

# Klein Bottle Horizontal Glued Comparison of Level 3 Eigenfunctions and Level 2 Eigenfunctions By Averaging (First 150)

SPUR 2016

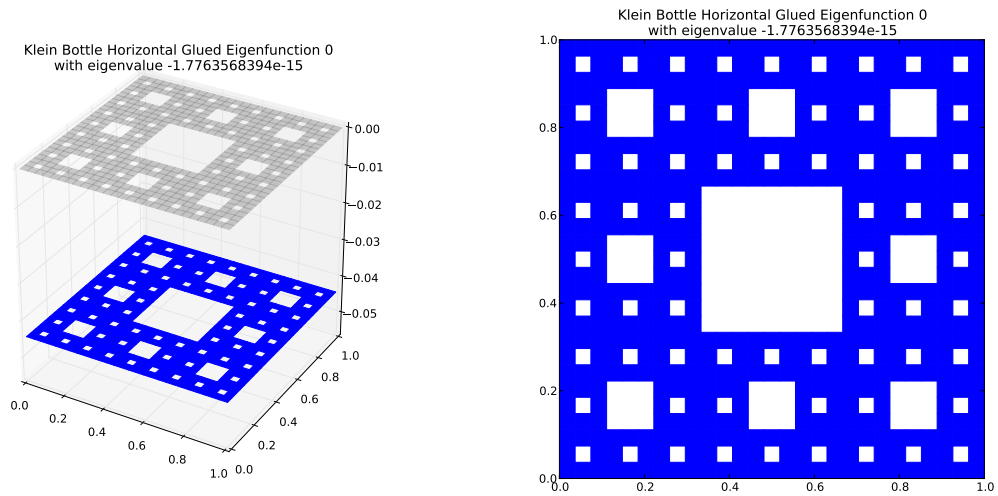
May 23, 2018

## **Key to Dot Value**

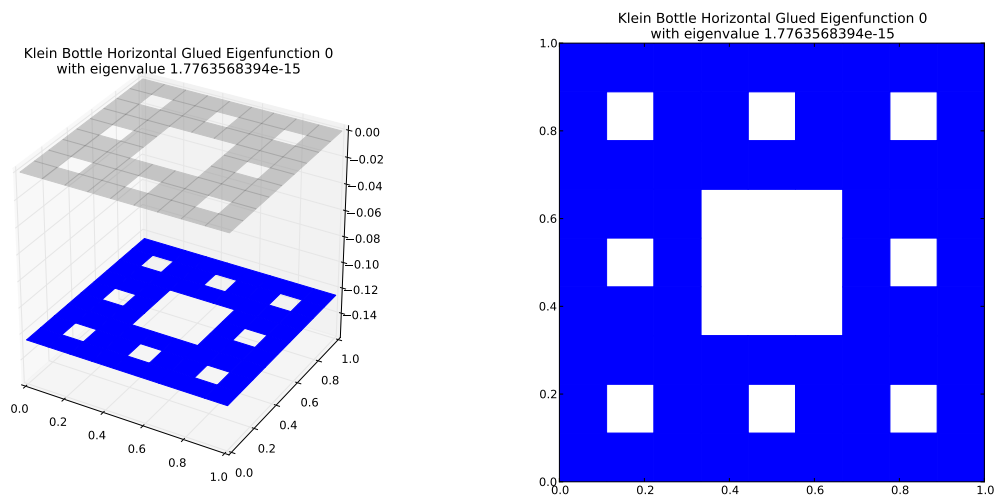
Dot values are in general between 0 and 1; those close to 0 are better matches, while those close to 1 are not good matches. Dot value 2 indicates the eigenvalue averages to the zero function. Dot value 3 indicates the projection onto the closest eigenspace is zero.

# 1 $M = 3$ Eigenfunction 0

$M = 3$  Eigenfunction 0 has eigenvalue  $2.67841304691e-15$



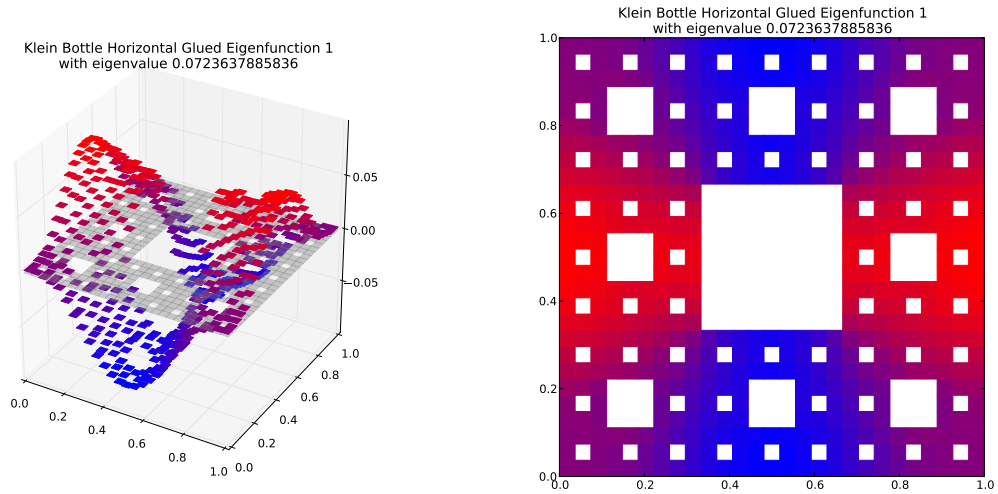
Compare to  $m = 2$  eigenspace with eigenvalue  $1.7763568394e-15$



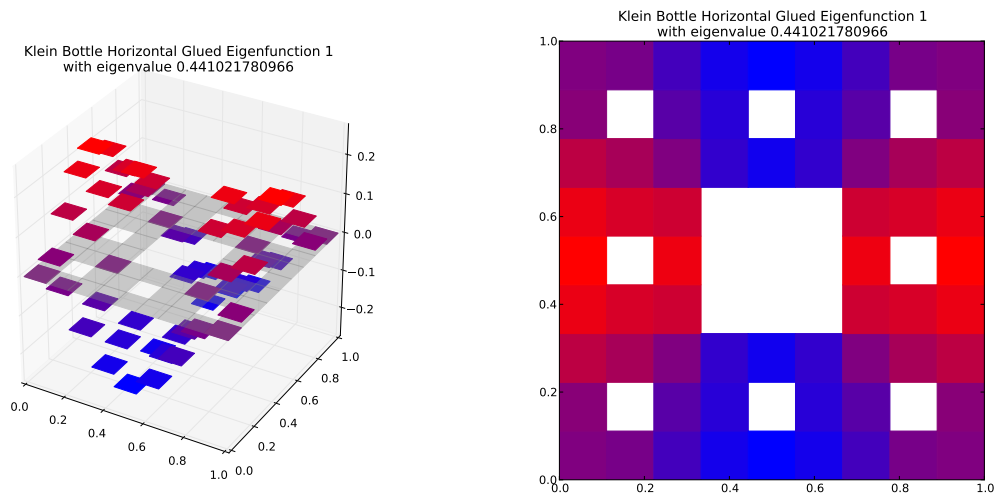
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 1.5078125$   
Dot Value: 0.0

## 2 $M = 3$ Eigenfunction 1

$M = 3$  Eigenfunction 1 has eigenvalue 0.0723637885836



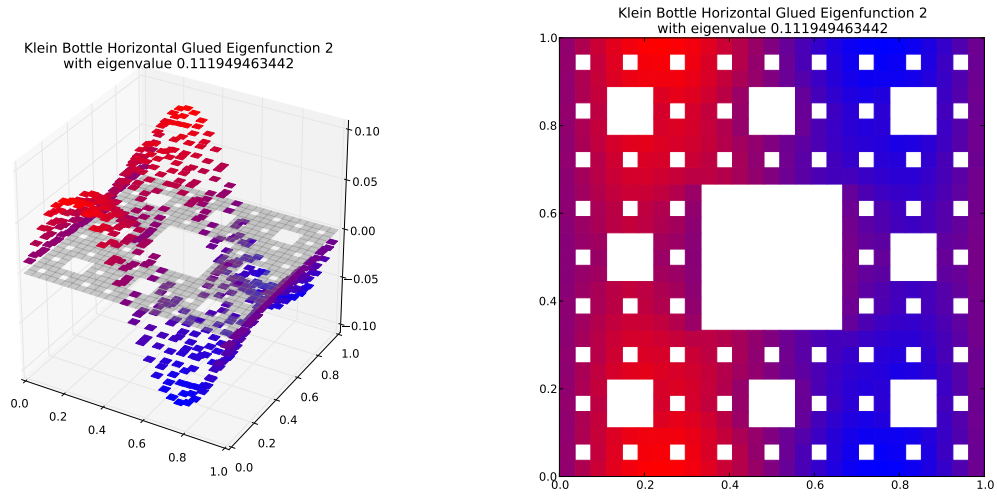
Compare to  $m = 2$  eigenspace with eigenvalue 0.441021780966



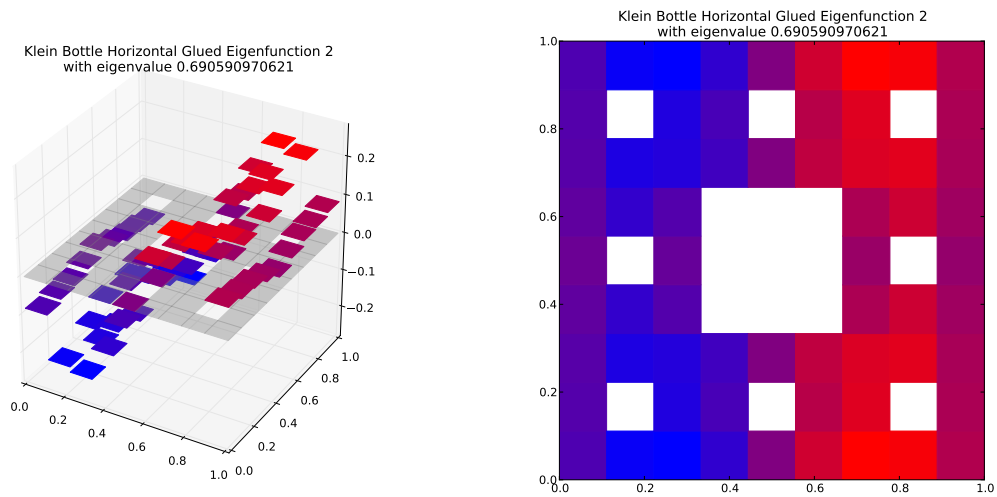
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.164082119539$   
Dot Value: 0.0004106827796719337

### 3 $M = 3$ Eigenfunction 2

$M = 3$  Eigenfunction 2 has eigenvalue 0.111949463442



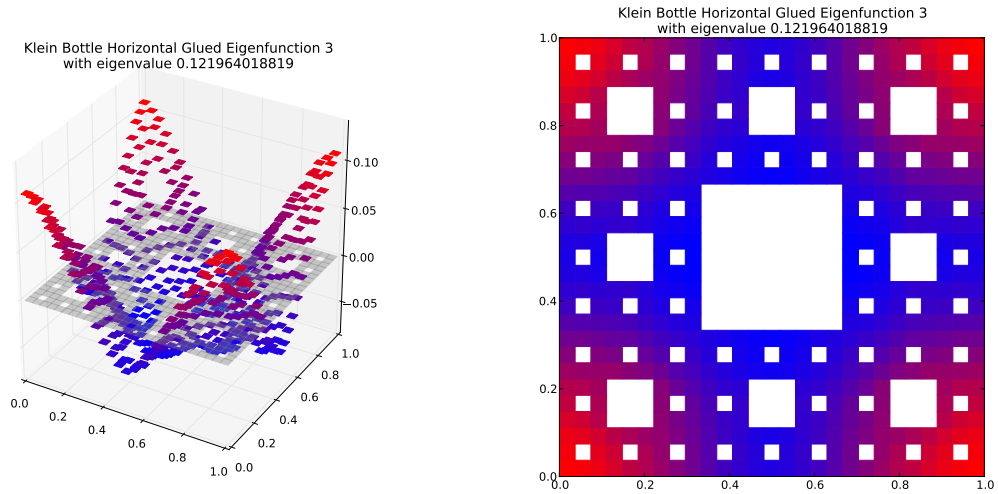
Compare to  $m = 2$  eigenspace with eigenvalue 0.690590970621



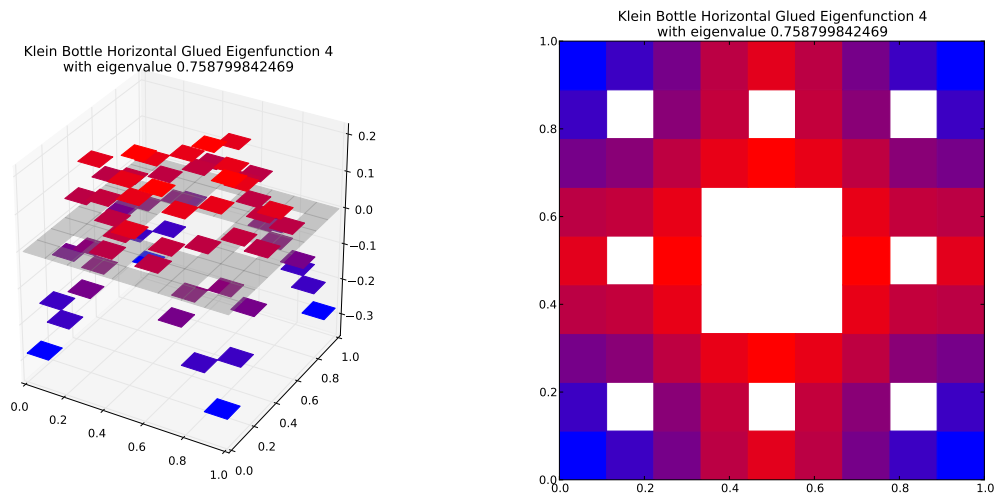
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.162106758131$   
Dot Value: 0.0008467422239333278

## 4 $M = 3$ Eigenfunction 3

$M = 3$  Eigenfunction 3 has eigenvalue 0.121964018819



Compare to  $m = 2$  eigenspace with eigenvalue 0.758799842469

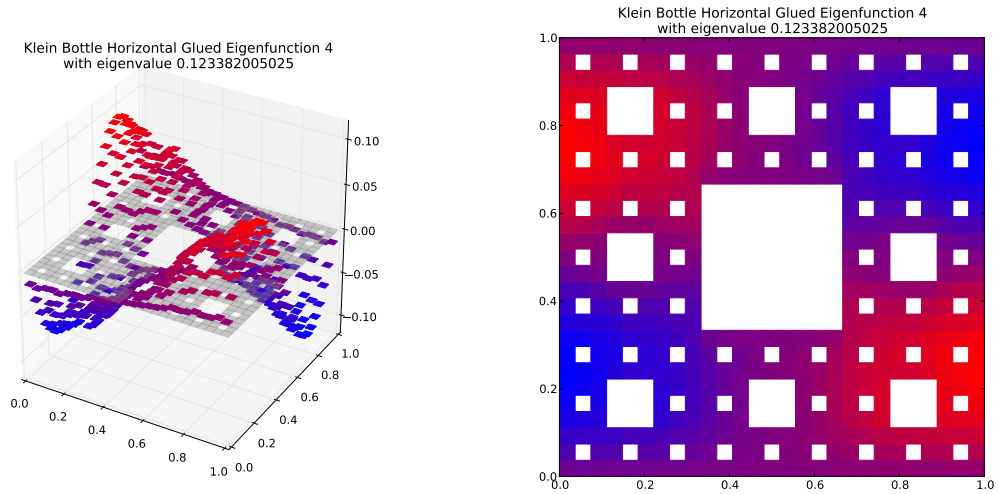


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.160732794069$

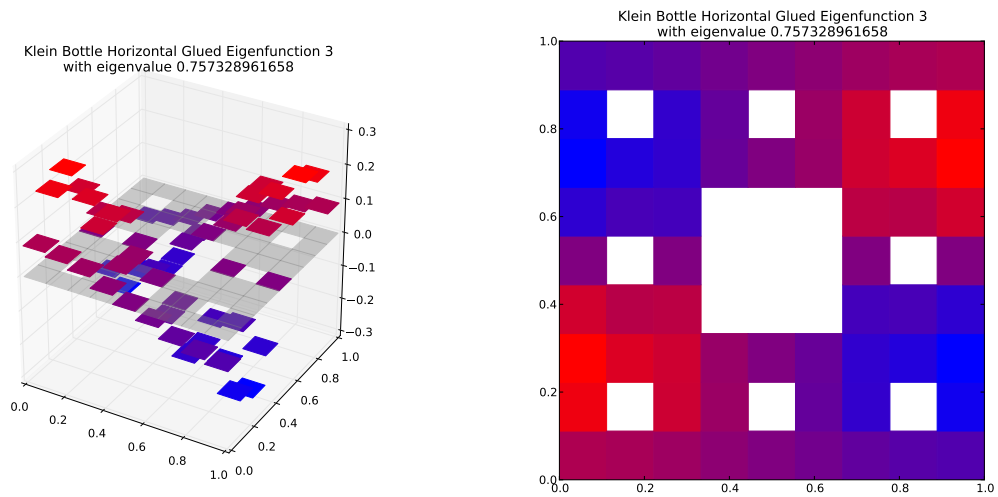
Dot Value: 0.001213690430604375

## 5 $M = 3$ Eigenfunction 4

$M = 3$  Eigenfunction 4 has eigenvalue 0.123382005025



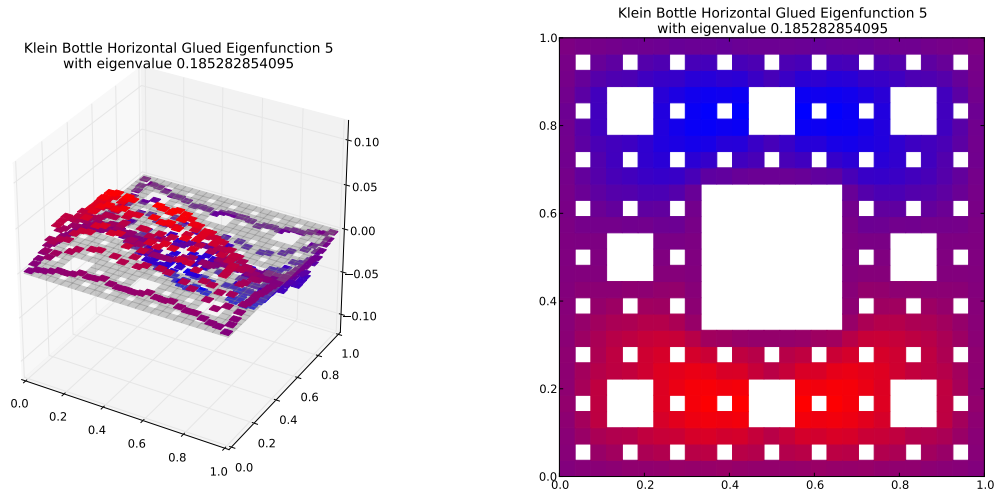
Compare to  $m = 2$  eigenspace with eigenvalue 0.757328961658



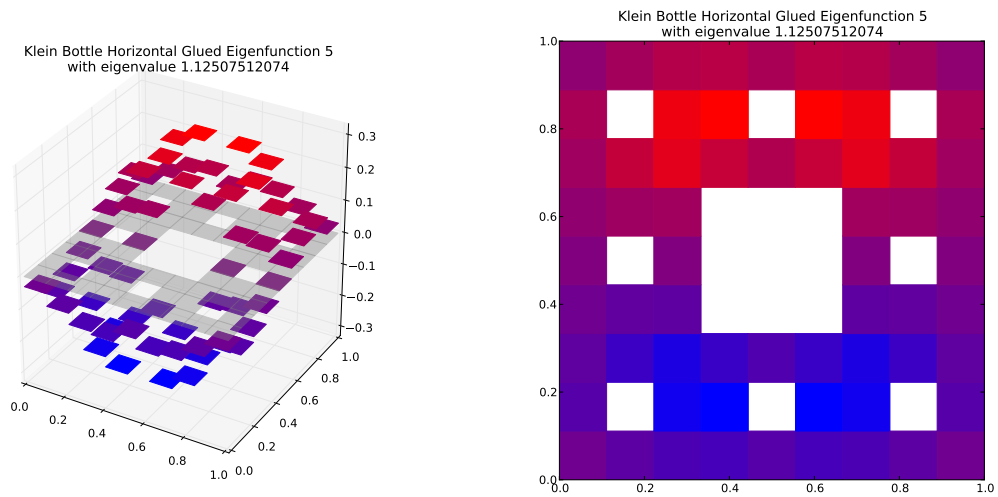
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.162917320308$   
Dot Value: 0.001695495860162266

