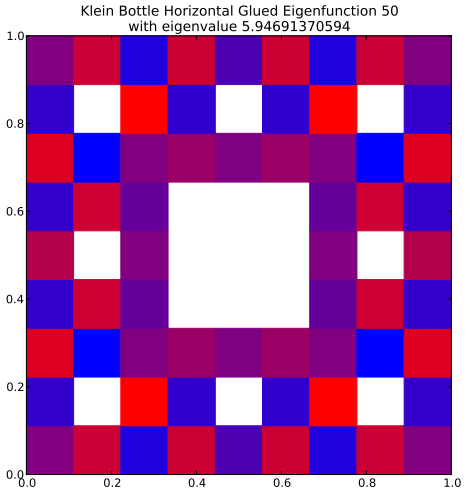


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.25032358187$
Dot Value: 0.1407250439999469

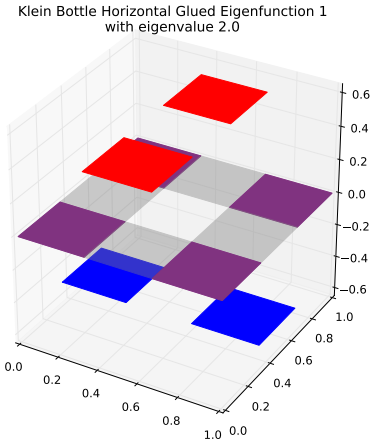
51 $M = 2$ Eigenfunction 50

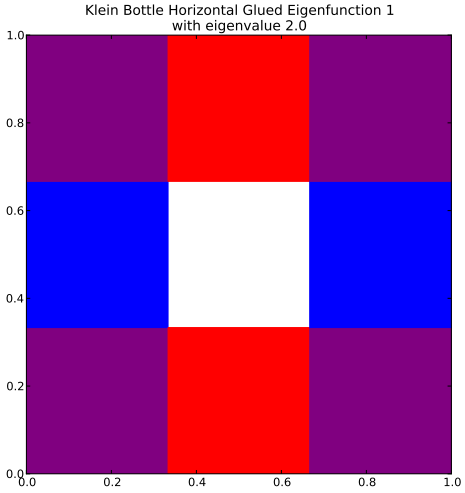
$M = 2$ Eigenfunction 50 has eigenvalue 5.94691370594





Compare to $m = 1$ eigenspace with eigenvalue 2.0

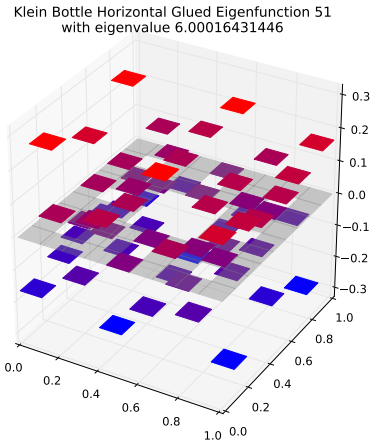


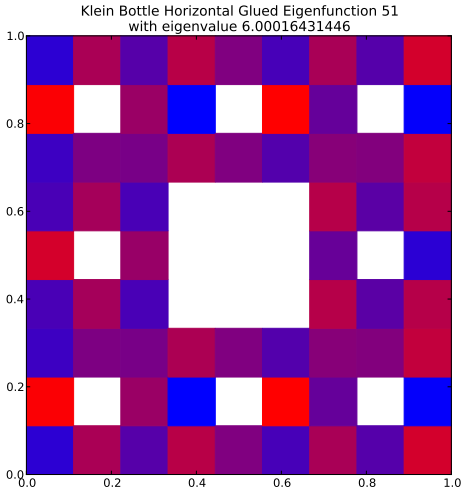


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 2.97345685297$
Dot Value: 2.220446049250313e-16

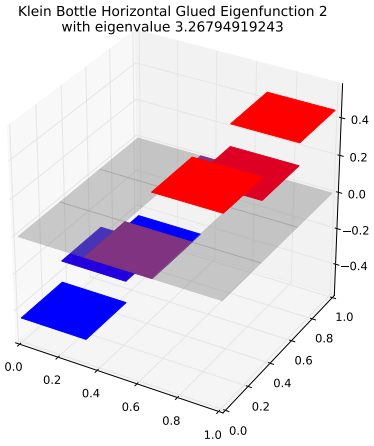
52 $M = 2$ Eigenfunction 51

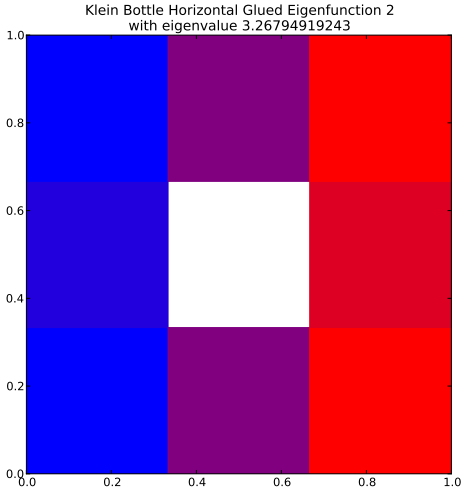
$M = 2$ Eigenfunction 51 has eigenvalue 6.00016431446