## 6 $M = 3$ Eigenfunction 5

$M = 3$  Eigenfunction 5 has eigenvalue 0.185282854095



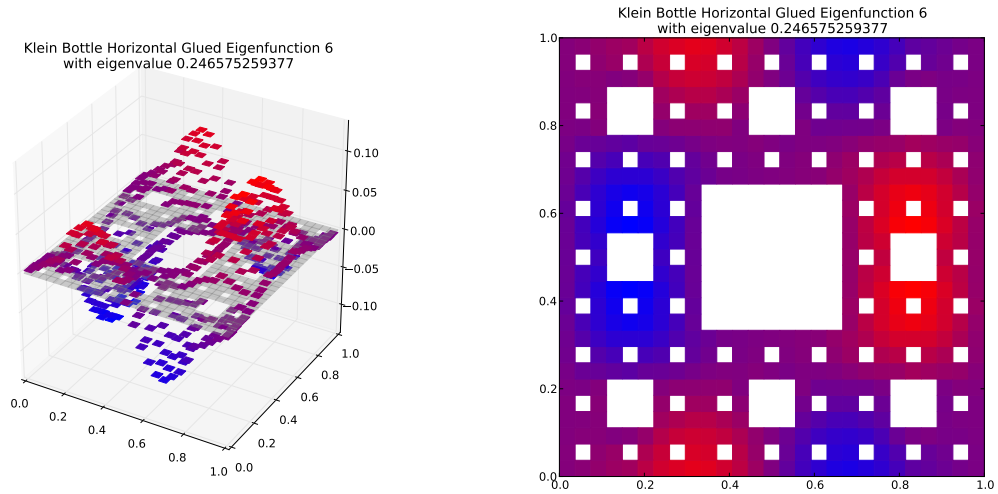
Compare to  $m = 2$  eigenspace with eigenvalue 1.12507512074



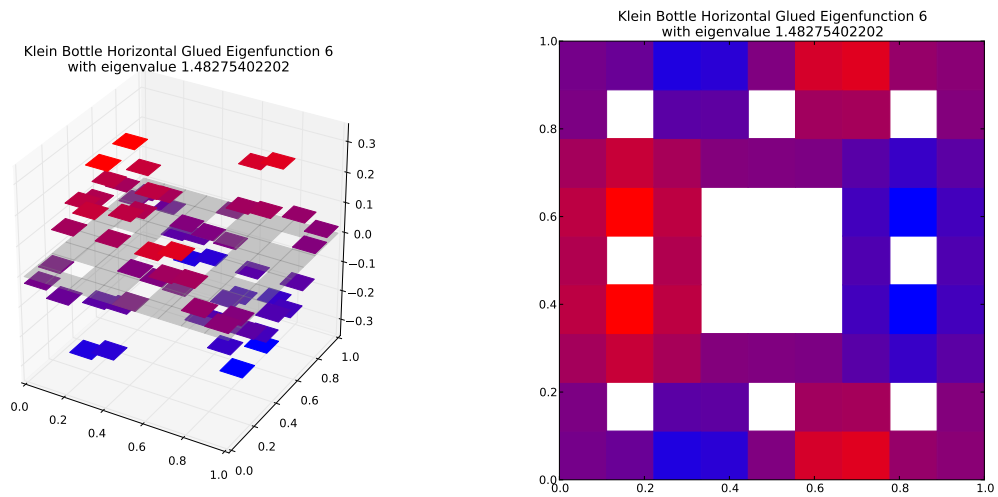
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.16468487364$   
Dot Value: 0.0013315311553242504

## 7 $M = 3$ Eigenfunction 6

$M = 3$  Eigenfunction 6 has eigenvalue 0.246575259377



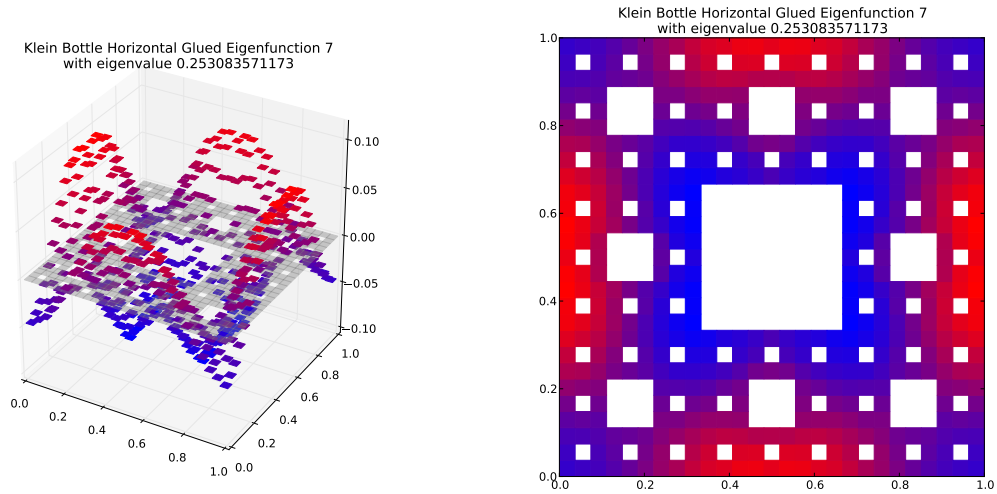
Compare to  $m = 2$  eigenspace with eigenvalue 1.48275402202



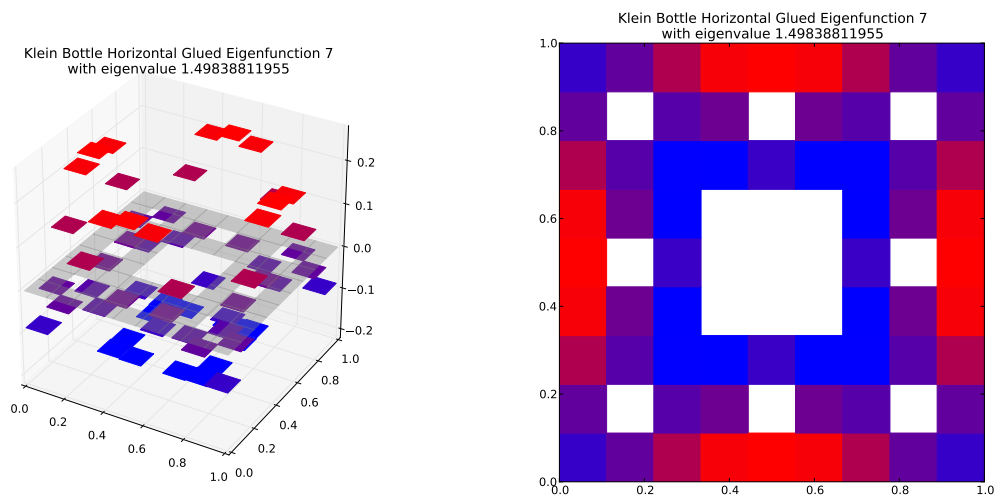
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.166295458124$   
Dot Value: 0.003761623212570231

## 8 $M = 3$ Eigenfunction 7

$M = 3$  Eigenfunction 7 has eigenvalue 0.253083571173



Compare to  $m = 2$  eigenspace with eigenvalue 1.49838811955

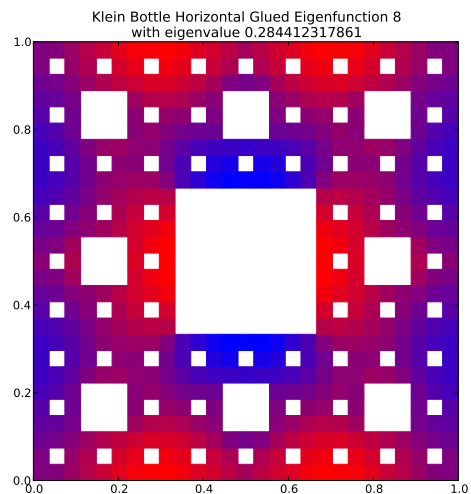
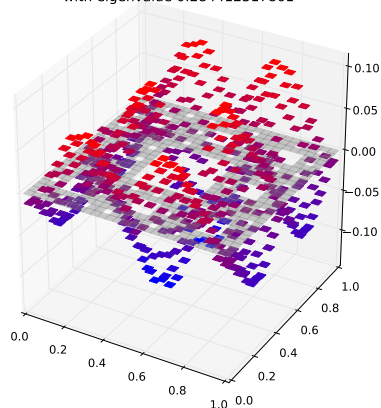


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.168903882693$   
Dot Value: 0.014718189368639667

## 9 $M = 3$ Eigenfunction 8

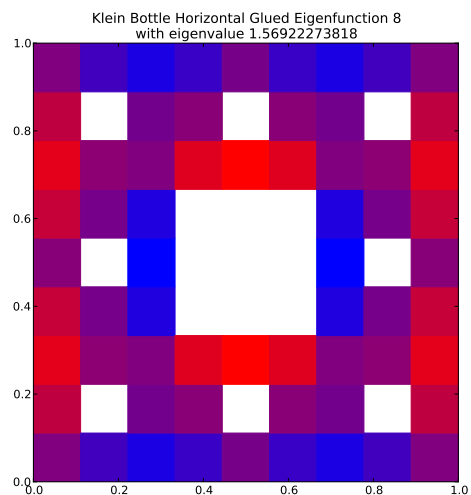
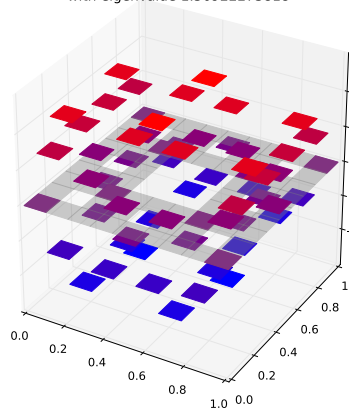
$M = 3$  Eigenfunction 8 has eigenvalue 0.284412317861

Klein Bottle Horizontal Glued Eigenfunction 8  
with eigenvalue 0.284412317861



Compare to  $m = 2$  eigenspace with eigenvalue 1.56922273818

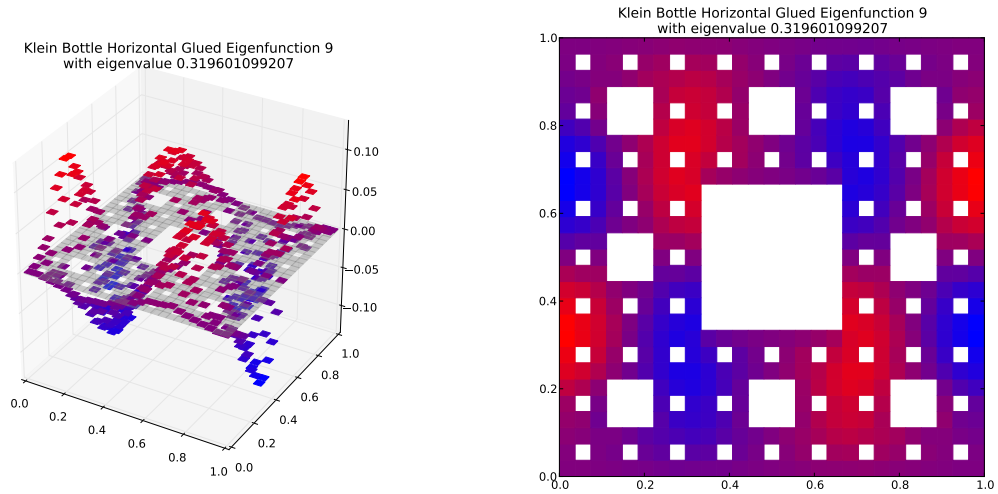
Klein Bottle Horizontal Glued Eigenfunction 8  
with eigenvalue 1.56922273818



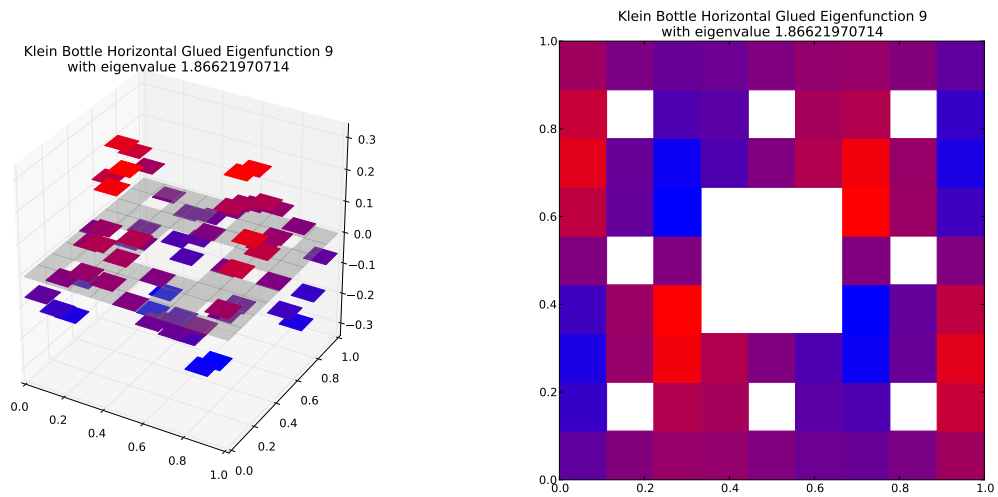
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.181244071311$   
Dot Value: 0.011905936095741754

## 10 $M = 3$ Eigenfunction 9

$M = 3$  Eigenfunction 9 has eigenvalue 0.319601099207



Compare to  $m = 2$  eigenspace with eigenvalue 1.86621970714

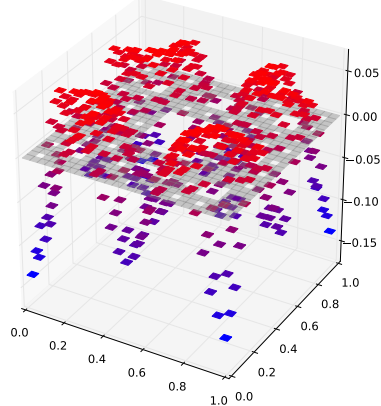


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.171255880529$   
Dot Value: 0.08498327717138932

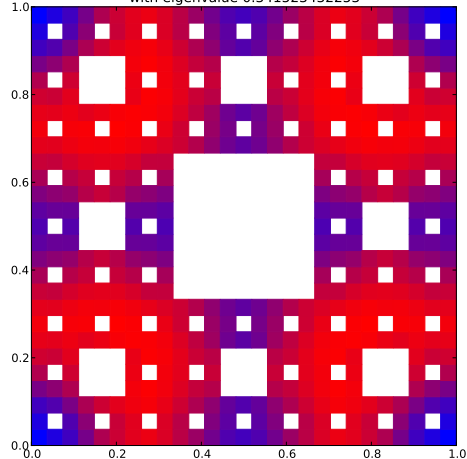
# 11 $M = 3$ Eigenfunction 10

$M = 3$  Eigenfunction 10 has eigenvalue 0.341523432253

Klein Bottle Horizontal Glued Eigenfunction 10  
with eigenvalue 0.341523432253

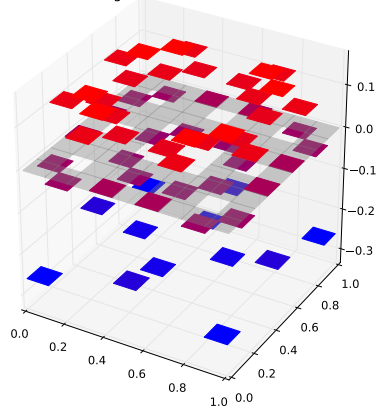


Klein Bottle Horizontal Glued Eigenfunction 10  
with eigenvalue 0.341523432253

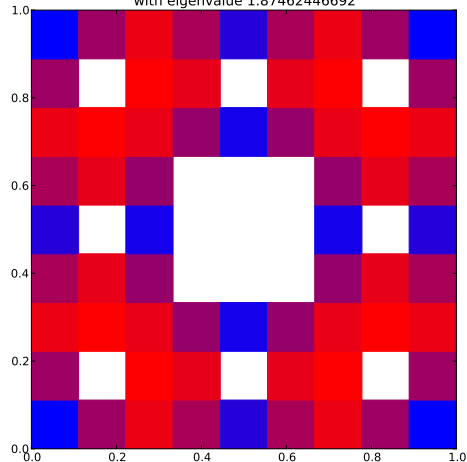


Compare to  $m = 2$  eigenspace with eigenvalue 1.87462446692

Klein Bottle Horizontal Glued Eigenfunction 10  
with eigenvalue 1.87462446692



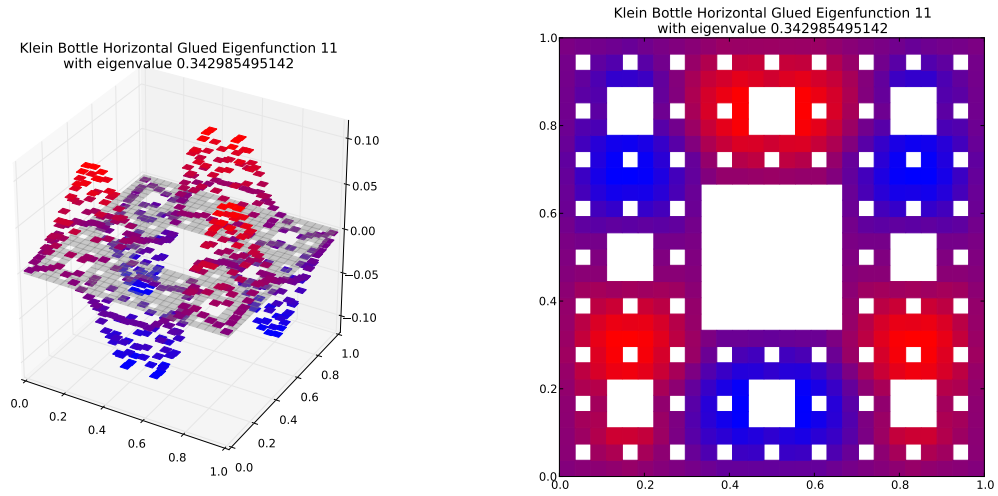
Klein Bottle Horizontal Glued Eigenfunction 10  
with eigenvalue 1.87462446692



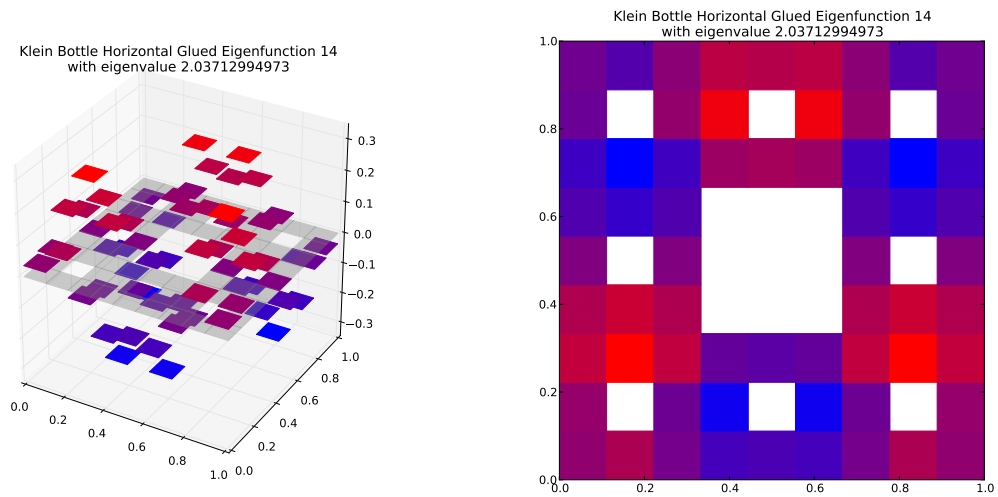
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.182182318795$   
Dot Value: 0.024030335828698846

## 12 $M = 3$ Eigenfunction 11

$M = 3$  Eigenfunction 11 has eigenvalue 0.342985495142



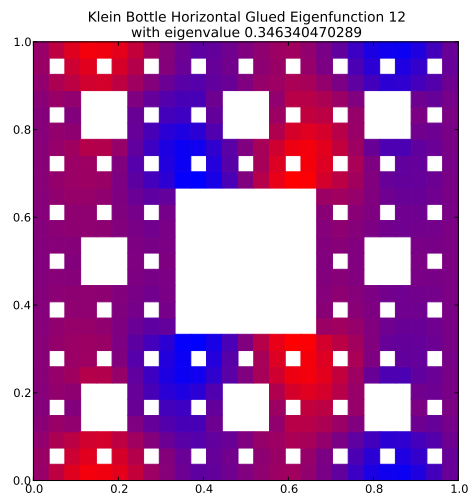
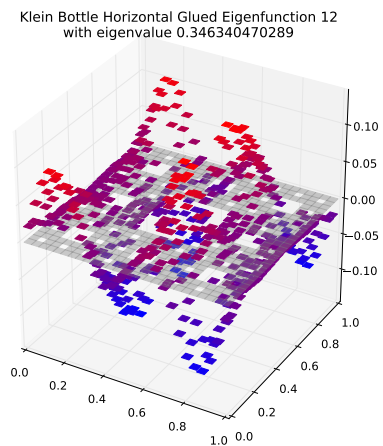
Compare to  $m = 2$  eigenspace with eigenvalue 2.03712994973



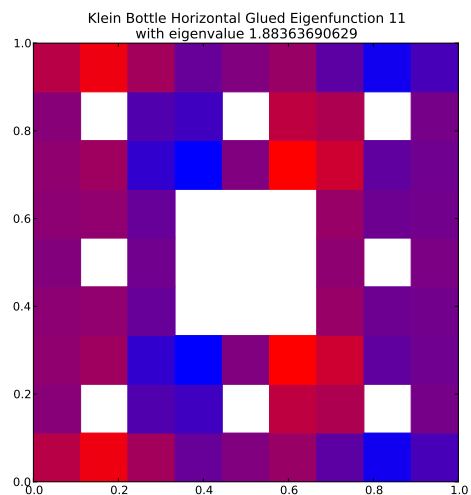
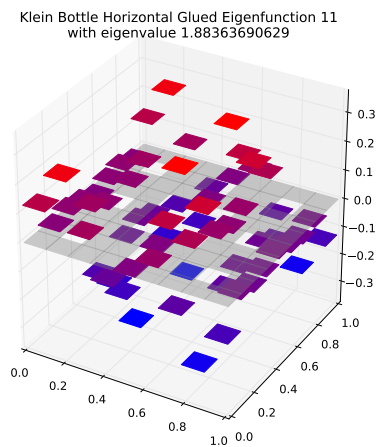
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.168367018111$   
Dot Value: 0.011037897483484604

### 13 $M = 3$ Eigenfunction 12

$M = 3$  Eigenfunction 12 has eigenvalue 0.346340470289



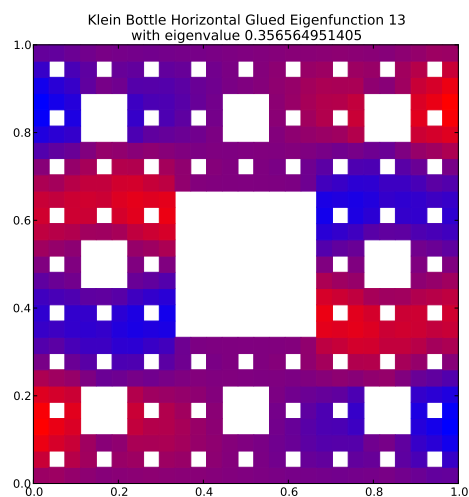
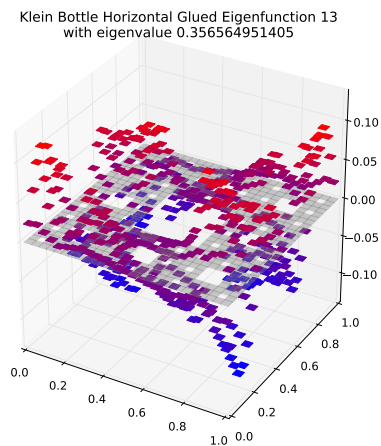
Compare to  $m = 2$  eigenspace with eigenvalue 1.88363690629



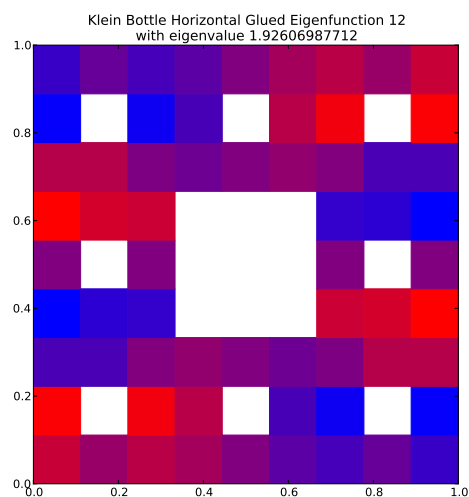
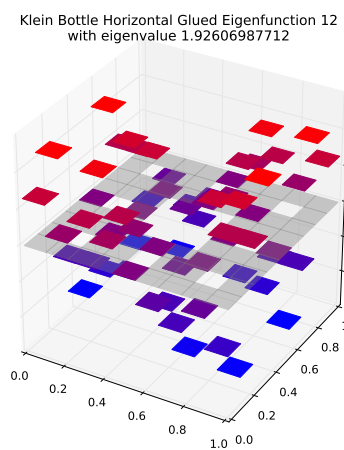
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.183867957318$   
Dot Value: 0.004090726671721723

## 14 $M = 3$ Eigenfunction 13

$M = 3$  Eigenfunction 13 has eigenvalue 0.356564951405



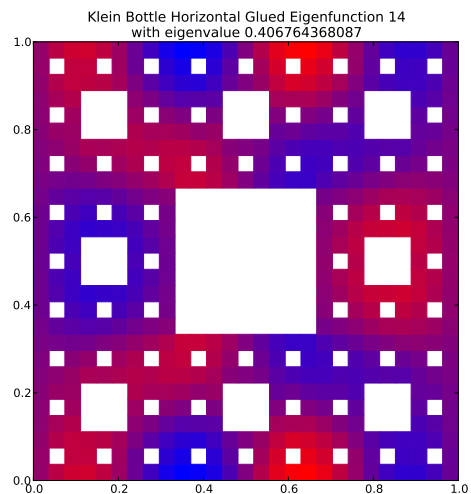
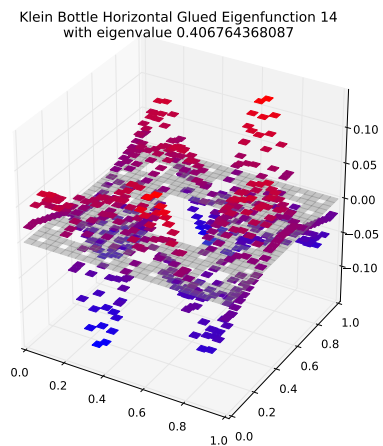
Compare to  $m = 2$  eigenspace with eigenvalue 1.92606987712



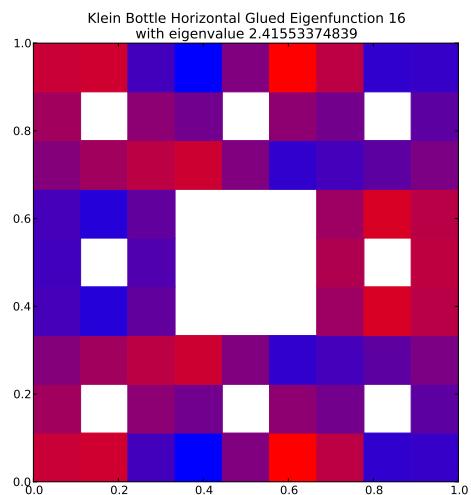
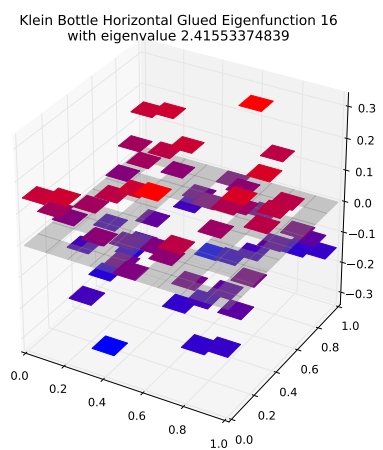
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.185125656987$   
Dot Value: 0.08910903449335483

# 15 $M = 3$ Eigenfunction 14

$M = 3$  Eigenfunction 14 has eigenvalue 0.406764368087



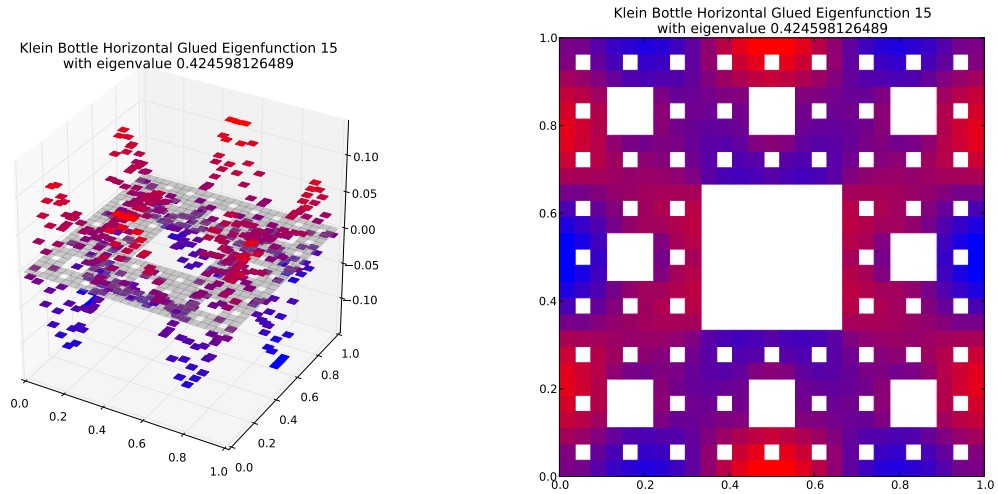
Compare to  $m = 2$  eigenspace with eigenvalue 2.41553374839



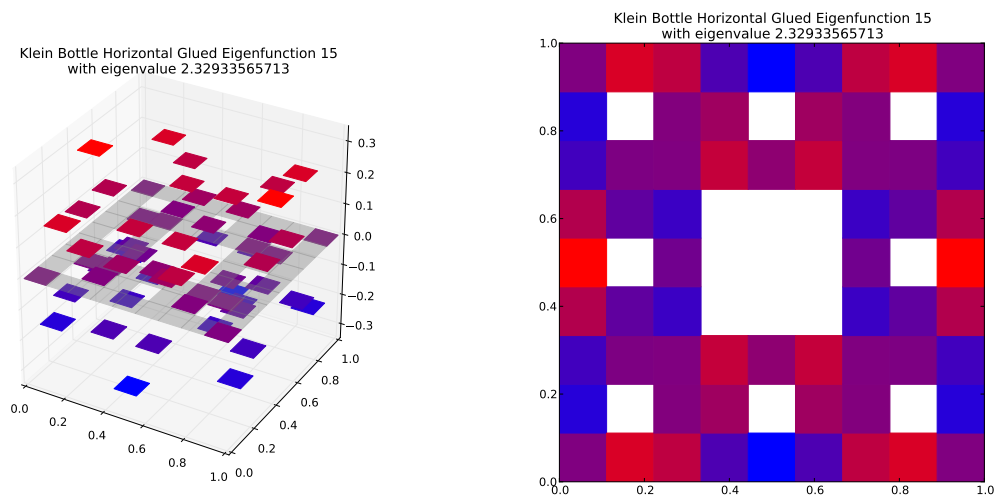
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.168395232879$   
Dot Value: 0.005514441531876635

## 16 $M = 3$ Eigenfunction 15

$M = 3$  Eigenfunction 15 has eigenvalue 0.424598126489



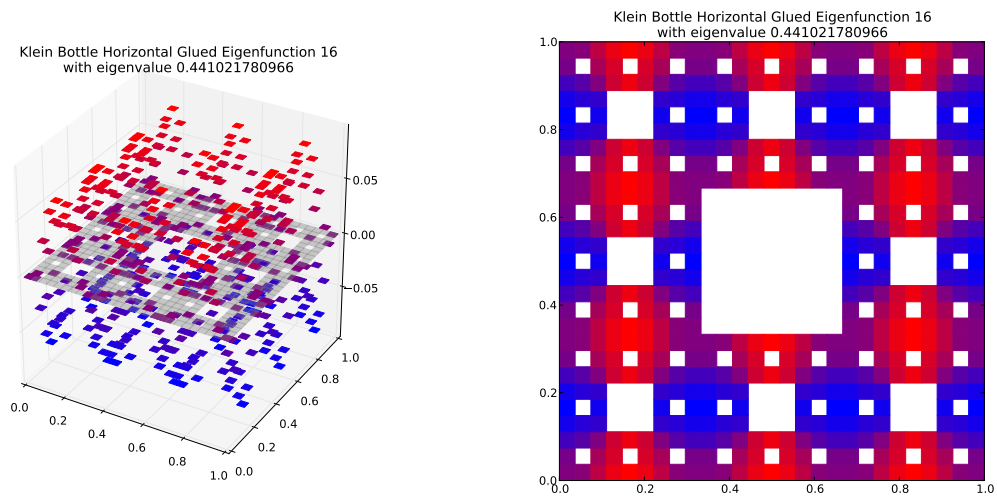
Compare to  $m = 2$  eigenspace with eigenvalue 2.32933565713



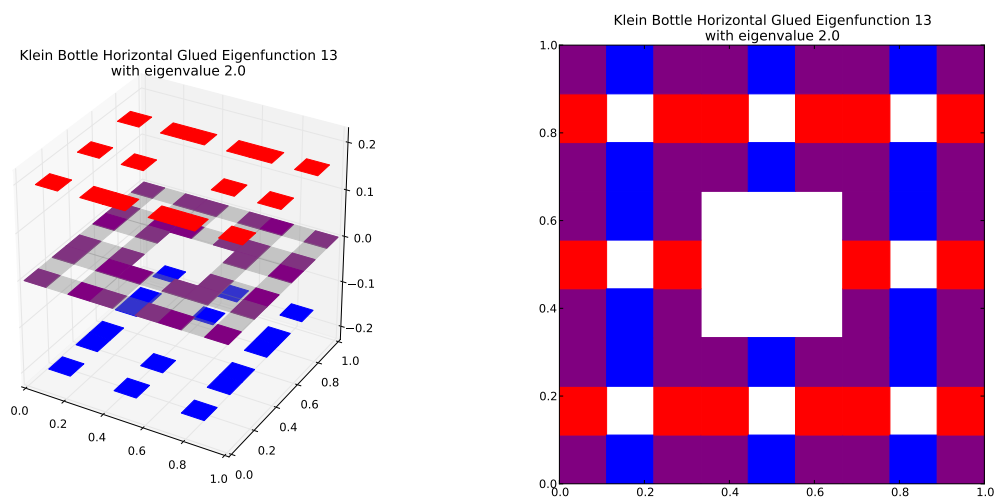
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.182282929122$   
Dot Value: 0.013337646831422134

# 17 $M = 3$ Eigenfunction 16

$M = 3$  Eigenfunction 16 has eigenvalue 0.441021780966



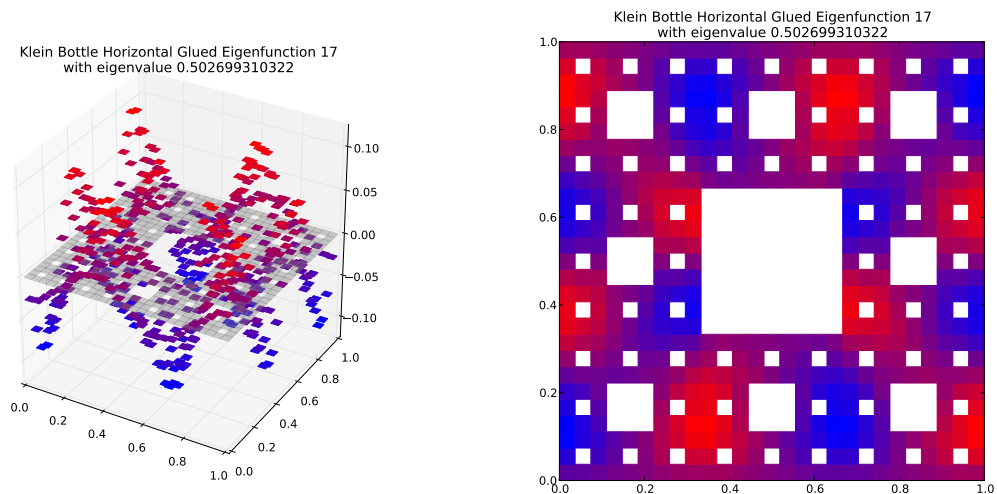
Compare to  $m = 2$  eigenspace with eigenvalue 2.0



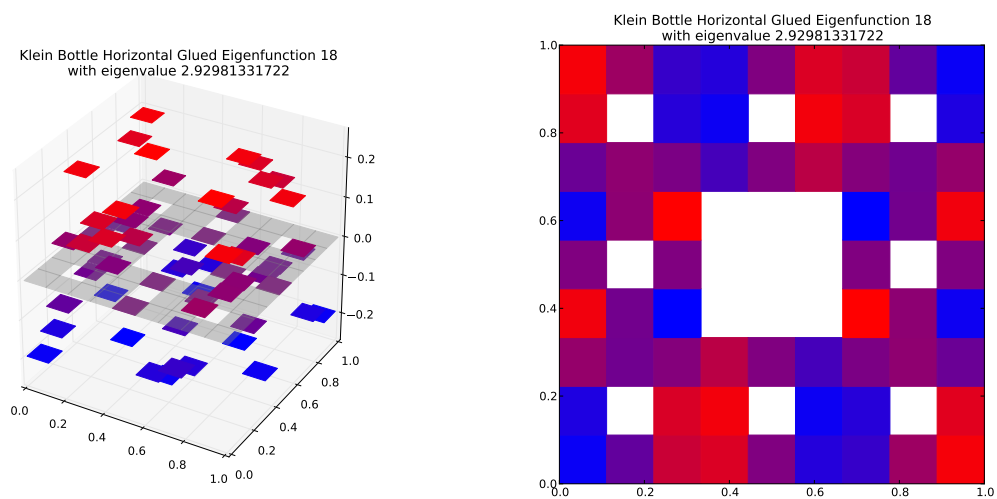
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.220510890483$   
Dot Value: 1.1102230246251565e-16

# 18 $M = 3$ Eigenfunction 17

$M = 3$  Eigenfunction 17 has eigenvalue 0.502699310322



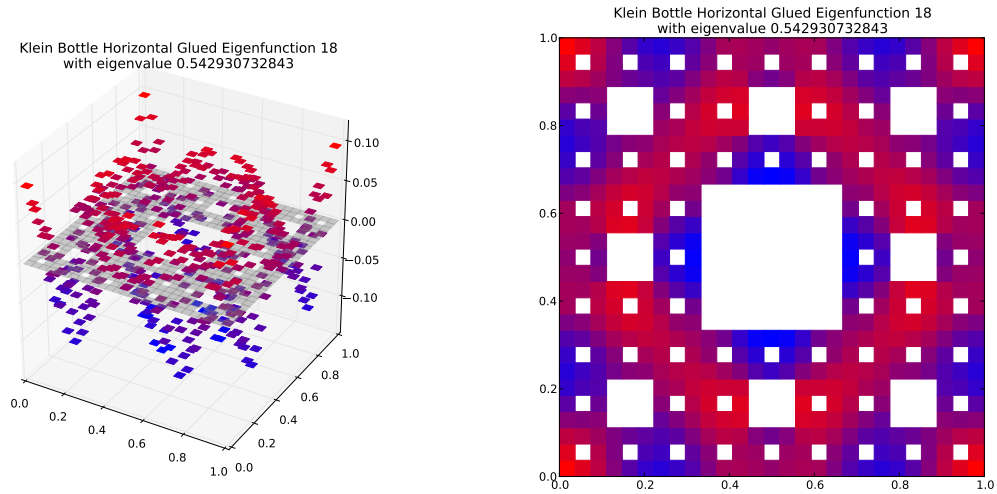
Compare to  $m = 2$  eigenspace with eigenvalue 2.92981331722



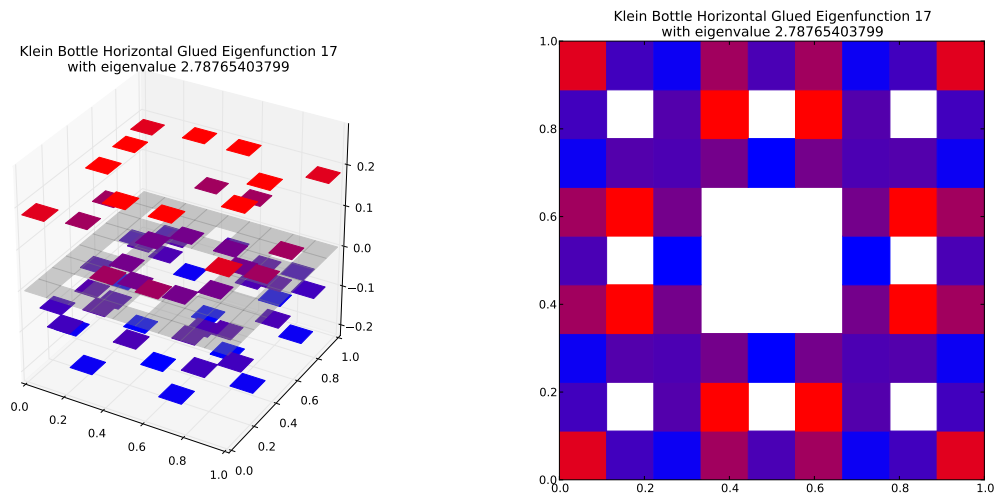
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.17158066262$   
Dot Value: 0.006241499711309606