Compare to $m = 1$ eigenspace with eigenvalue 3.26794919243

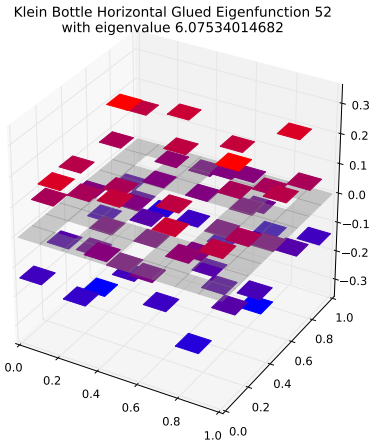


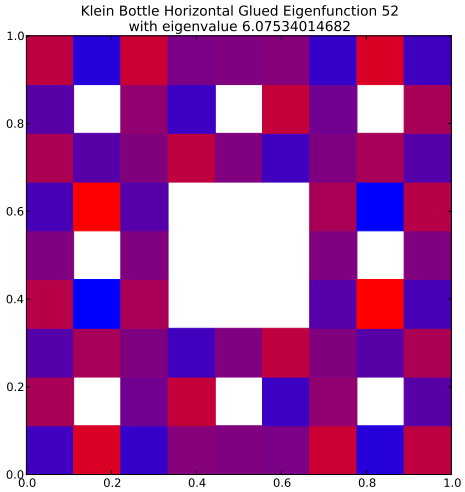


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.83606413721$
Dot Value: 0.25778074451684296

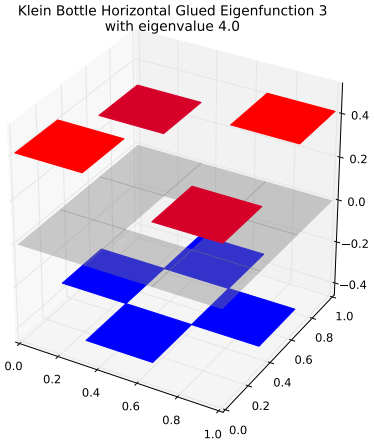
53 $M = 2$ Eigenfunction 52

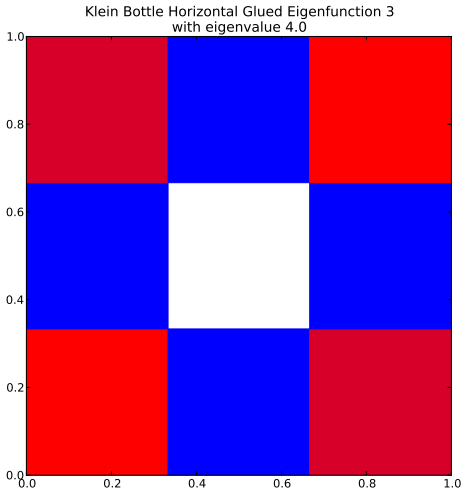
$M = 2$ Eigenfunction 52 has eigenvalue 6.07534014682