## 19 $M = 3$ Eigenfunction 18

$M = 3$  Eigenfunction 18 has eigenvalue 0.542930732843



Compare to  $m = 2$  eigenspace with eigenvalue 2.78765403799

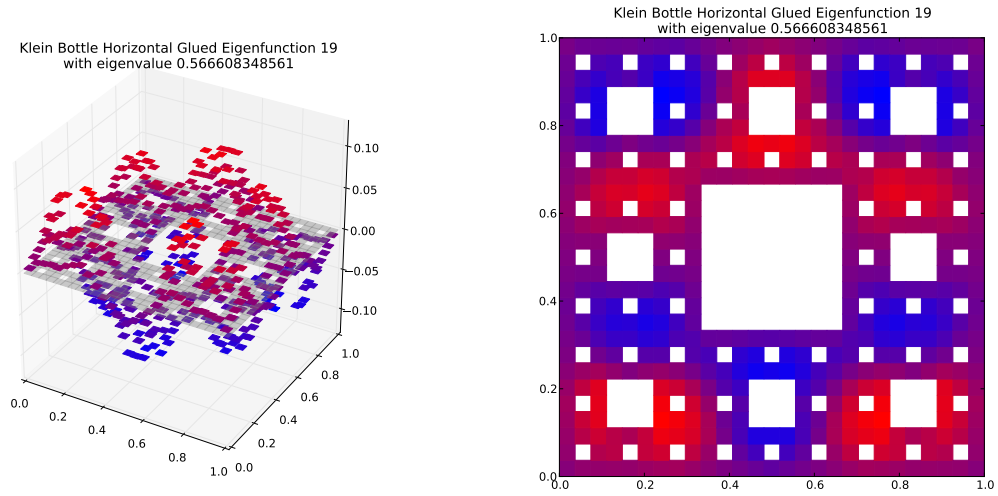


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.194762594441$

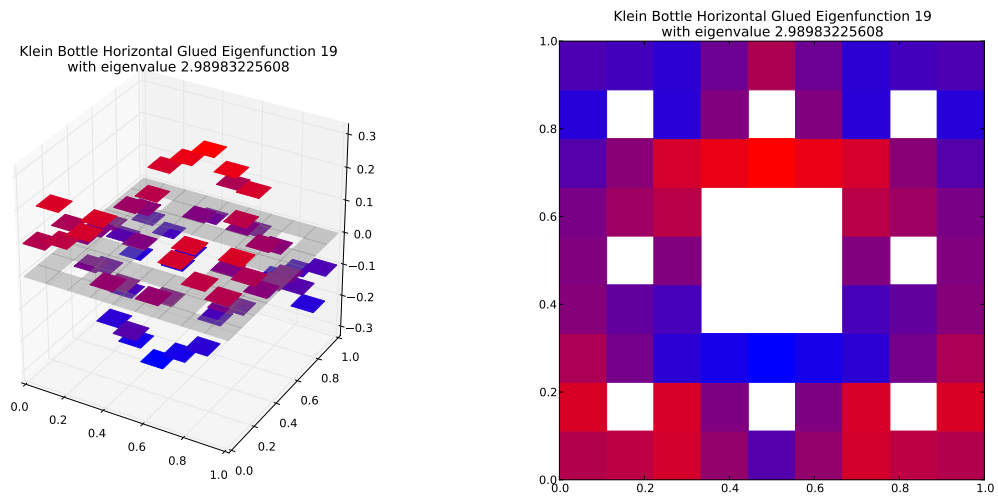
Dot Value: 0.11830647910249703

## 20 $M = 3$ Eigenfunction 19

$M = 3$  Eigenfunction 19 has eigenvalue 0.566608348561



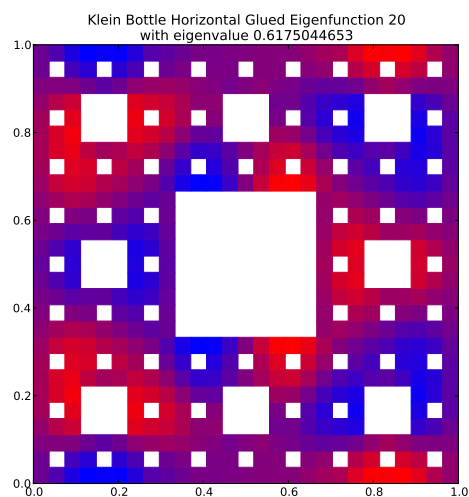
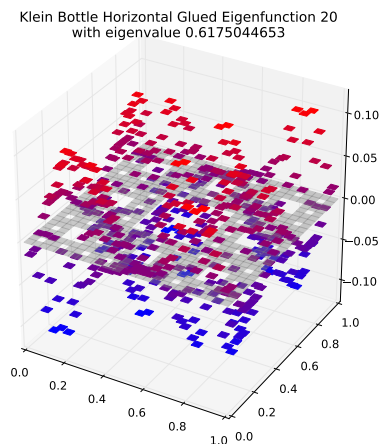
Compare to  $m = 2$  eigenspace with eigenvalue 2.98983225608



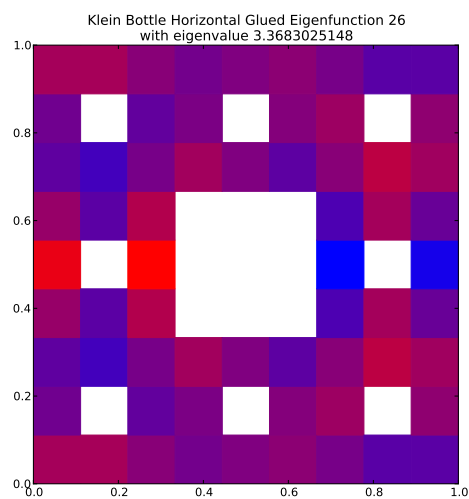
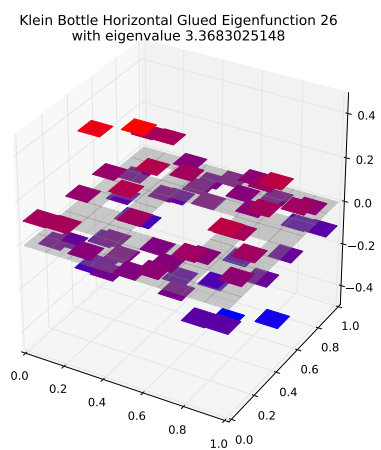
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.189511751841$   
Dot Value: 0.13082409185296662

## 21 $M = 3$ Eigenfunction 20

$M = 3$  Eigenfunction 20 has eigenvalue 0.6175044653



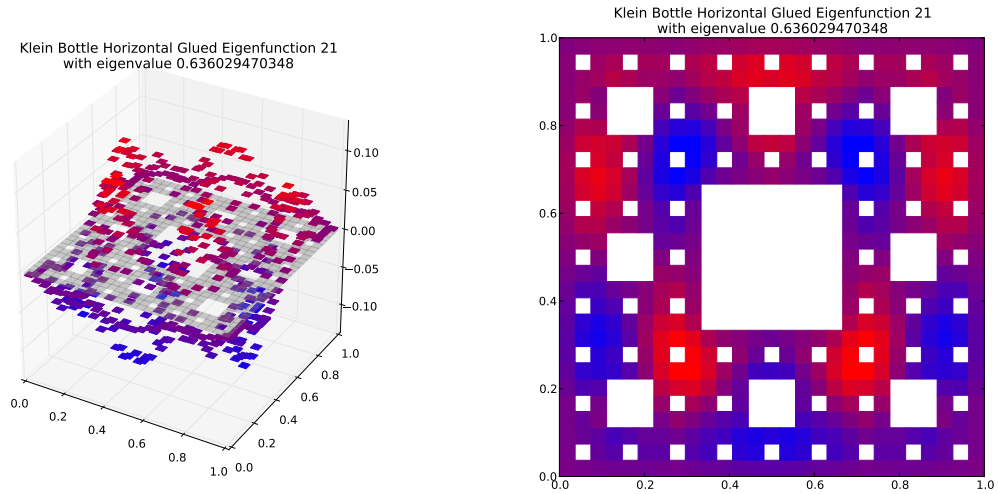
Compare to  $m = 2$  eigenspace with eigenvalue 3.3683025148



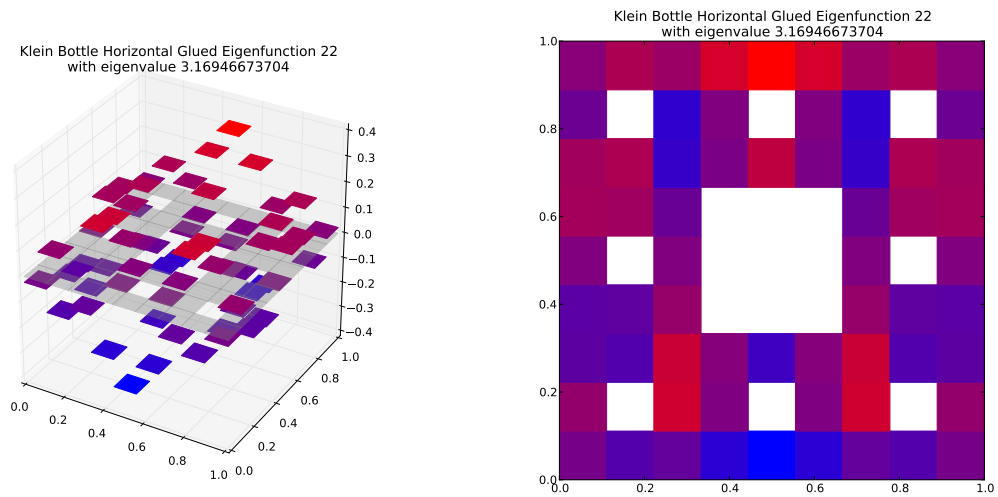
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.18332808962$   
Dot Value: 0.25247824050527956

## 22 $M = 3$ Eigenfunction 21

$M = 3$  Eigenfunction 21 has eigenvalue 0.636029470348



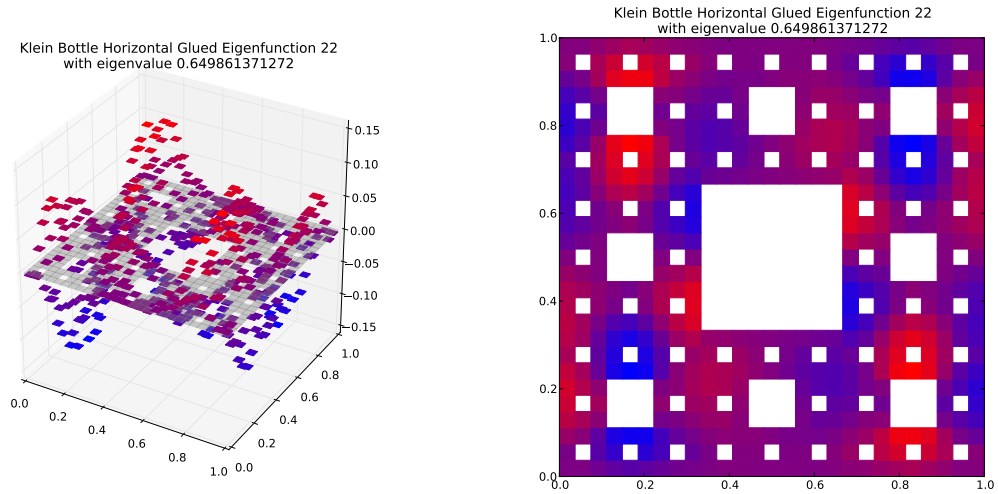
Compare to  $m = 2$  eigenspace with eigenvalue 3.16946673704



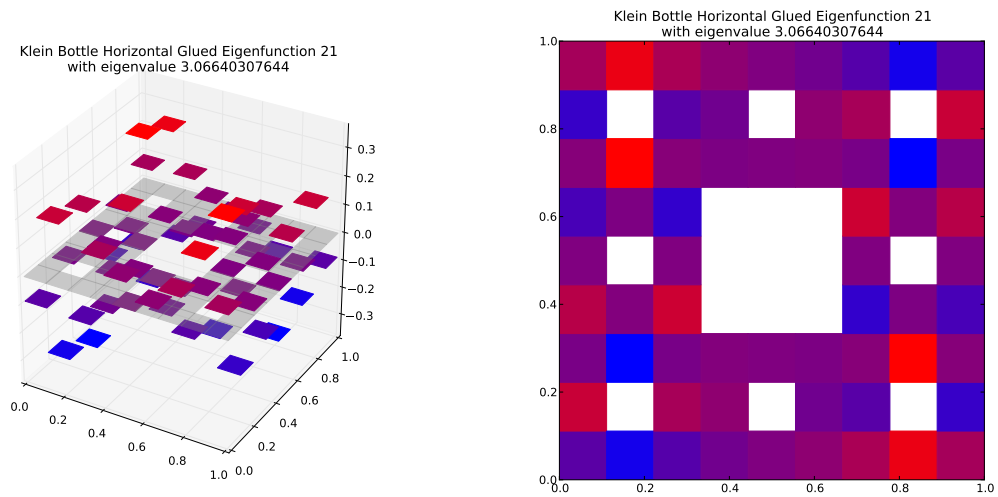
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.200673969193$   
Dot Value: 0.15402010369246466

## 23 $M = 3$ Eigenfunction 22

$M = 3$  Eigenfunction 22 has eigenvalue 0.649861371272



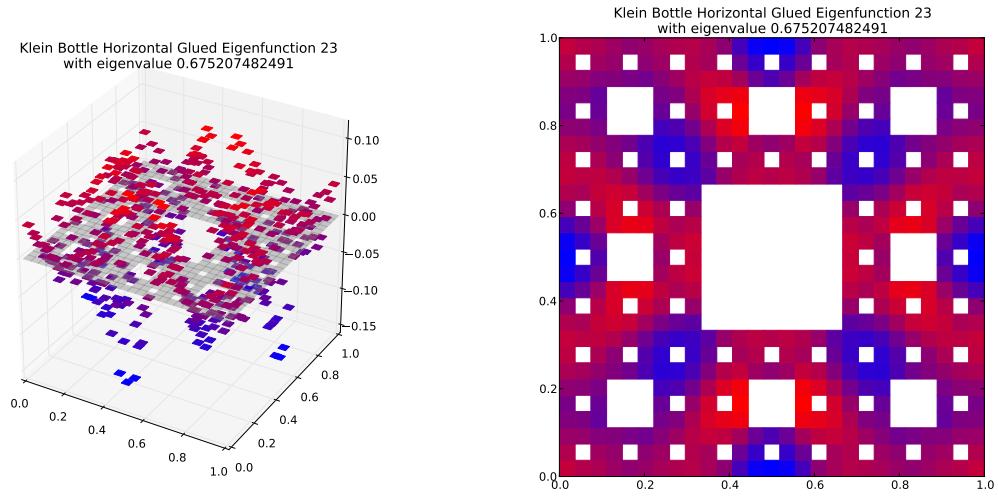
Compare to  $m = 2$  eigenspace with eigenvalue 3.06640307644



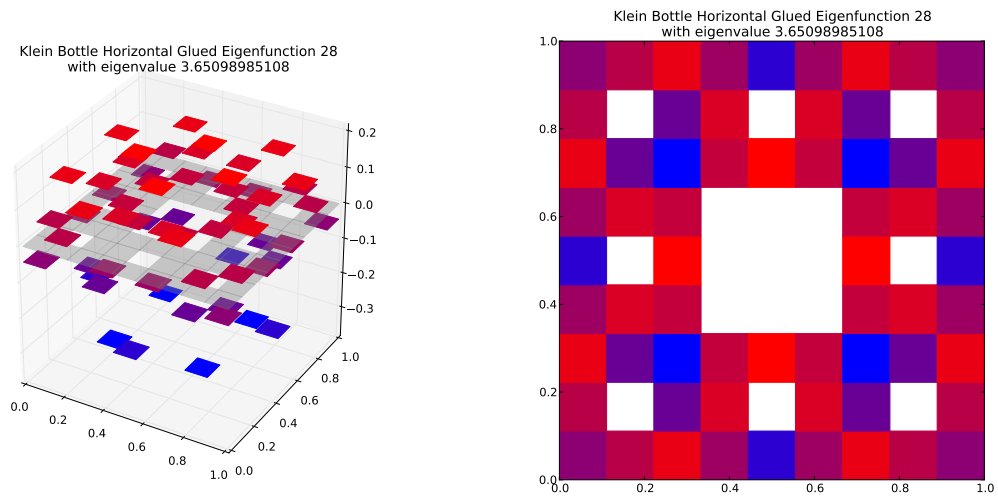
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.211929532769$   
Dot Value: 0.061781236190881494

## 24 $M = 3$ Eigenfunction 23

$M = 3$  Eigenfunction 23 has eigenvalue 0.675207482491



Compare to  $m = 2$  eigenspace with eigenvalue 3.65098985108

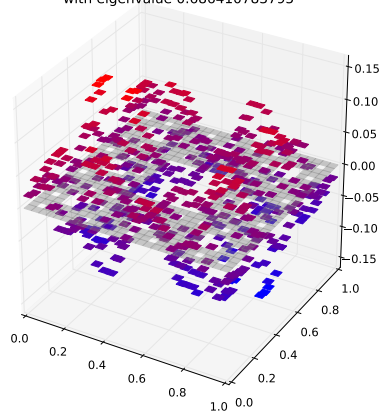


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.184938197594$   
Dot Value: 0.1350729871903632

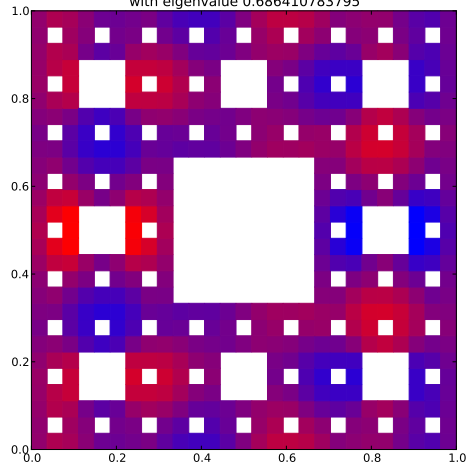
## 25 $M = 3$ Eigenfunction 24

$M = 3$  Eigenfunction 24 has eigenvalue 0.686410783795

Klein Bottle Horizontal Glued Eigenfunction 24  
with eigenvalue 0.686410783795

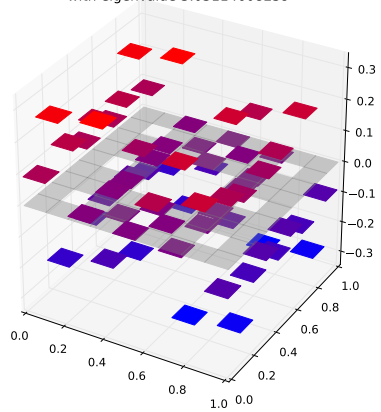


Klein Bottle Horizontal Glued Eigenfunction 24  
with eigenvalue 0.686410783795

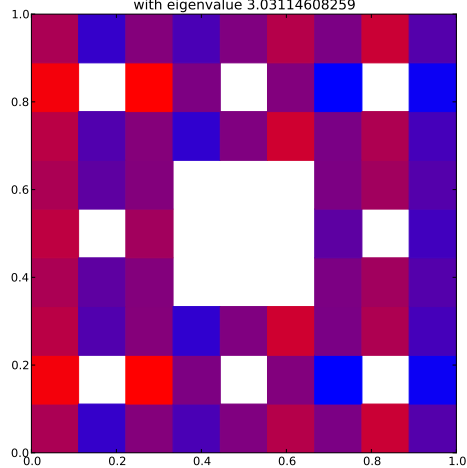


Compare to  $m = 2$  eigenspace with eigenvalue 3.03114608259

Klein Bottle Horizontal Glued Eigenfunction 20  
with eigenvalue 3.03114608259



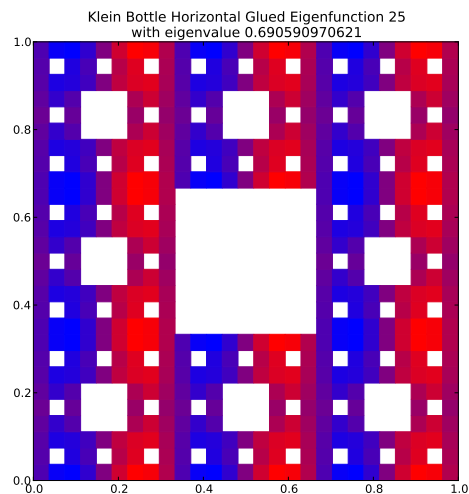
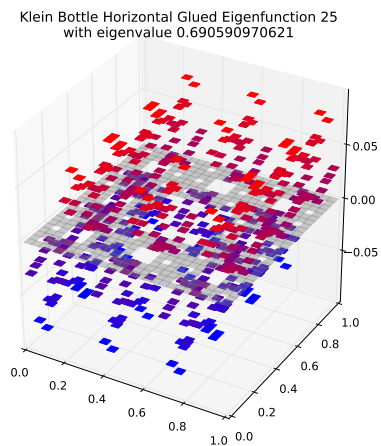
Klein Bottle Horizontal Glued Eigenfunction 20  
with eigenvalue 3.03114608259