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

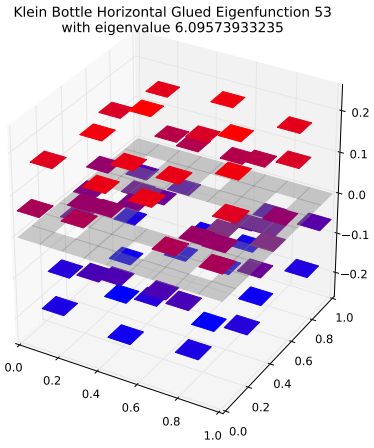


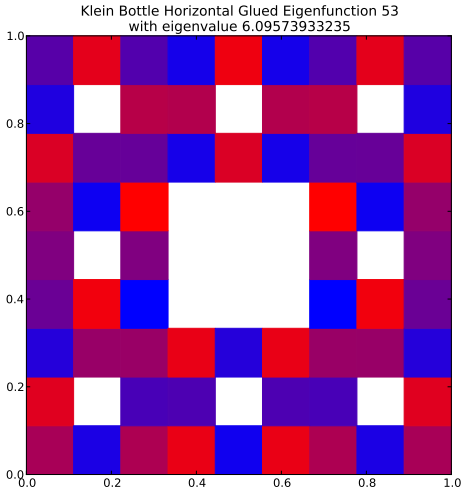


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.5188350367$
Dot Value: 0.0

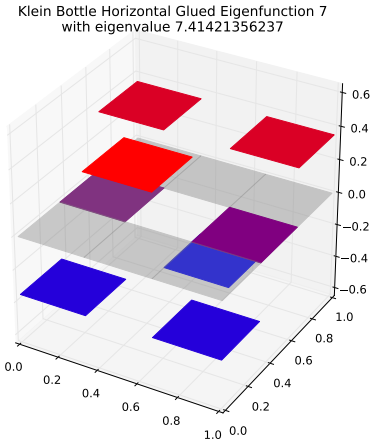
54 $M = 2$ Eigenfunction 53

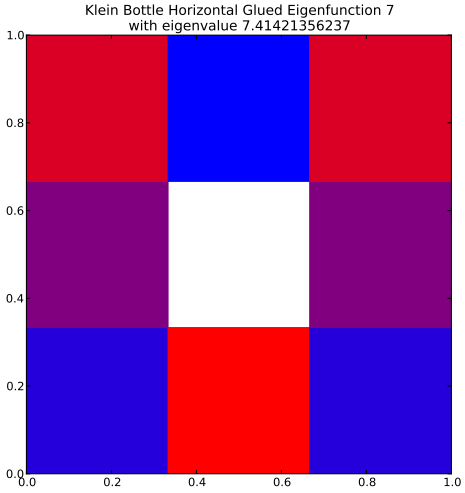
$M = 2$ Eigenfunction 53 has eigenvalue 6.09573933235





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

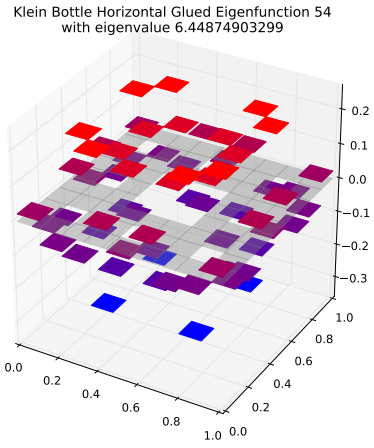


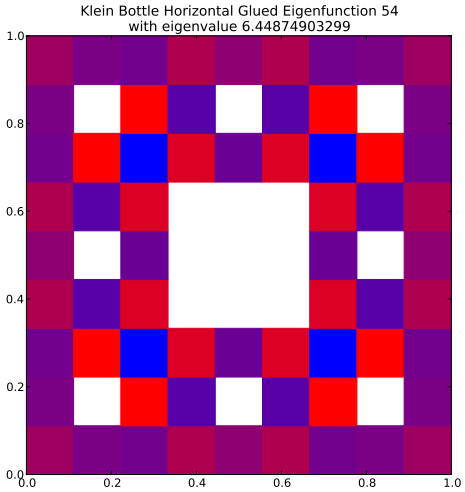


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.822169375223$
Dot Value: 0.18510190067000099

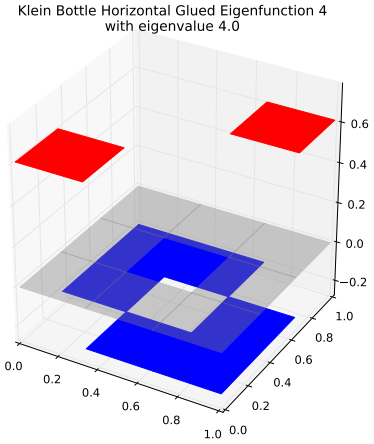
55 $M = 2$ Eigenfunction 54

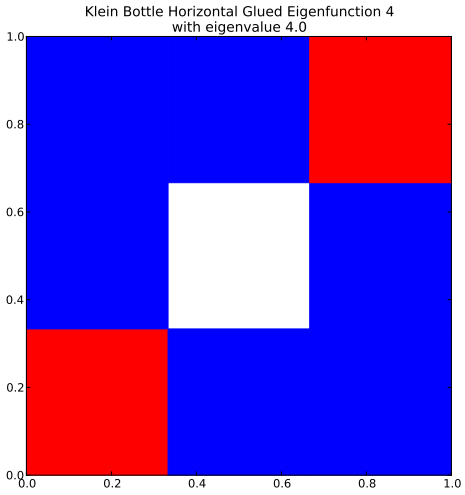
$M = 2$ Eigenfunction 54 has eigenvalue 6.44874903299





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

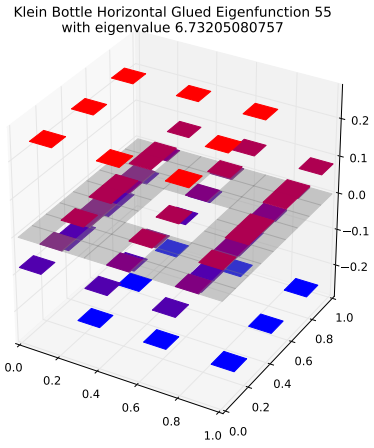


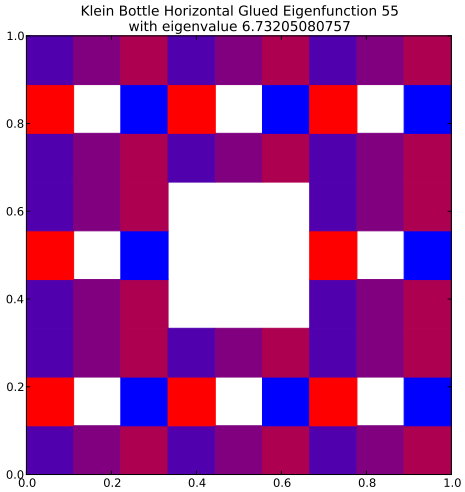


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.61218725825$
Dot Value: 0.0