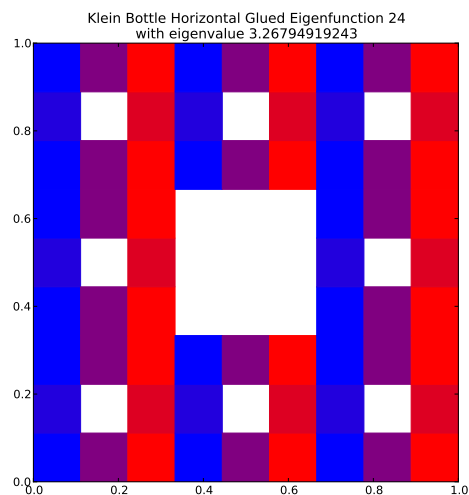
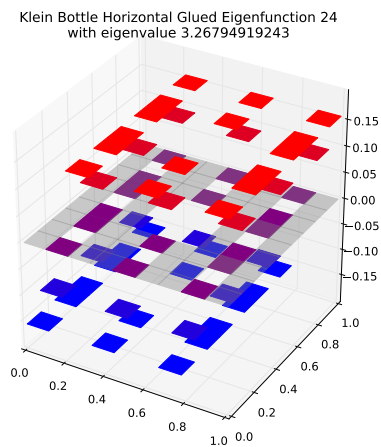
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.226452557908$   
Dot Value: 0.23316645227194455

## 26 $M = 3$ Eigenfunction 25

$M = 3$  Eigenfunction 25 has eigenvalue 0.690590970621



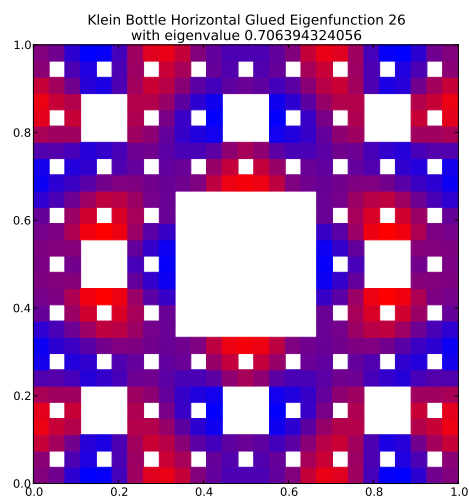
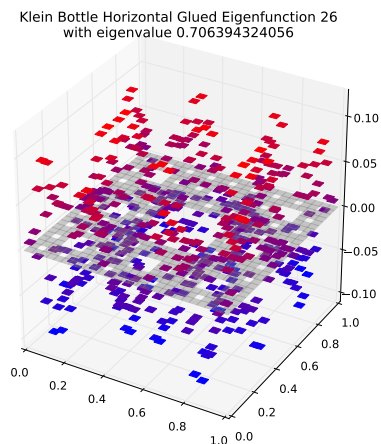
Compare to  $m = 2$  eigenspace with eigenvalue 3.26794919243



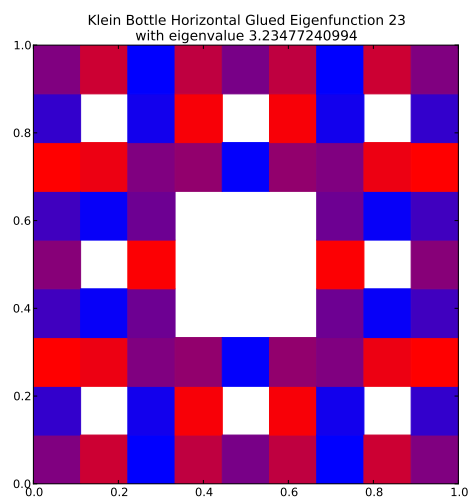
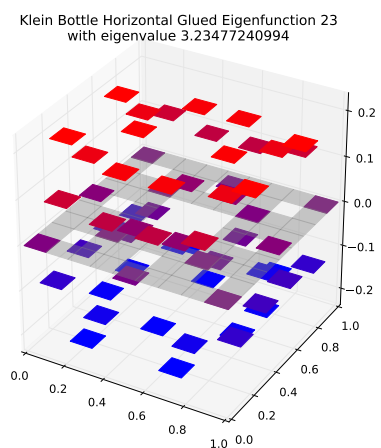
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.211322431885$   
Dot Value: 0.007208422818863469

## 27 $M = 3$ Eigenfunction 26

$M = 3$  Eigenfunction 26 has eigenvalue 0.706394324056



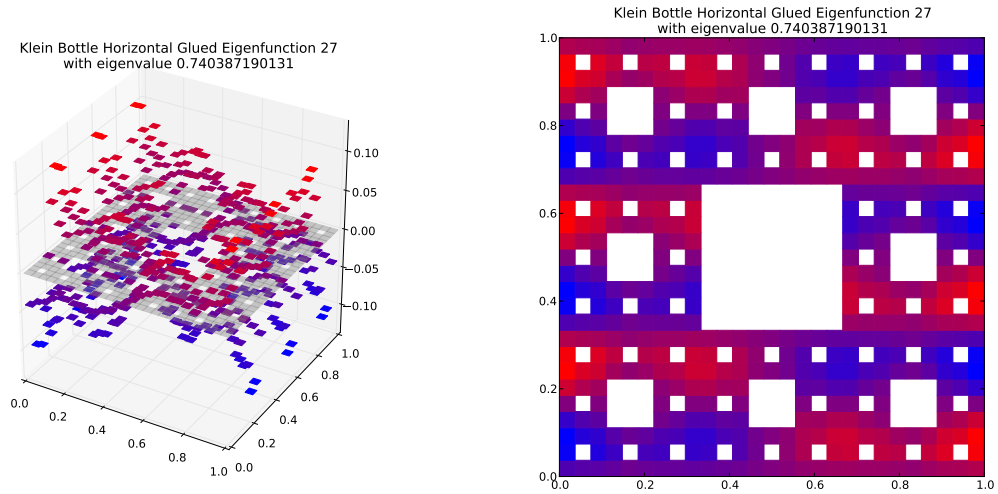
Compare to  $m = 2$  eigenspace with eigenvalue 3.23477240994



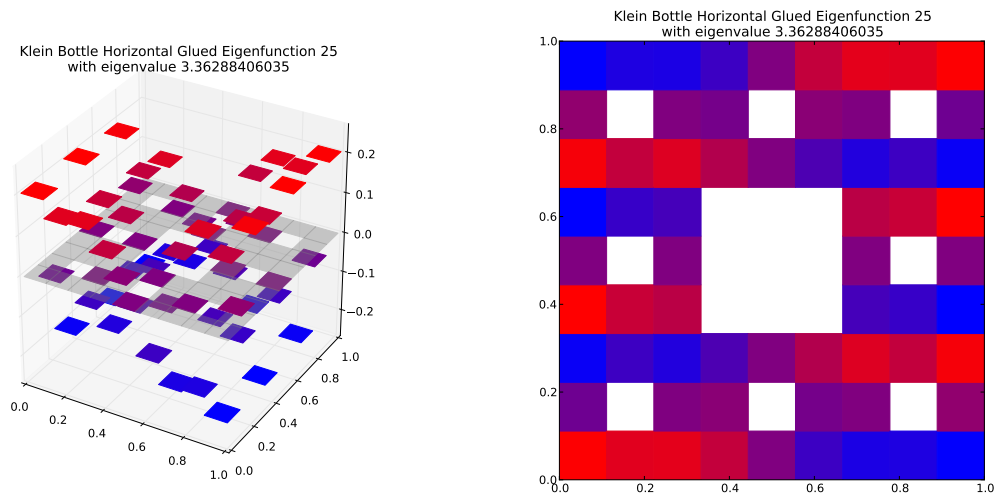
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.218375277928$   
Dot Value: 0.10046186052377393

## 28 $M = 3$ Eigenfunction 27

$M = 3$  Eigenfunction 27 has eigenvalue 0.740387190131



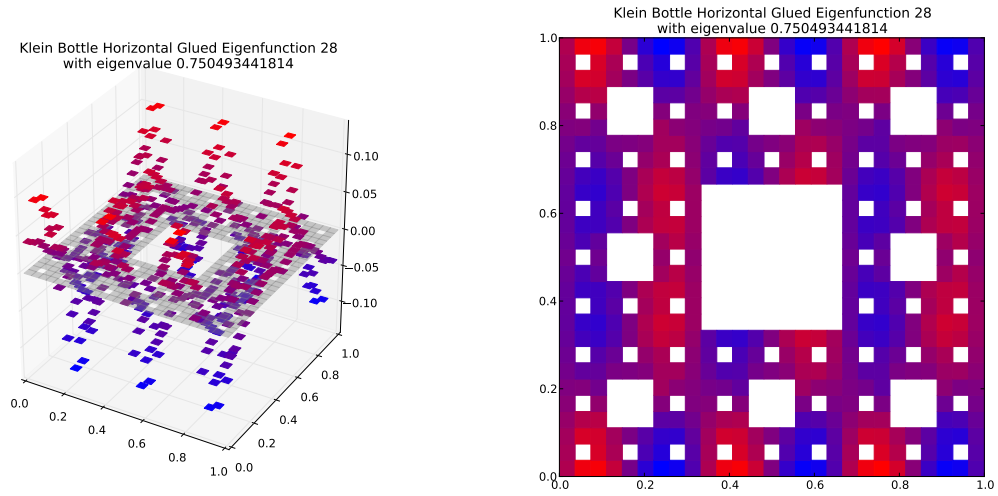
Compare to  $m = 2$  eigenspace with eigenvalue 3.36288406035



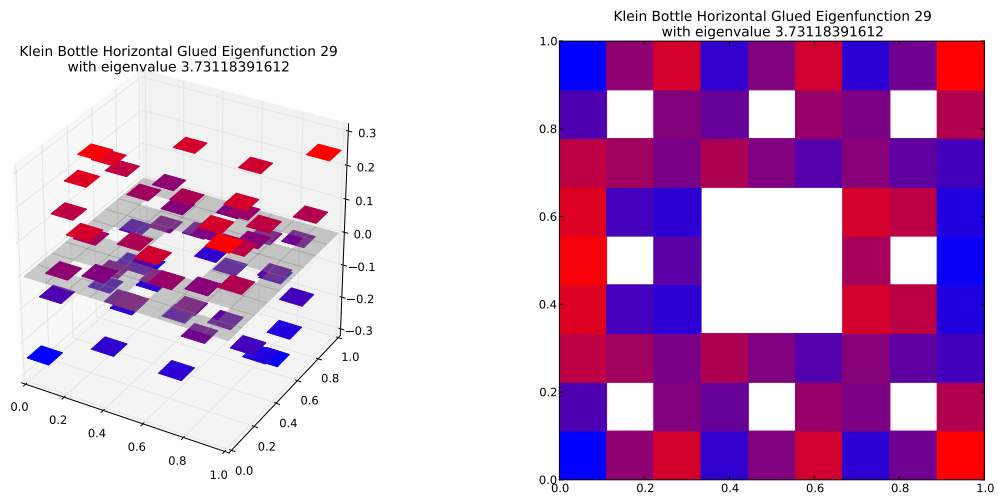
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.22016435204$   
Dot Value: 0.019977467290337114

## 29 $M = 3$ Eigenfunction 28

$M = 3$  Eigenfunction 28 has eigenvalue 0.750493441814



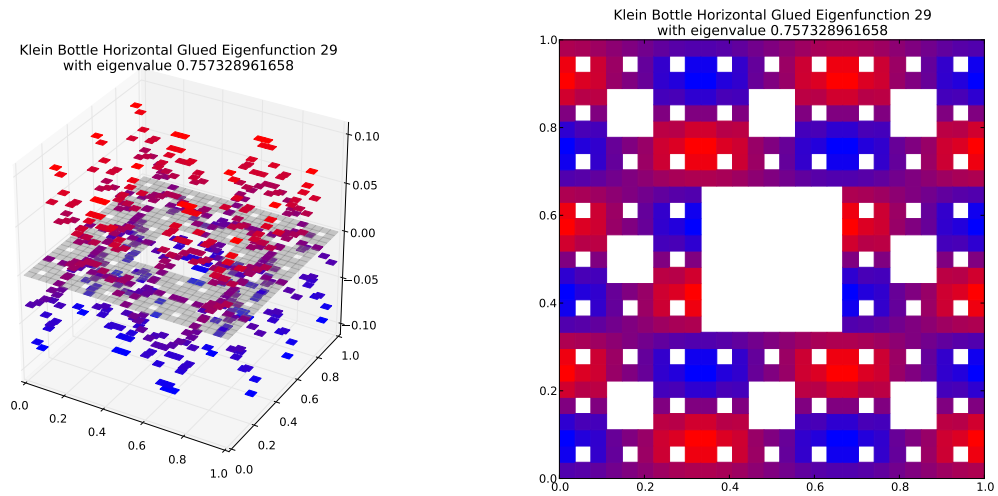
Compare to  $m = 2$  eigenspace with eigenvalue 3.73118391612



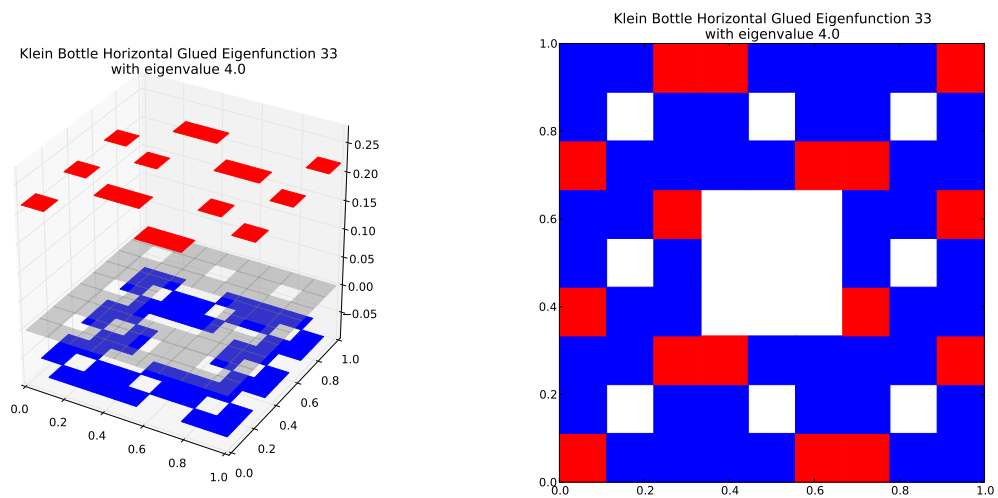
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.201140833228$   
Dot Value: 0.09871017803940663

### 30 $M = 3$ Eigenfunction 29

$M = 3$  Eigenfunction 29 has eigenvalue 0.757328961658



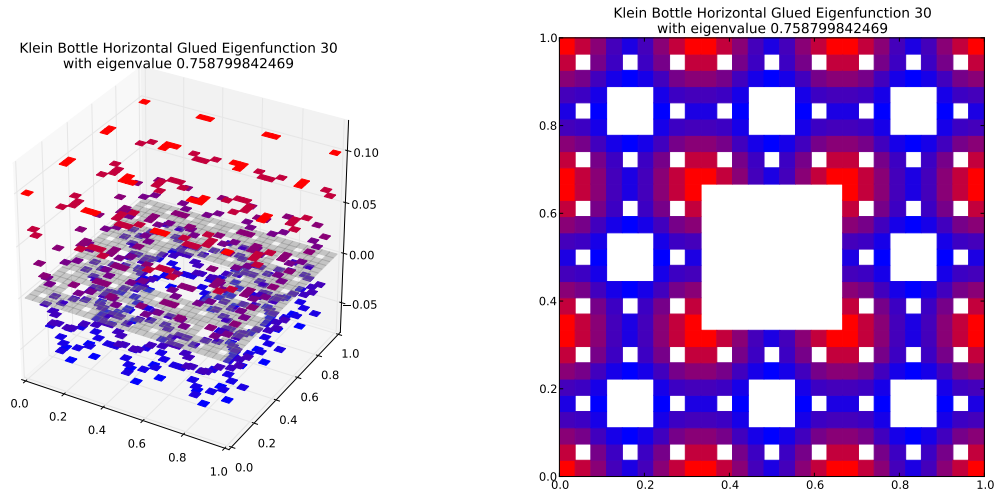
Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )



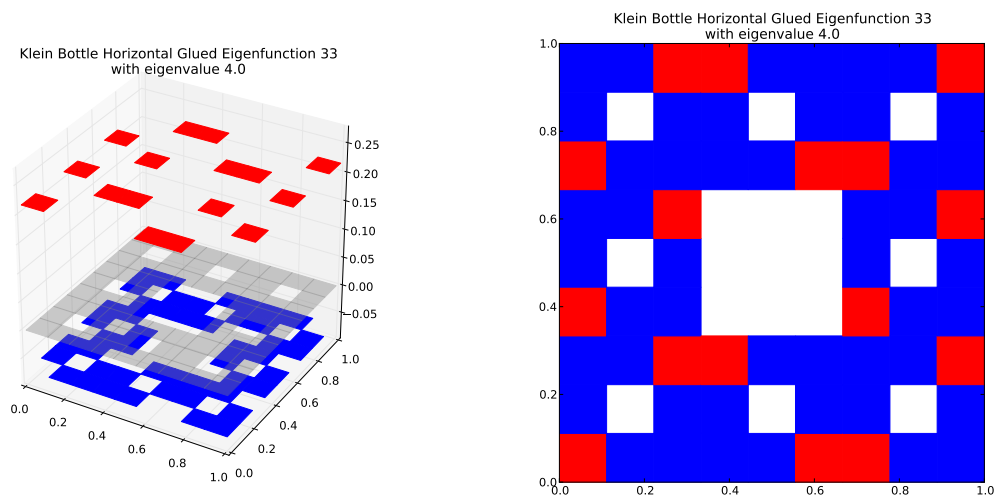
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.189332240415$   
Dot Value: 0.0

### 31 $M = 3$ Eigenfunction 30

$M = 3$  Eigenfunction 30 has eigenvalue 0.758799842469



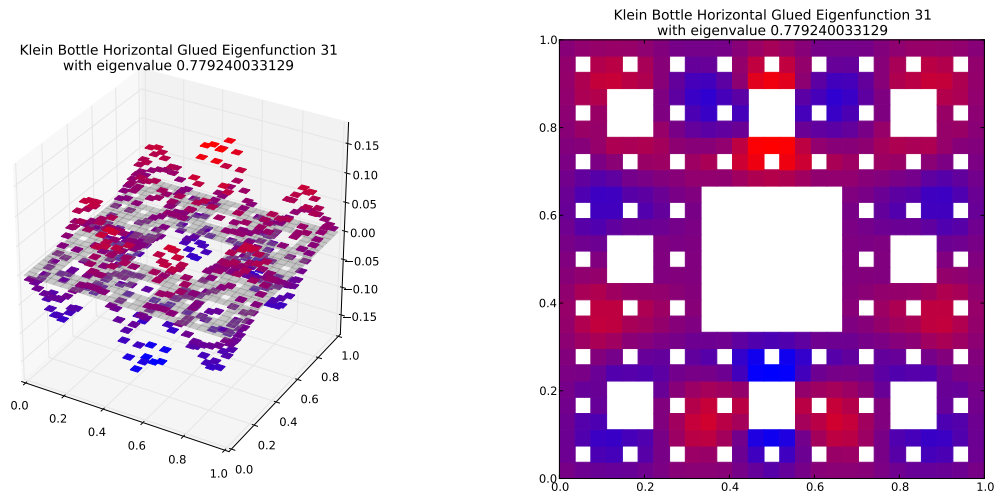
Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )



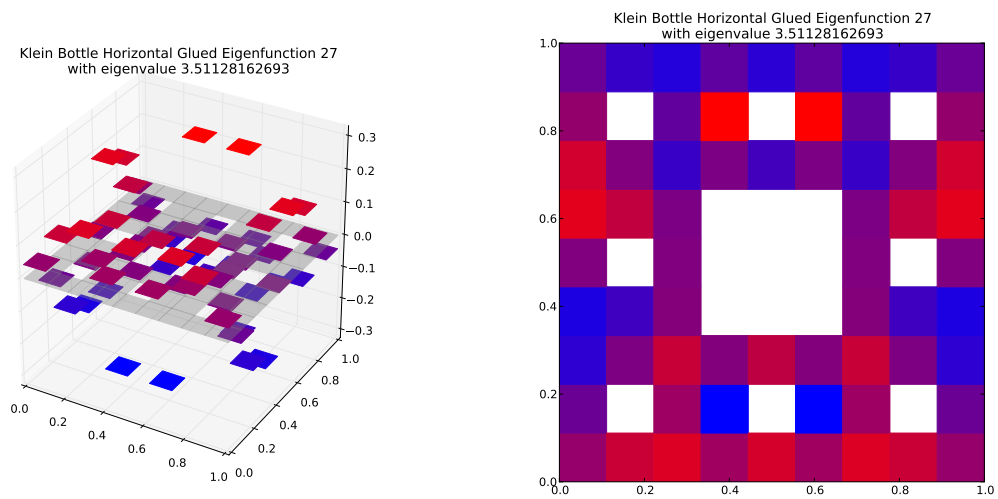
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.189699960617$   
Dot Value: 0.0