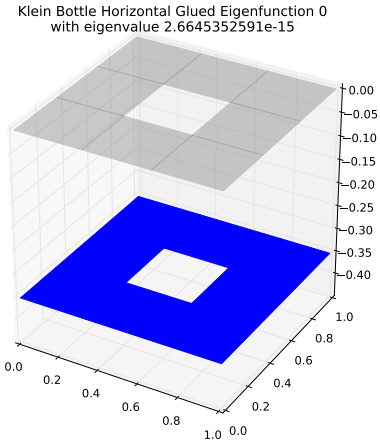
56 $M = 2$ Eigenfunction 55

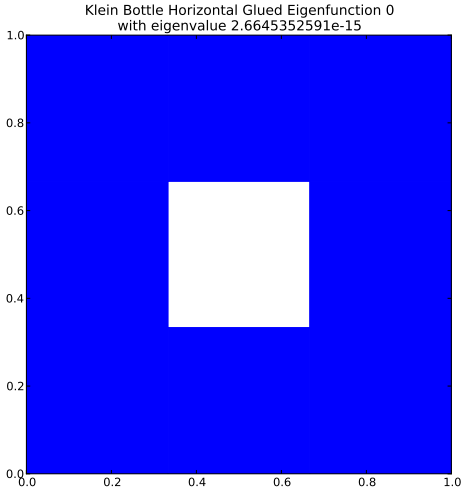
$M = 2$ Eigenfunction 55 has eigenvalue 6.73205080757





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15

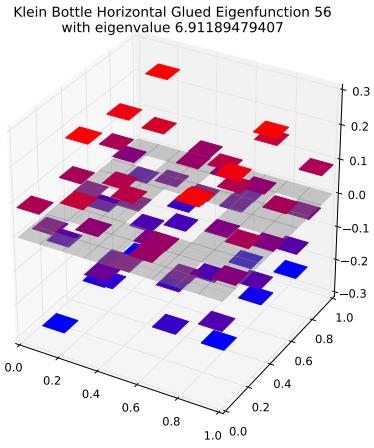


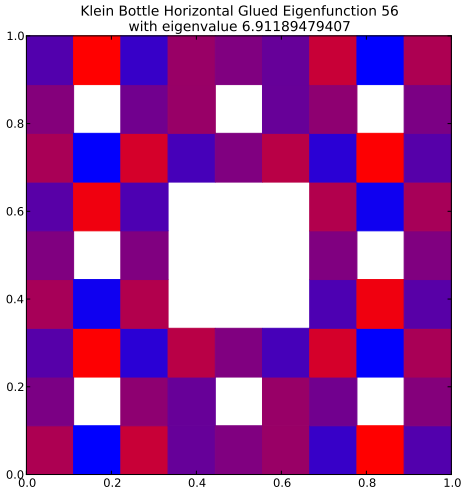


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 2.52653845903e + 15$
Dot Value: 2

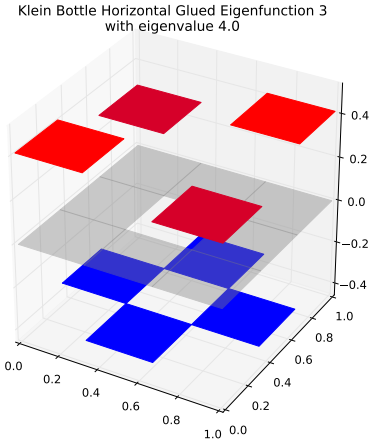
57 $M = 2$ Eigenfunction 56

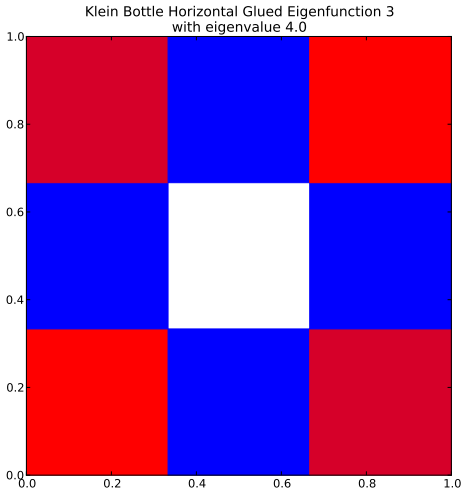
$M = 2$ Eigenfunction 56 has eigenvalue 6.91189479407