## 32 $M = 3$ Eigenfunction 31

$M = 3$  Eigenfunction 31 has eigenvalue 0.779240033129



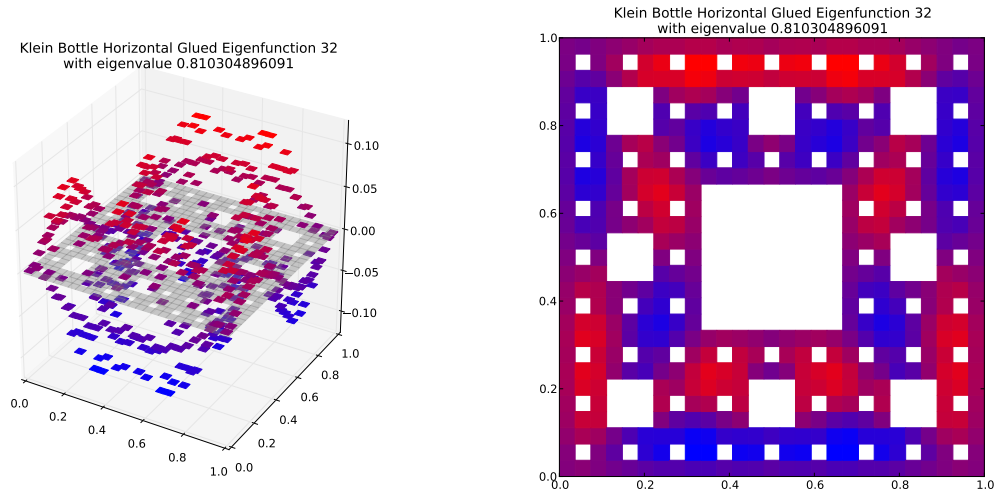
Compare to  $m = 2$  eigenspace with eigenvalue 3.51128162693



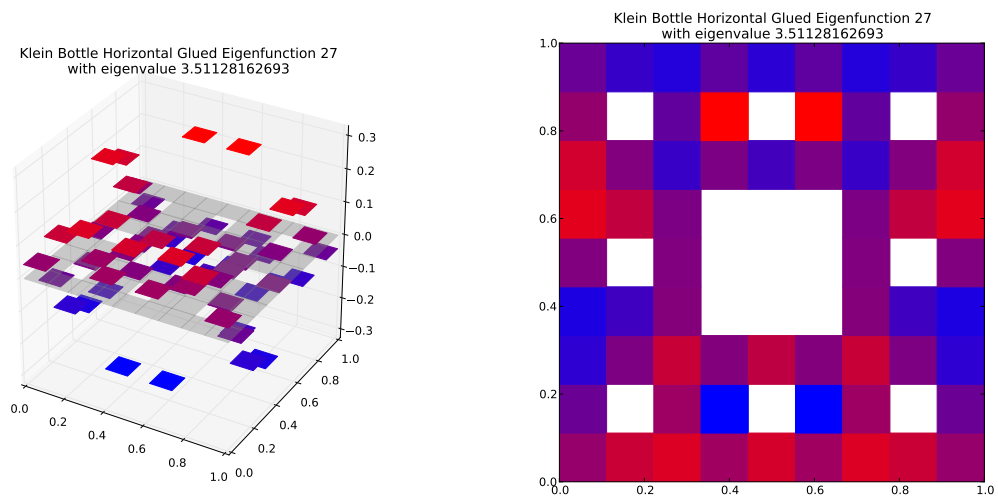
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.221924674783$   
Dot Value: 0.37599425933930164

### 33 $M = 3$ Eigenfunction 32

$M = 3$  Eigenfunction 32 has eigenvalue 0.810304896091



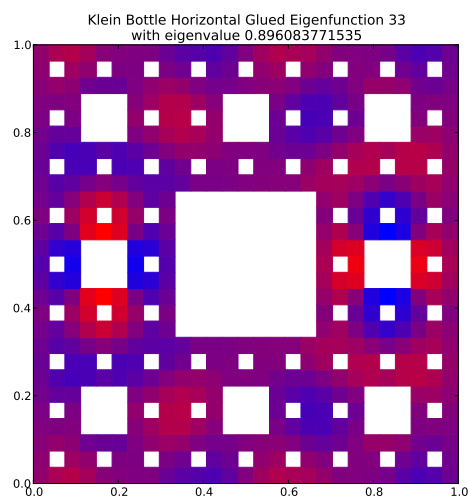
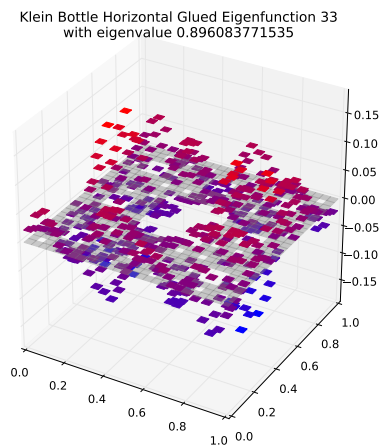
Compare to  $m = 2$  eigenspace with eigenvalue 3.51128162693



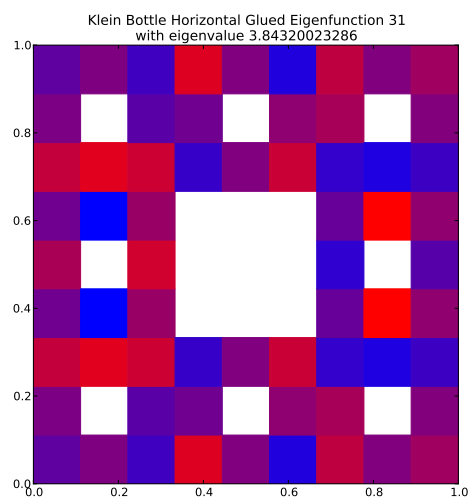
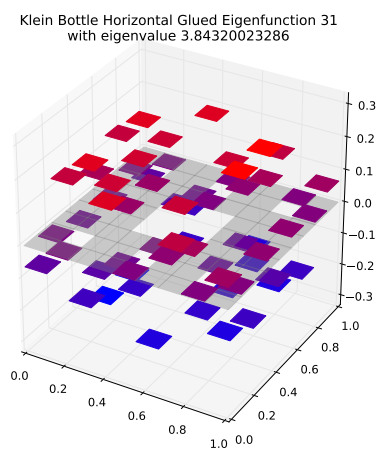
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.230771832677$   
Dot Value: 0.37998690472455043

### 34 $M = 3$ Eigenfunction 33

$M = 3$  Eigenfunction 33 has eigenvalue 0.896083771535



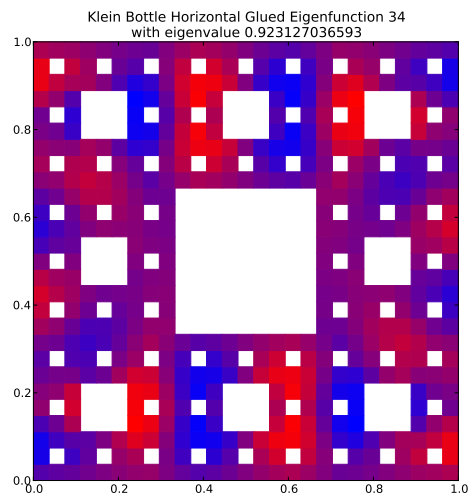
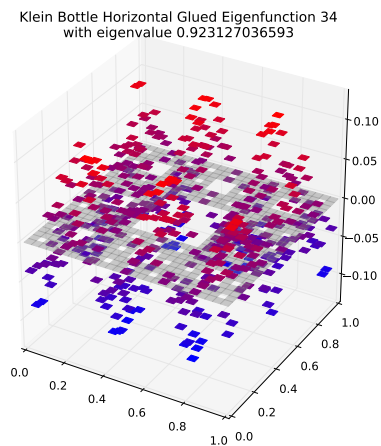
Compare to  $m = 2$  eigenspace with eigenvalue 3.84320023286



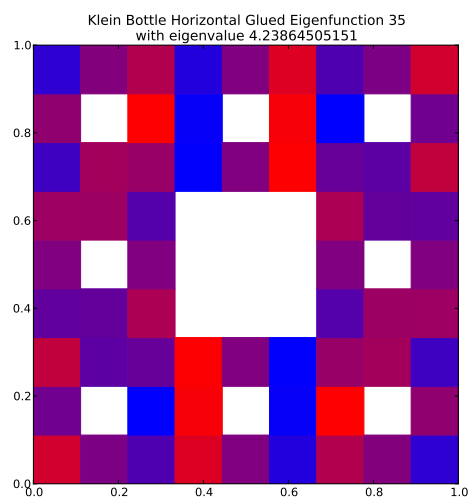
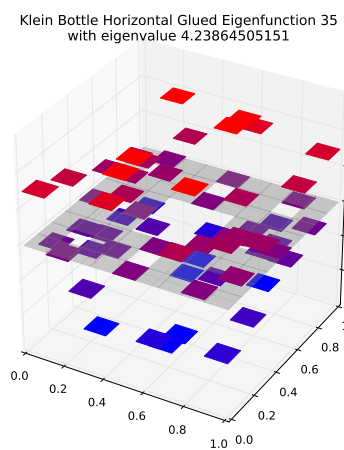
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.233160834003$   
Dot Value: 0.19248424337543024

### 35 $M = 3$ Eigenfunction 34

$M = 3$  Eigenfunction 34 has eigenvalue 0.923127036593



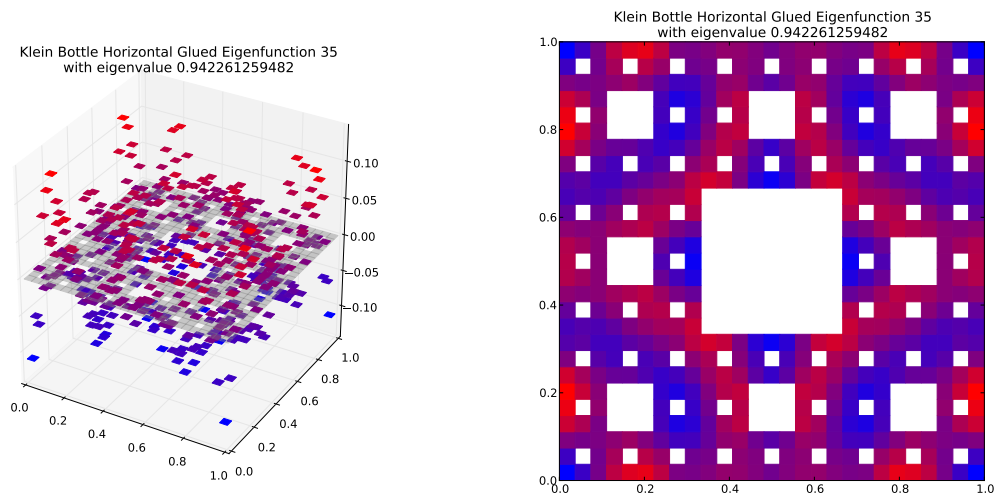
Compare to  $m = 2$  eigenspace with eigenvalue 4.23864505151



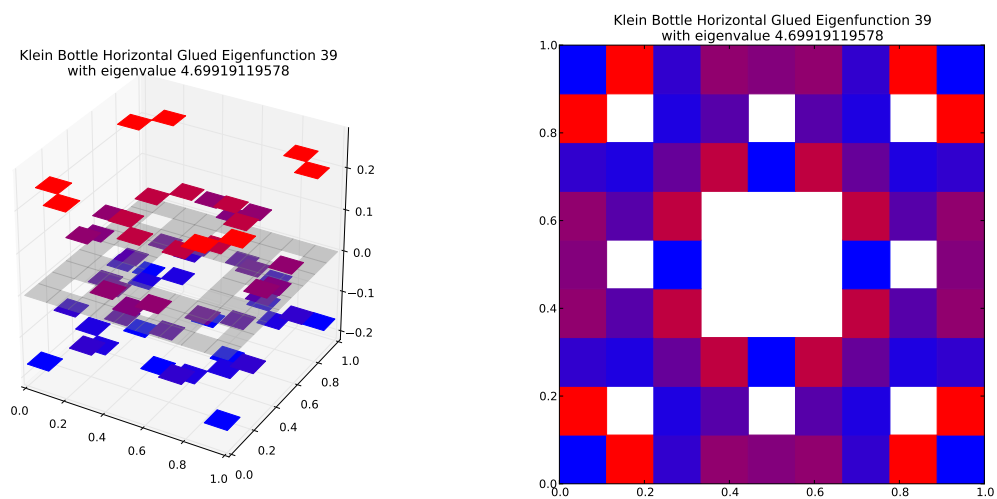
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.217788237839$   
Dot Value: 0.11199410507997432

### 36 $M = 3$ Eigenfunction 35

$M = 3$  Eigenfunction 35 has eigenvalue 0.942261259482



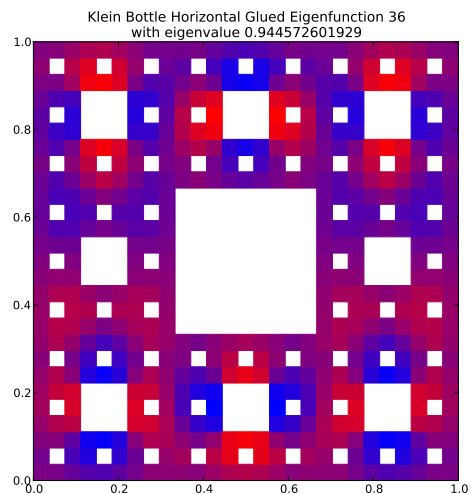
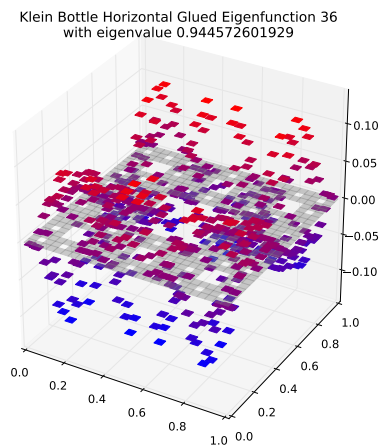
Compare to  $m = 2$  eigenspace with eigenvalue 4.69919119578



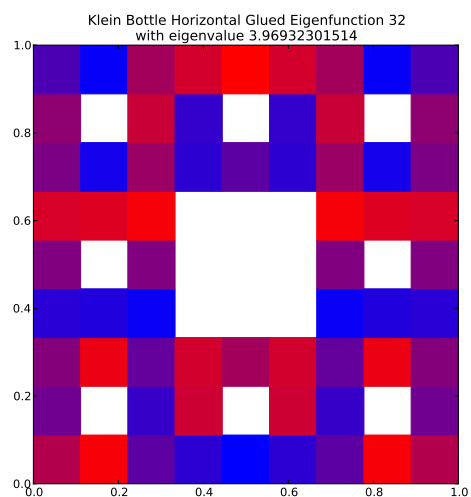
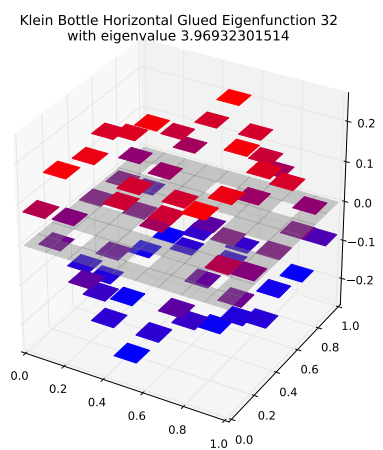
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.200515624971$   
Dot Value: 0.16409481134191473

### 37 $M = 3$ Eigenfunction 36

$M = 3$  Eigenfunction 36 has eigenvalue 0.944572601929



Compare to  $m = 2$  eigenspace with eigenvalue 3.96932301514

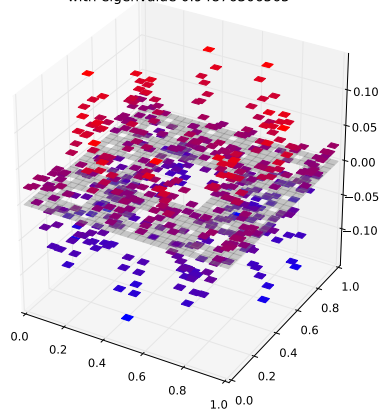


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.2379681871$   
Dot Value: 0.18684722051624647

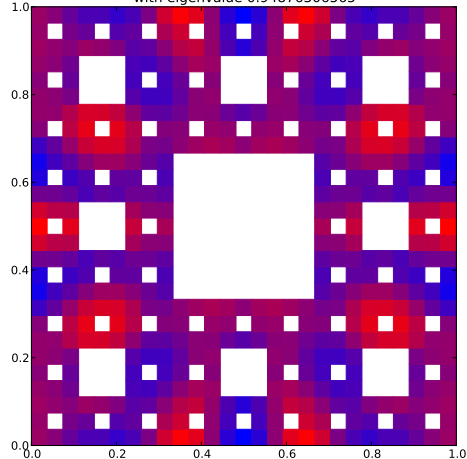
### 38 $M = 3$ Eigenfunction 37

$M = 3$  Eigenfunction 37 has eigenvalue 0.94876306565

Klein Bottle Horizontal Glued Eigenfunction 37  
with eigenvalue 0.94876306565

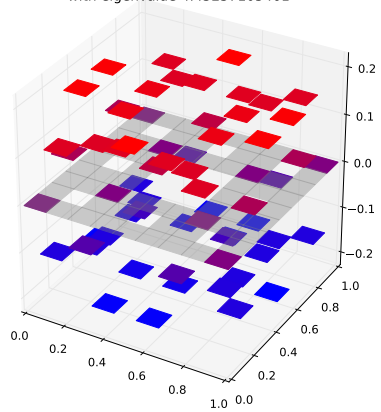


Klein Bottle Horizontal Glued Eigenfunction 37  
with eigenvalue 0.94876306565

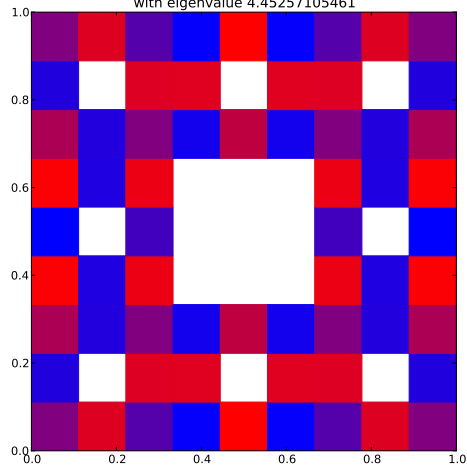


Compare to  $m = 2$  eigenspace with eigenvalue 4.45257105461

Klein Bottle Horizontal Glued Eigenfunction 36  
with eigenvalue 4.45257105461



Klein Bottle Horizontal Glued Eigenfunction 36  
with eigenvalue 4.45257105461

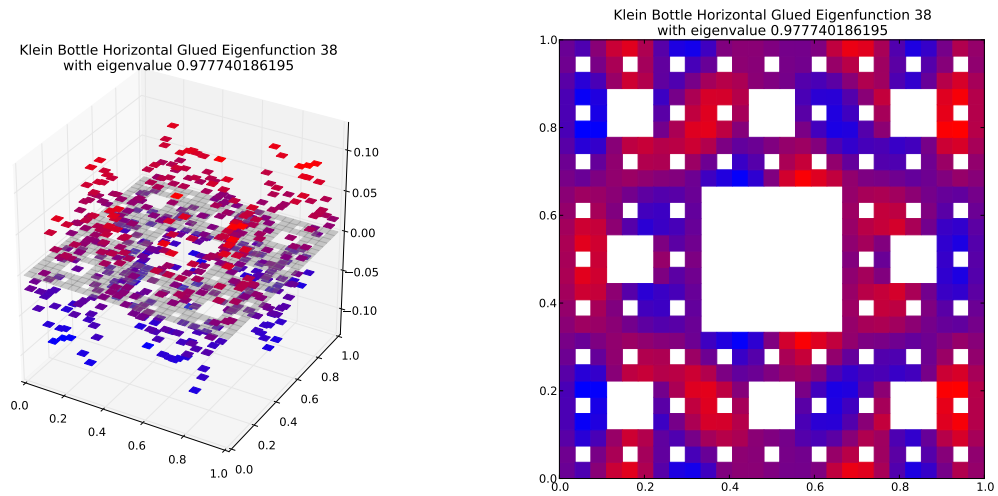


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.213082071912$

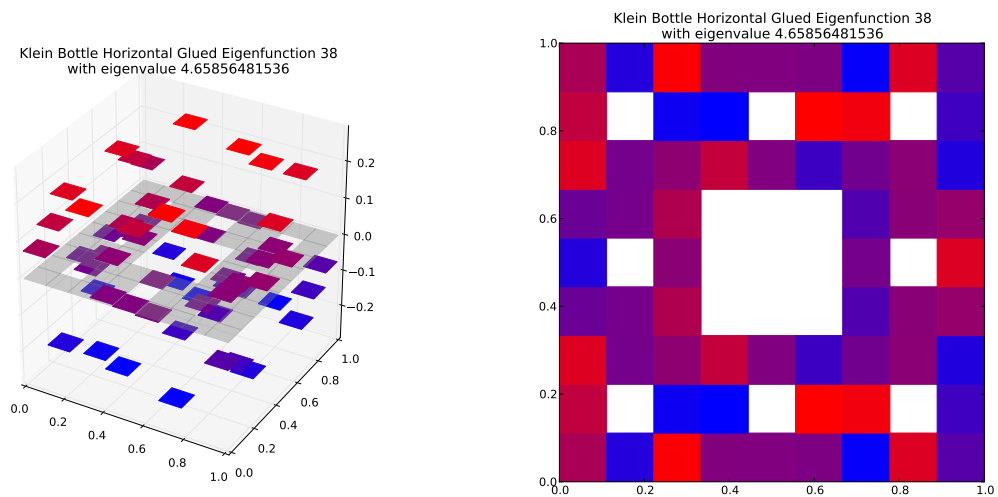
Dot Value: 0.2726867736389439

### 39 $M = 3$ Eigenfunction 38

$M = 3$  Eigenfunction 38 has eigenvalue 0.977740186195



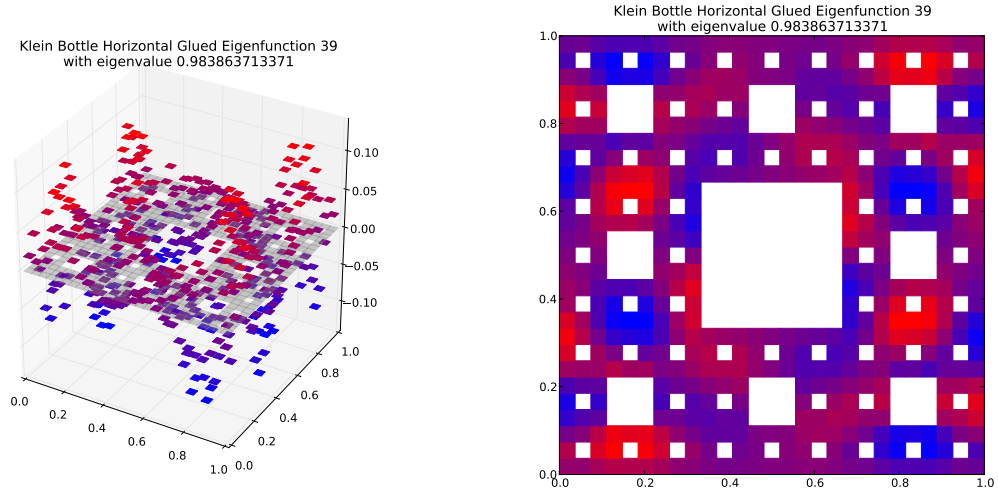
Compare to  $m = 2$  eigenspace with eigenvalue 4.65856481536



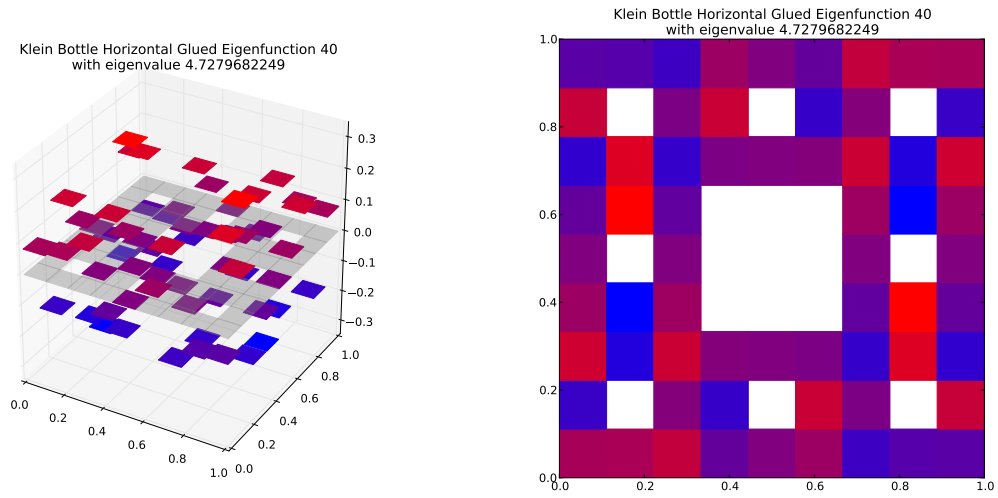
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.209880129385$   
Dot Value: 0.1834840592926278

## 40 $M = 3$ Eigenfunction 39

$M = 3$  Eigenfunction 39 has eigenvalue 0.983863713371



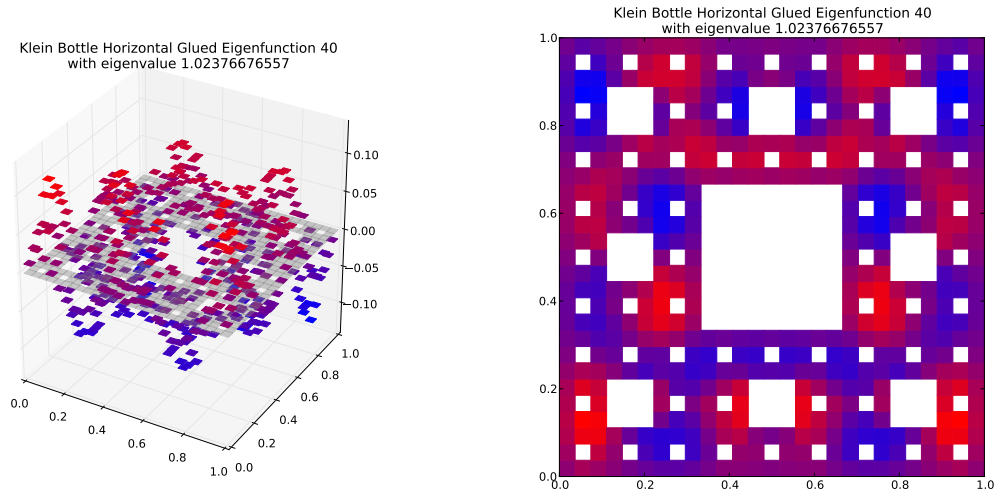
Compare to  $m = 2$  eigenspace with eigenvalue 4.7279682249



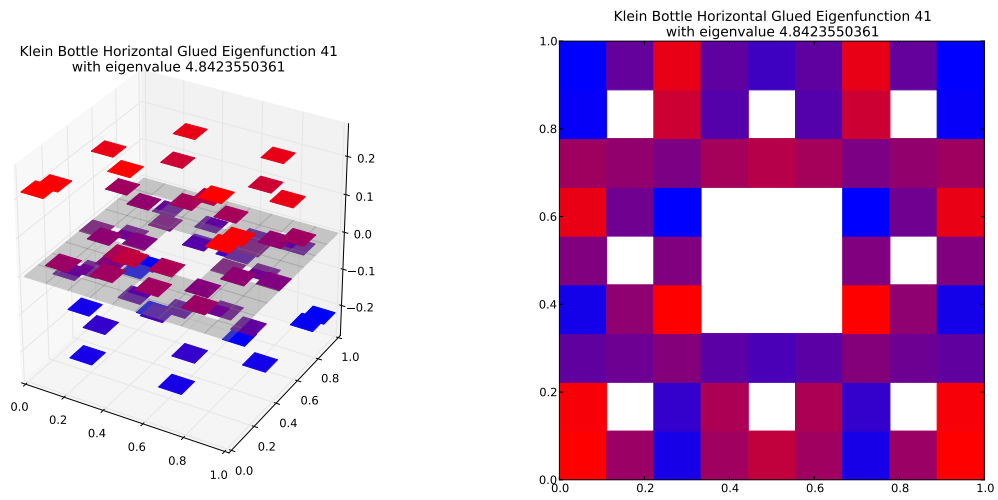
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.208094400506$   
Dot Value: 0.22084566563423513

# 41 $M = 3$ Eigenfunction 40

$M = 3$  Eigenfunction 40 has eigenvalue 1.02376676557



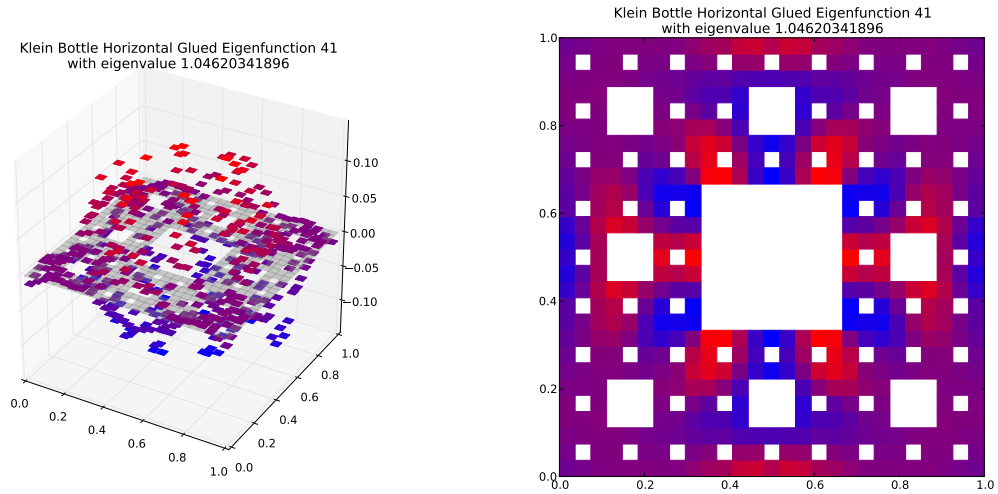
Compare to  $m = 2$  eigenspace with eigenvalue 4.8423550361



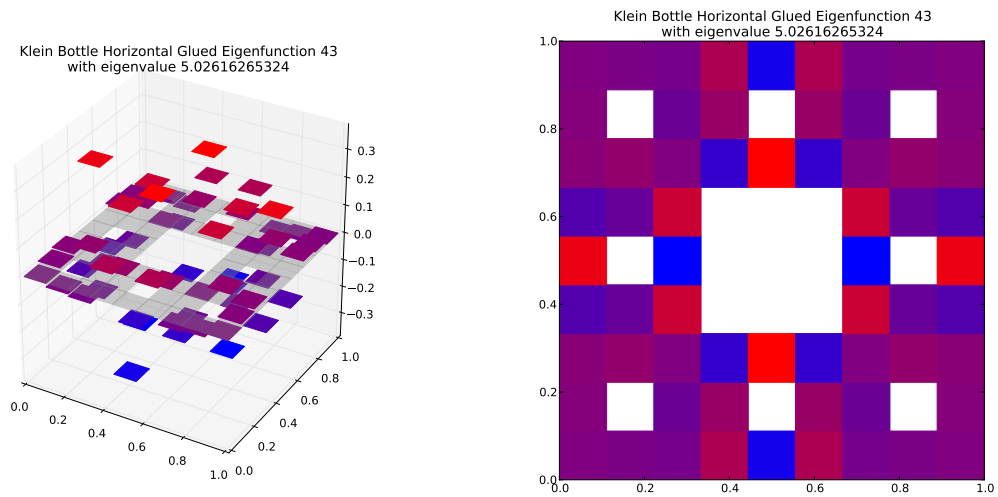
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.211419187139$   
Dot Value: 0.09879751700644401

## 42 $M = 3$ Eigenfunction 41

$M = 3$  Eigenfunction 41 has eigenvalue 1.04620341896



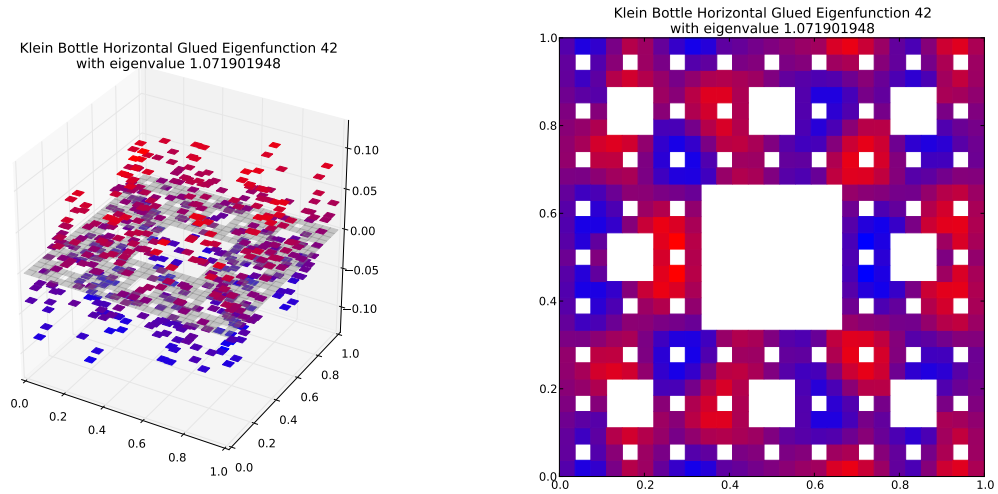
Compare to  $m = 2$  eigenspace with eigenvalue 5.02616265324



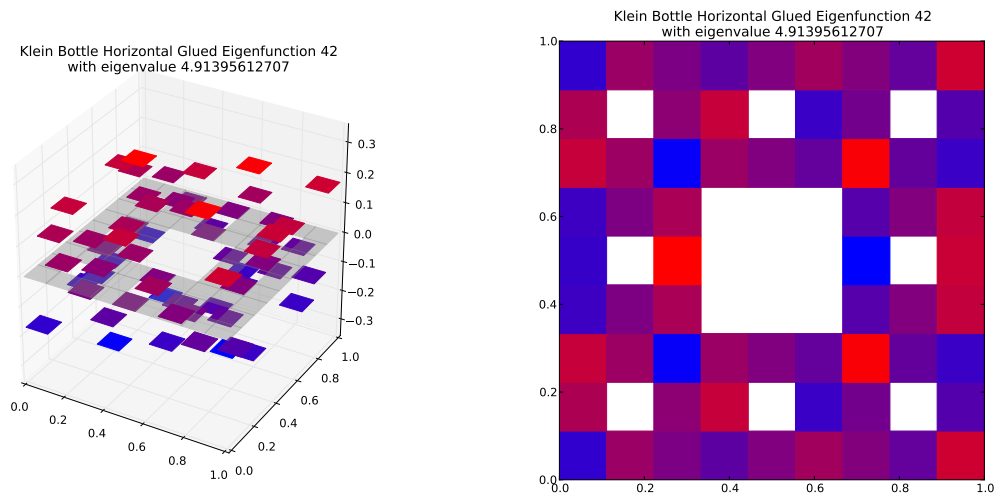
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.20815152456$   
Dot Value: 0.19302480660366017

### 43 $M = 3$ Eigenfunction 42

$M = 3$  Eigenfunction 42 has eigenvalue 1.071901948



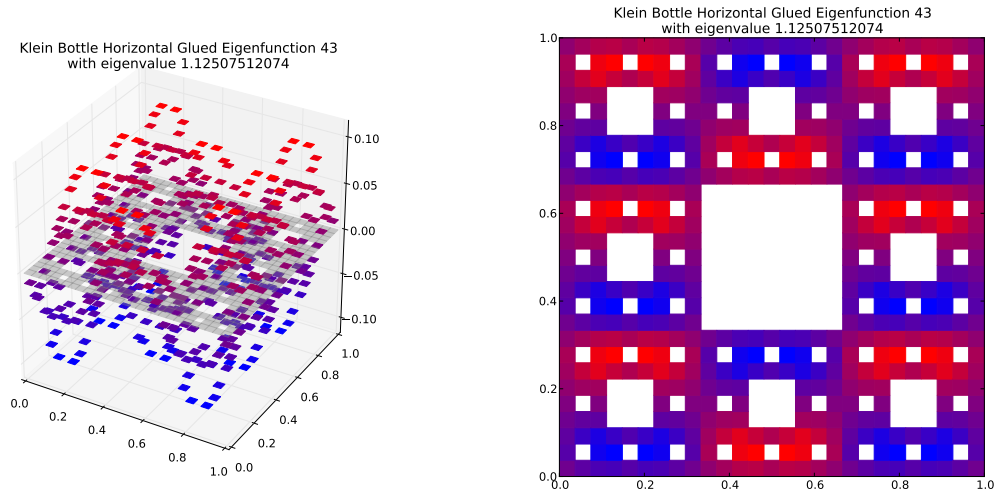
Compare to  $m = 2$  eigenspace with eigenvalue 4.91395612707



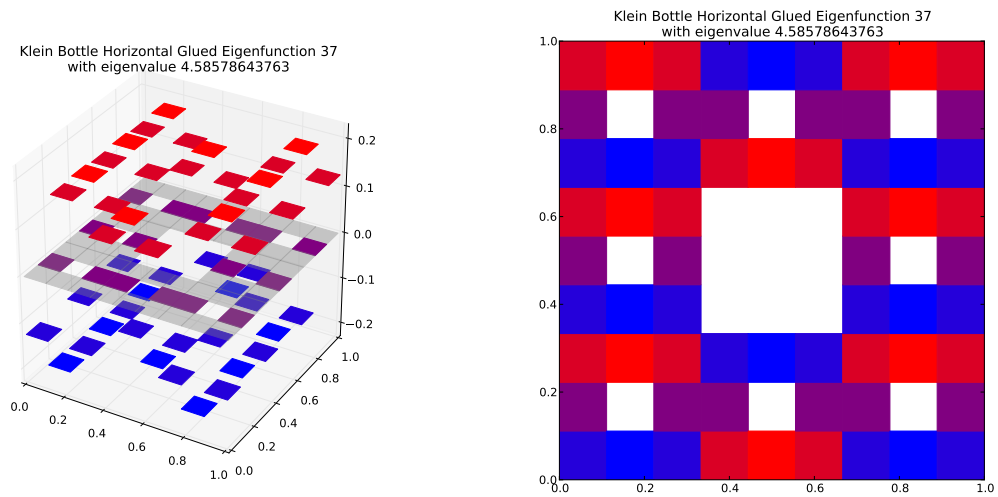
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.218134212086$   
Dot Value: 0.07007729698159326

## 44 $M = 3$ Eigenfunction 43

$M = 3$  Eigenfunction 43 has eigenvalue 1.12507512074



Compare to  $m = 2$  eigenspace with eigenvalue 4.58578643763

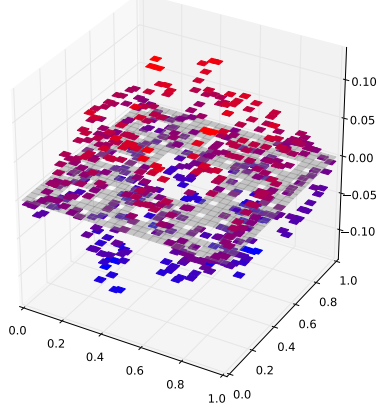


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.245339624085$   
Dot Value: 0.0006505301638103367