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

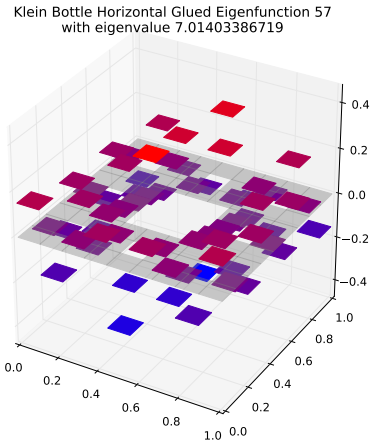


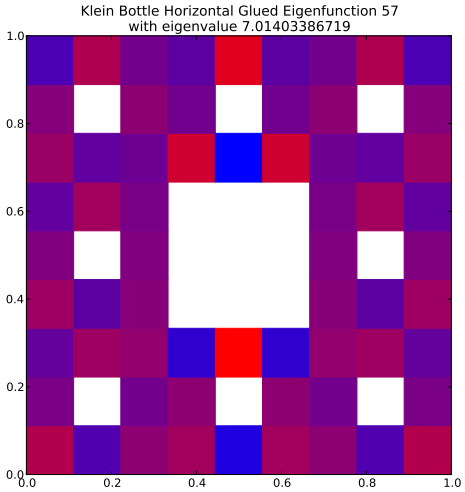


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.72797369852$
Dot Value: 0.0

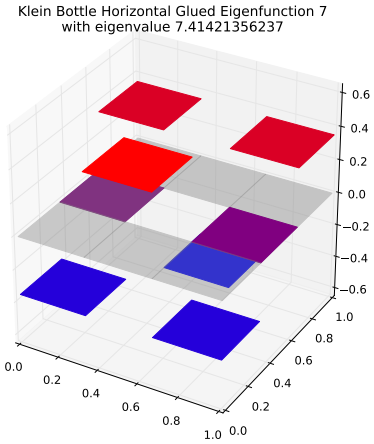
58 $M = 2$ Eigenfunction 57

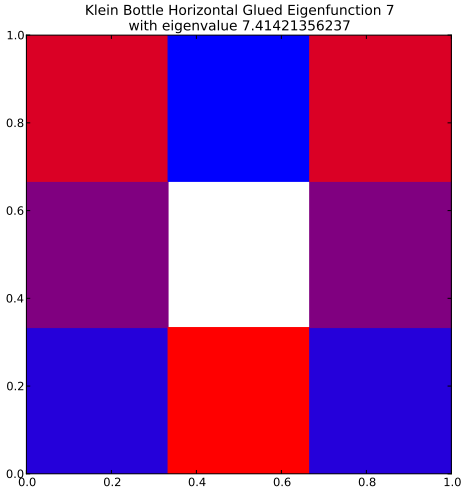
$M = 2$ Eigenfunction 57 has eigenvalue 7.01403386719





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

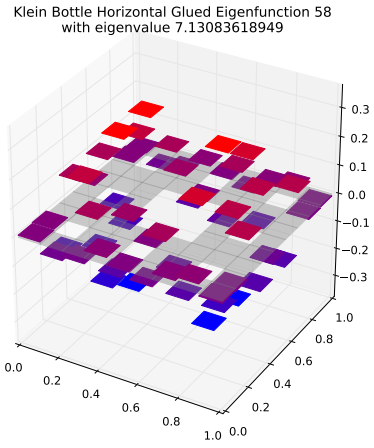


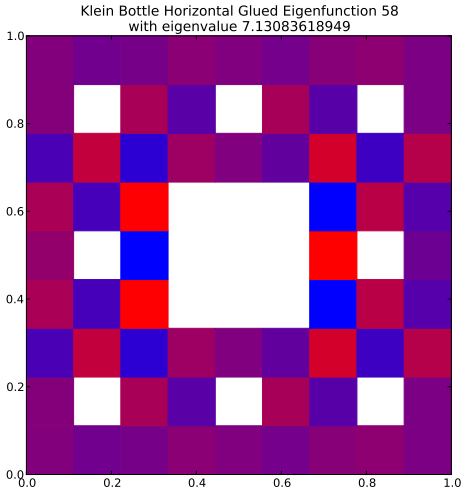


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.946025334742$
Dot Value: 0.0015552241298304459

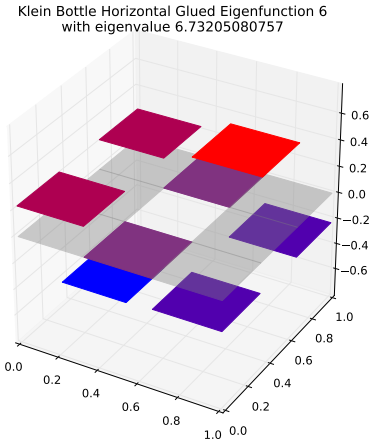
59 $M = 2$ Eigenfunction 58

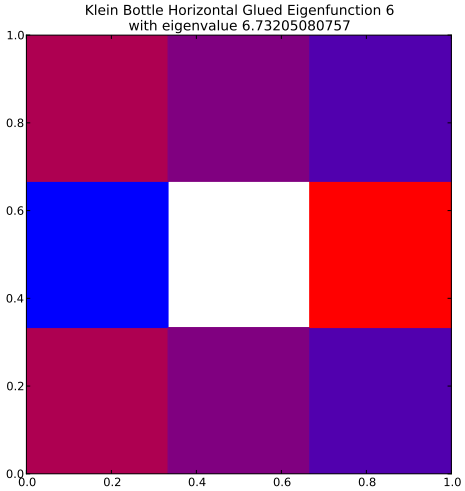
$M = 2$ Eigenfunction 58 has eigenvalue 7.13083618949





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

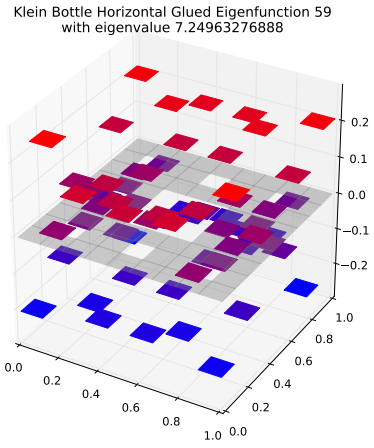


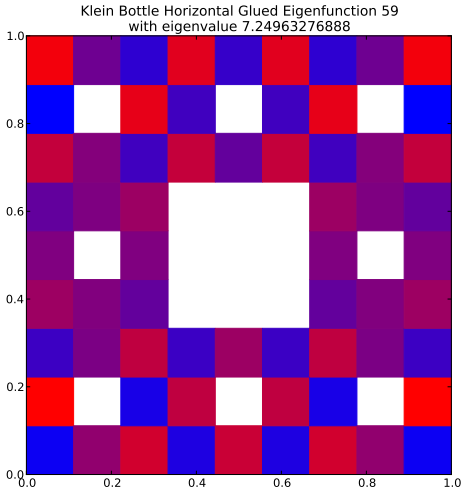


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.05923683485$
Dot Value: 0.0006700670542103548

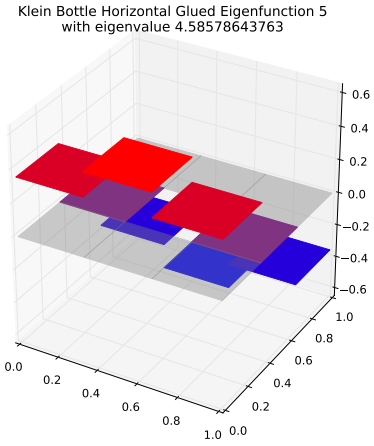
60 $M = 2$ Eigenfunction 59

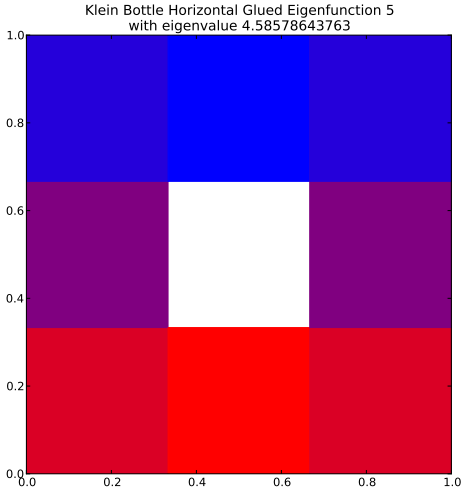
$M = 2$ Eigenfunction 59 has eigenvalue 7.24963276888





Compare to $m = 1$ eigenspace with eigenvalue 4.58578643763

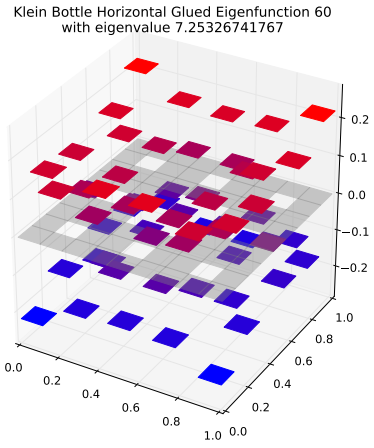


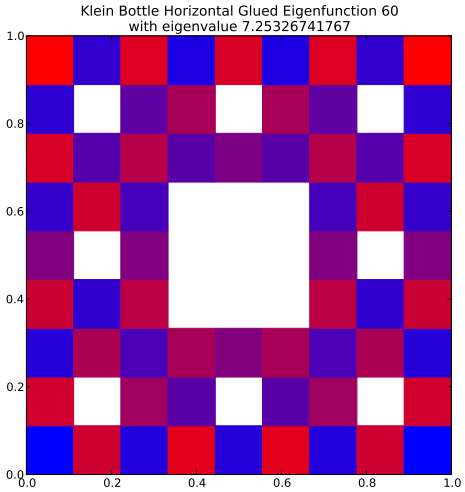


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.58089192933$
Dot Value: 0.27530814227198264

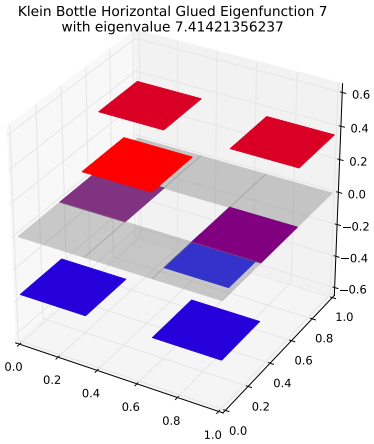
61 $M = 2$ Eigenfunction 60

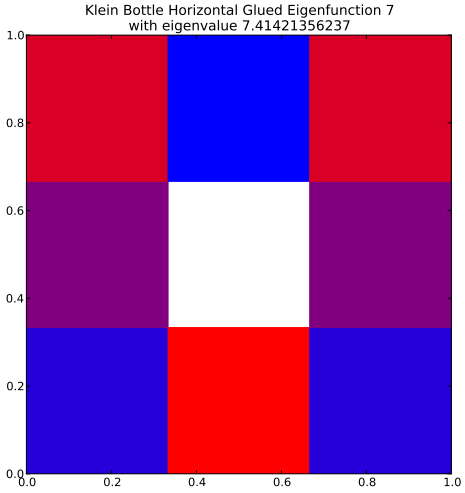
$M = 2$ Eigenfunction 60 has eigenvalue 7.25326741767





Compare to $m = 1$ eigenspace with eigenvalue 7.41421356237

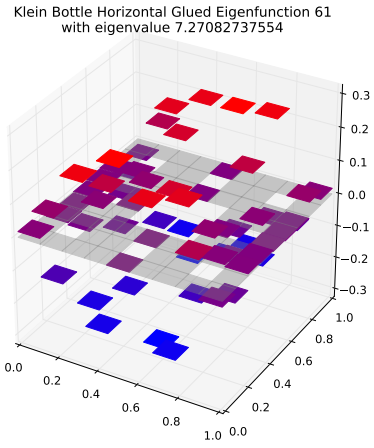


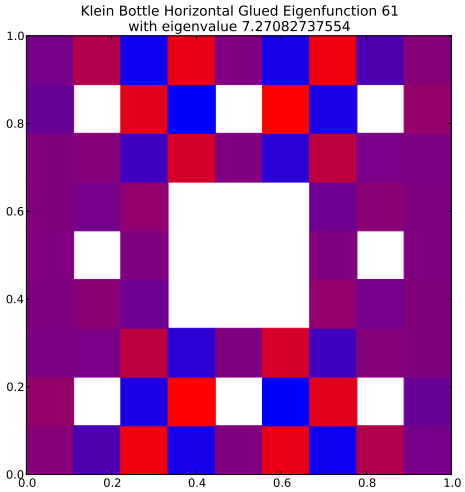


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 0.978292216249$
Dot Value: 0.025723692140125265

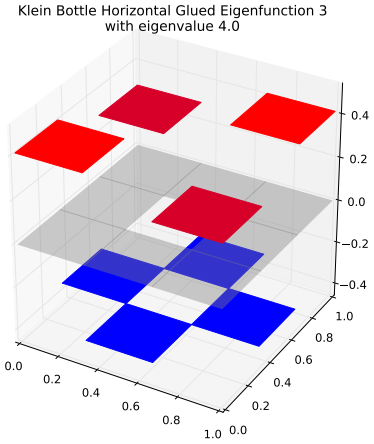
62 $M = 2$ Eigenfunction 61

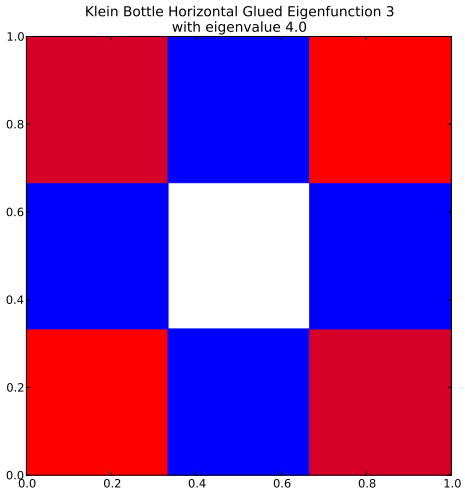
$M = 2$ Eigenfunction 61 has eigenvalue 7.27082737554





Compare to $m = 1$ eigenspace with eigenvalue 4.0
(Note: Eigenspace Dimension > 1)

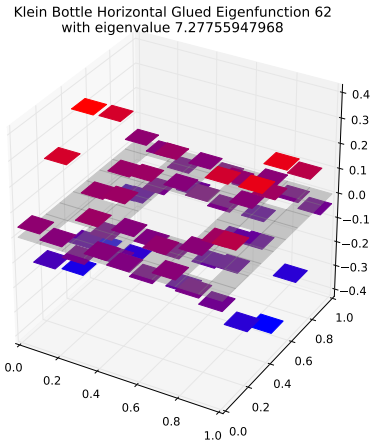


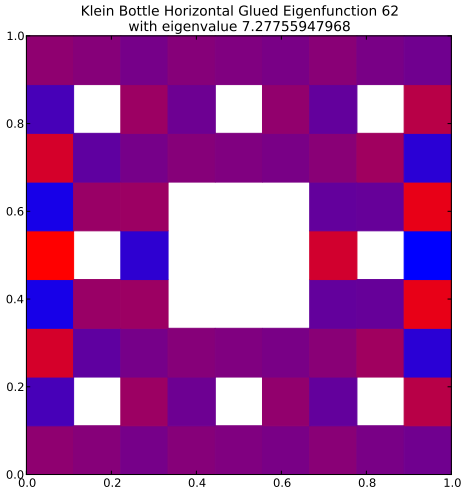


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.81770684389$
Dot Value: 2.220446049250313e-16

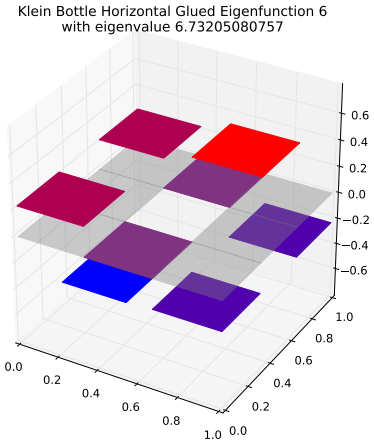
63 $M = 2$ Eigenfunction 62

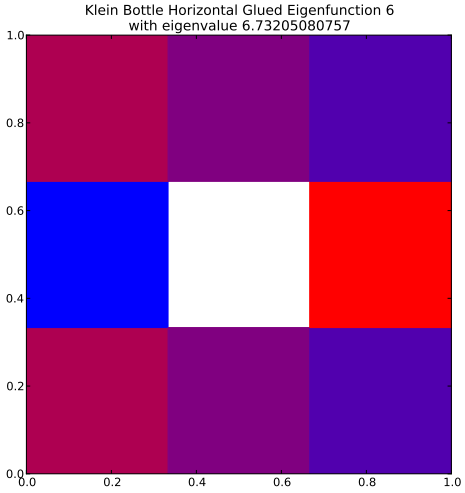
$M = 2$ Eigenfunction 62 has eigenvalue 7.27755947968





Compare to $m = 1$ eigenspace with eigenvalue 6.73205080757

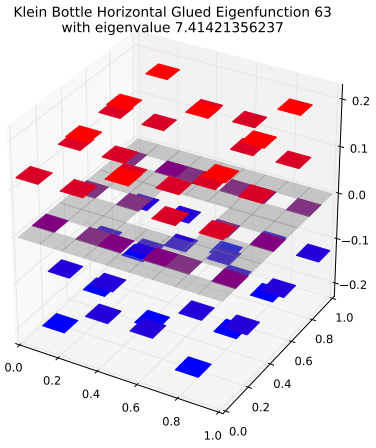


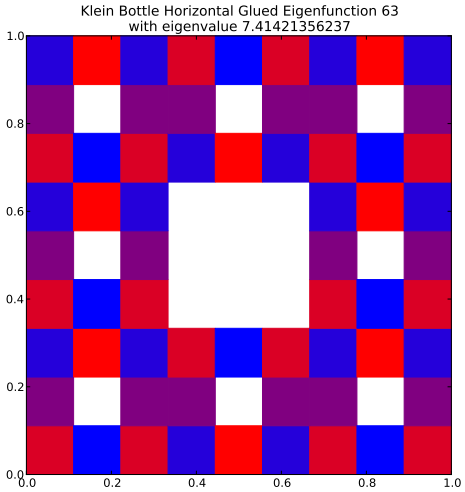


Eigenvalue Ratio: $\lambda_2/\lambda_1 = 1.08103157384$
Dot Value: 0.008519342893785309

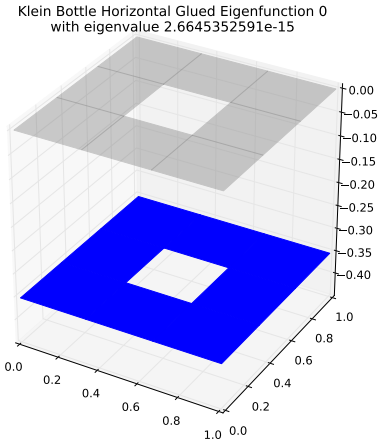
64 $M = 2$ Eigenfunction 63

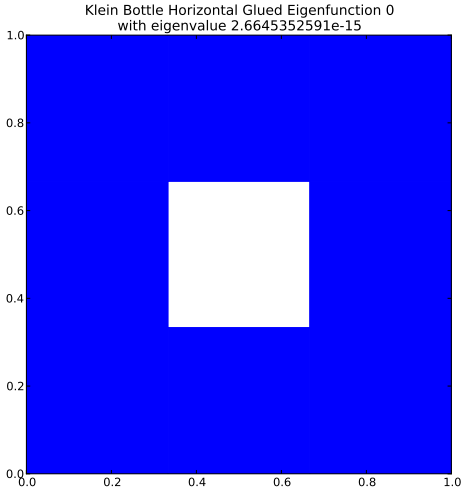
$M = 2$ Eigenfunction 63 has eigenvalue 7.41421356237





Compare to $m = 1$ eigenspace with eigenvalue 2.6645352591e-15





Eigenvalue Ratio: $\lambda_2/\lambda_1 = 2.78255411973e + 15$
Dot Value: 2