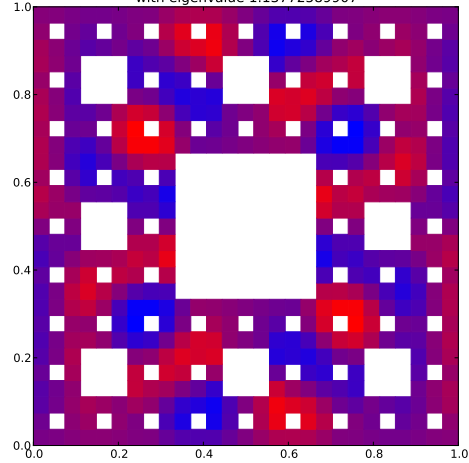
# 45 $M = 3$ Eigenfunction 44

$M = 3$  Eigenfunction 44 has eigenvalue 1.13772589907

Klein Bottle Horizontal Glued Eigenfunction 44  
with eigenvalue 1.13772589907

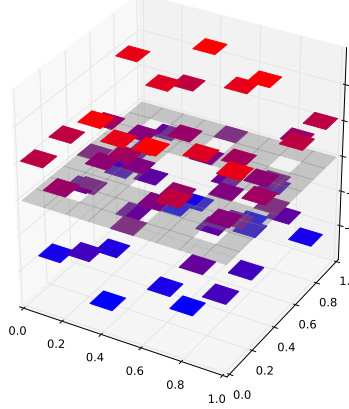


Klein Bottle Horizontal Glued Eigenfunction 44  
with eigenvalue 1.13772589907

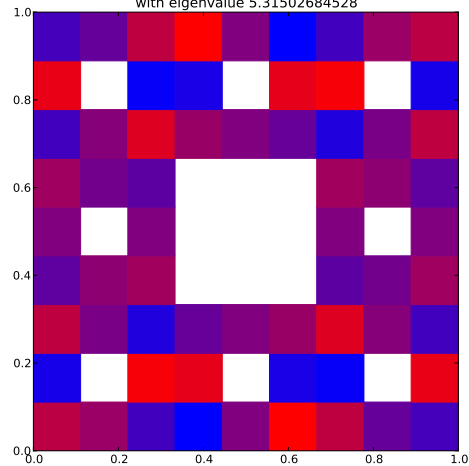


Compare to  $m = 2$  eigenspace with eigenvalue 5.31502684528

Klein Bottle Horizontal Glued Eigenfunction 46  
with eigenvalue 5.31502684528



Klein Bottle Horizontal Glued Eigenfunction 46  
with eigenvalue 5.31502684528

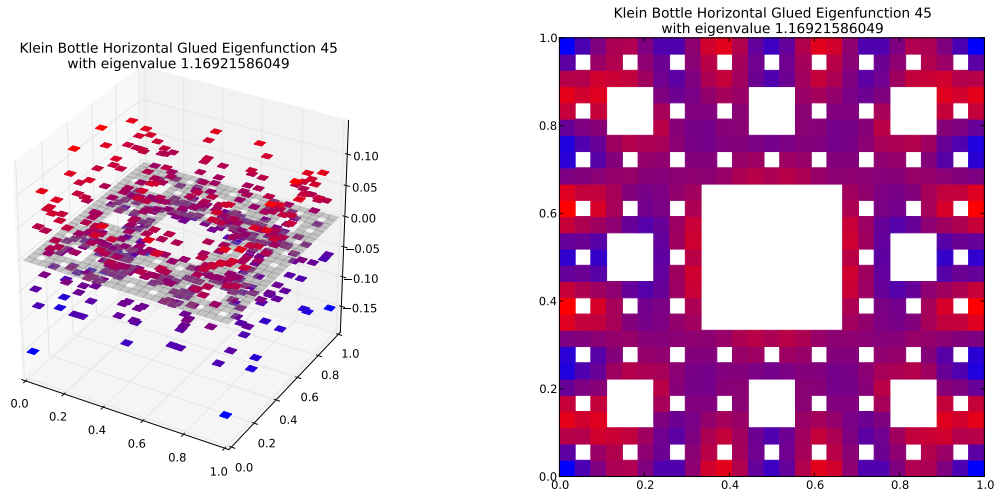


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.214058354208$

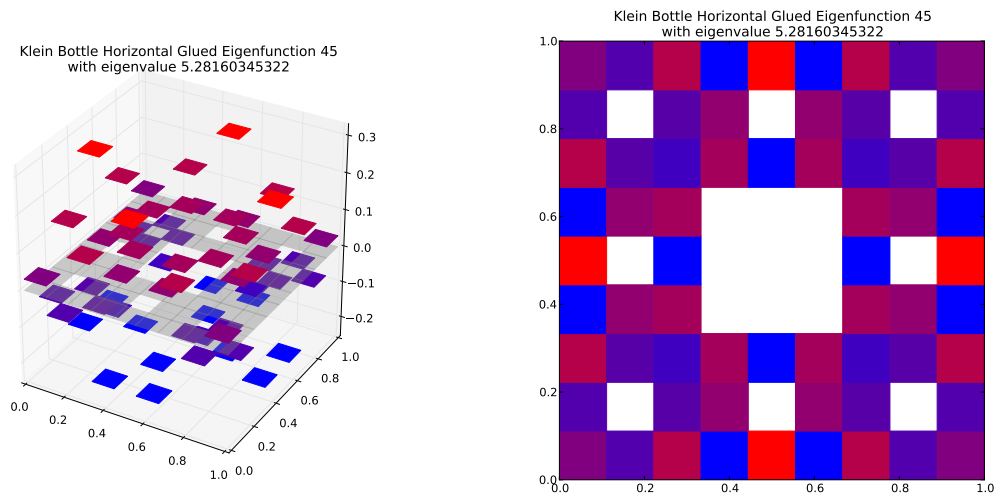
Dot Value: 0.2958935896209792

# 46 $M = 3$ Eigenfunction 45

$M = 3$  Eigenfunction 45 has eigenvalue 1.16921586049



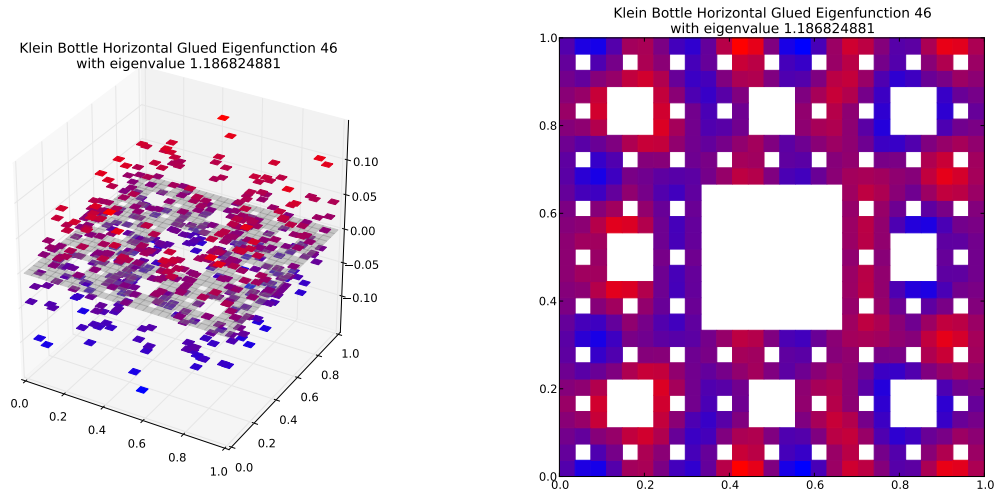
Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322



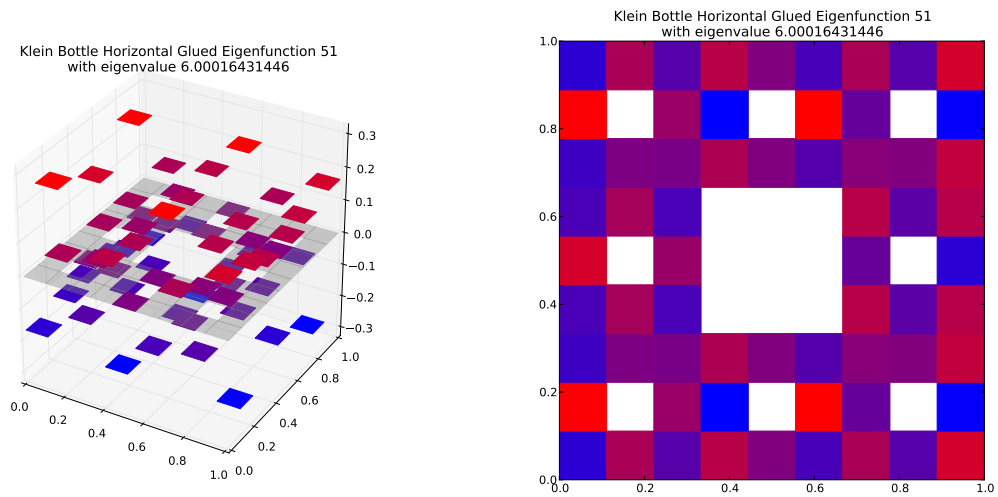
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.221375169652$   
Dot Value: 0.18000713951216196

# 47 $M = 3$ Eigenfunction 46

$M = 3$  Eigenfunction 46 has eigenvalue 1.186824881



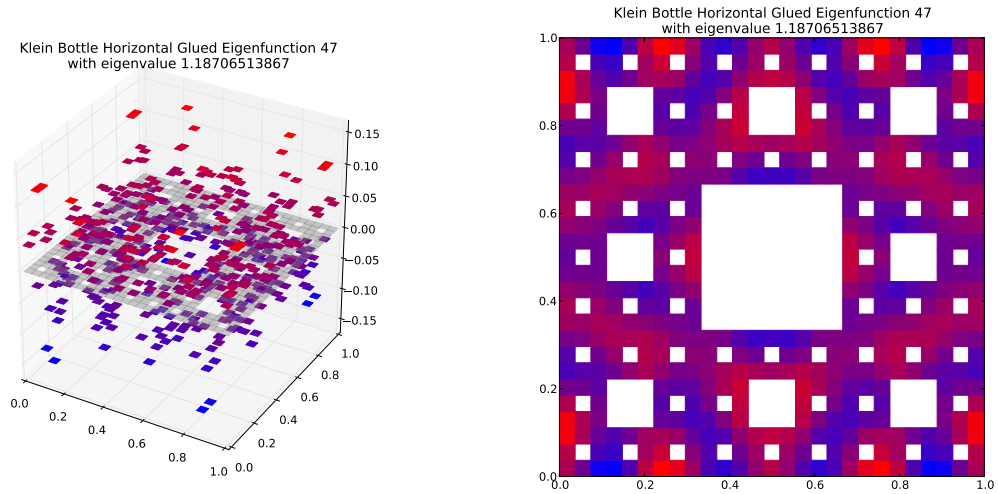
Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446



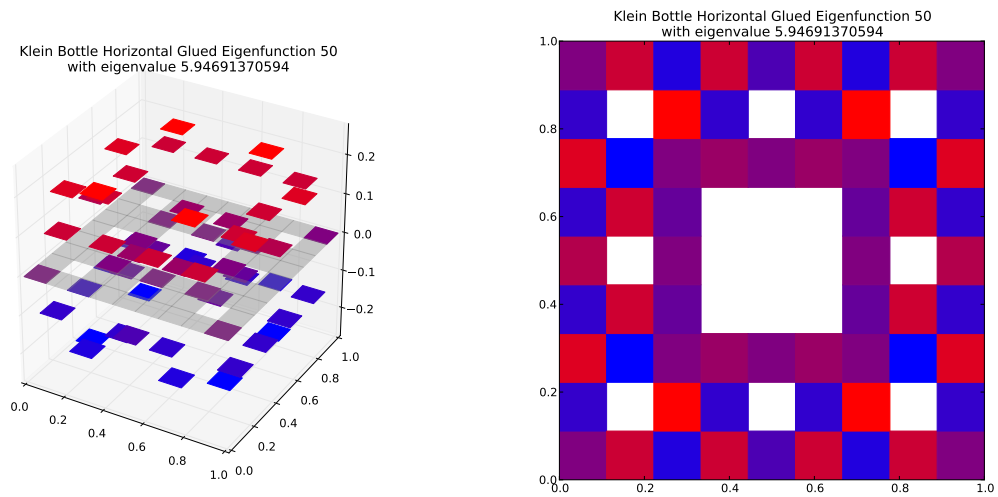
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.197798729969$   
Dot Value: 0.1297109917508924

# 48 $M = 3$ Eigenfunction 47

$M = 3$  Eigenfunction 47 has eigenvalue 1.18706513867



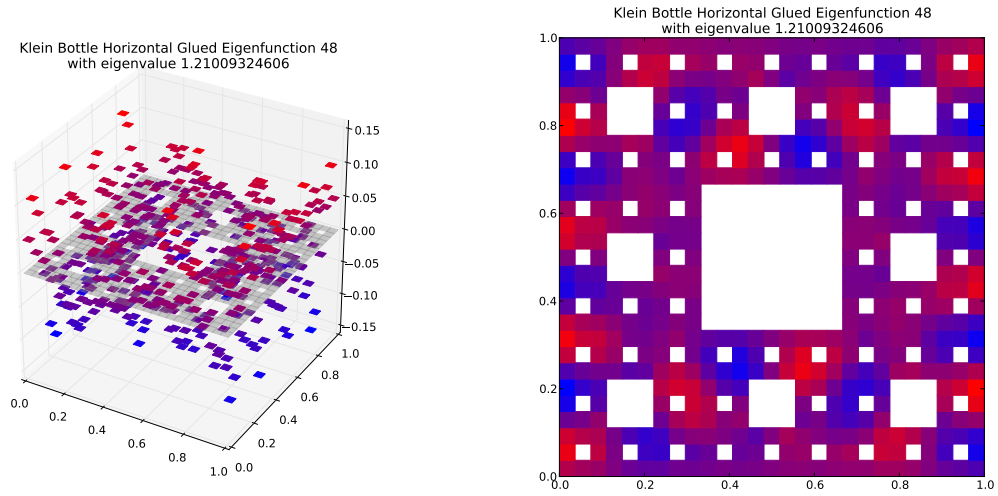
Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594



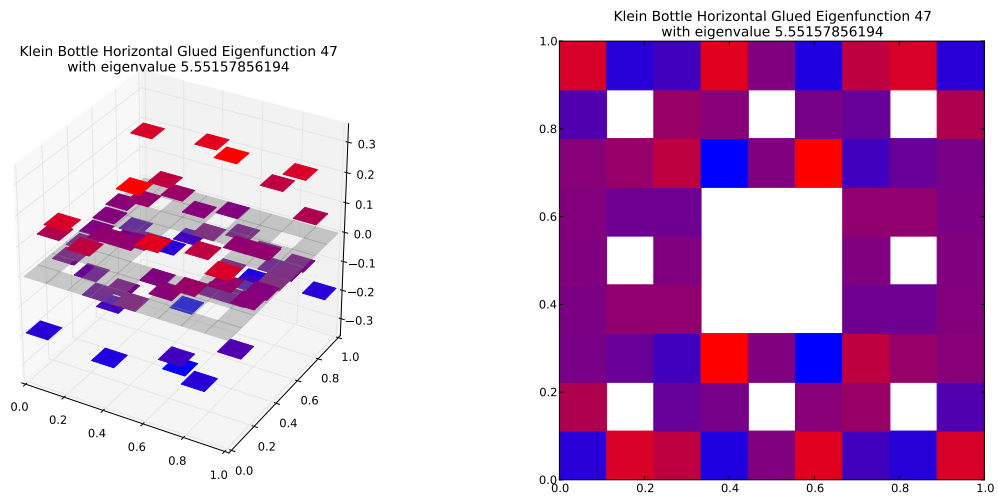
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.199610284825$   
Dot Value: 0.13102095348176845

## 49 $M = 3$ Eigenfunction 48

$M = 3$  Eigenfunction 48 has eigenvalue 1.21009324606



Compare to  $m = 2$  eigenspace with eigenvalue 5.55157856194

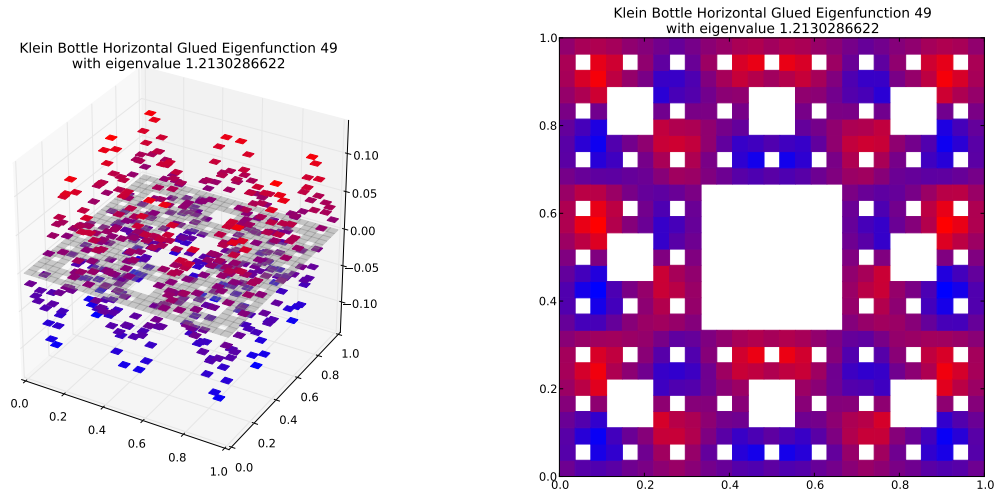


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.217972822065$

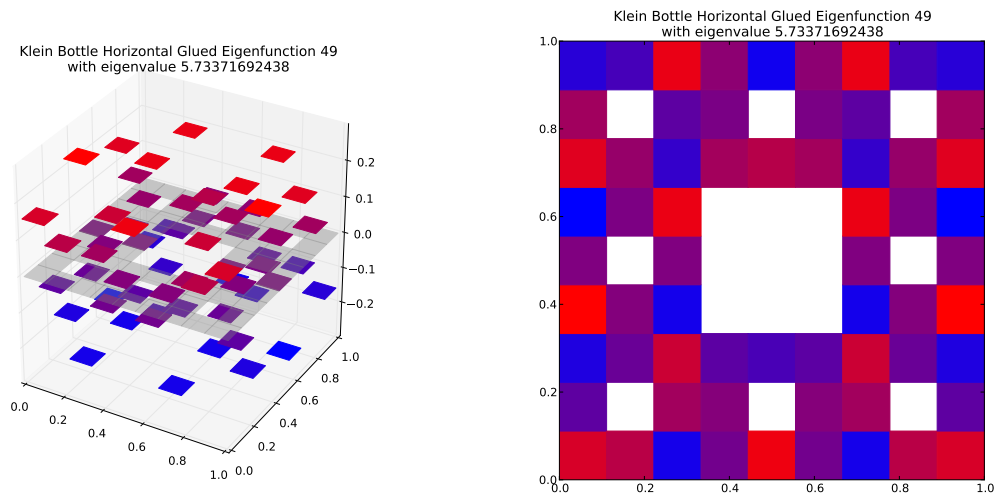
Dot Value: 0.2703801922201232

## 50 $M = 3$ Eigenfunction 49

$M = 3$  Eigenfunction 49 has eigenvalue 1.2130286622



Compare to  $m = 2$  eigenspace with eigenvalue 5.73371692438

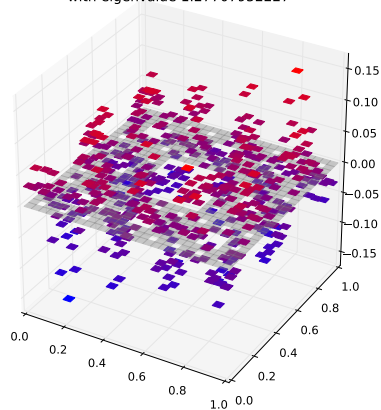


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.211560612112$   
Dot Value: 0.10435968304780596

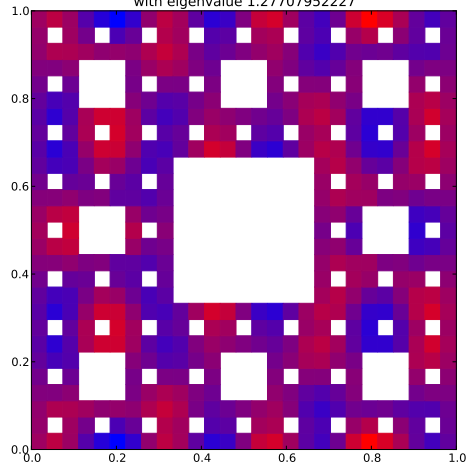
# 51 $M = 3$ Eigenfunction 50

$M = 3$  Eigenfunction 50 has eigenvalue 1.27707952227

Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 1.27707952227

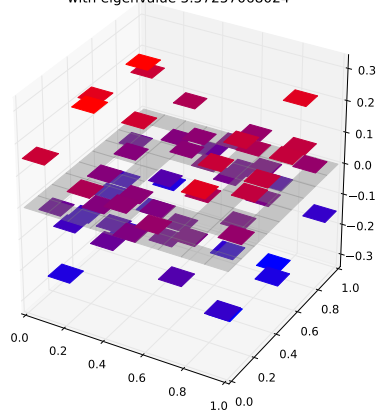


Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 1.27707952227

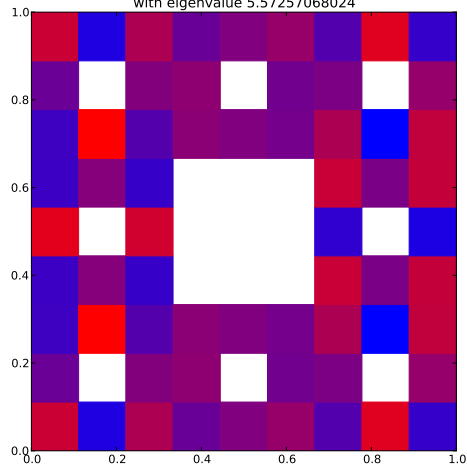


Compare to  $m = 2$  eigenspace with eigenvalue 5.57257068024

Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024



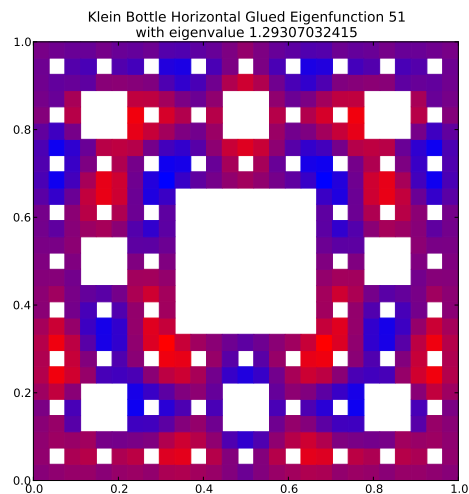
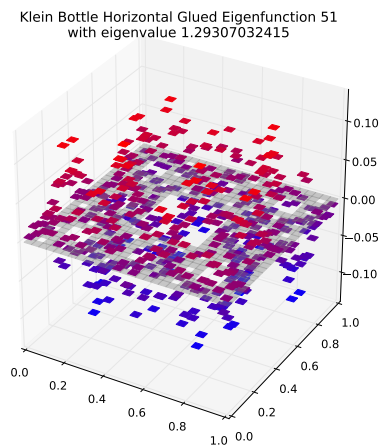
Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024



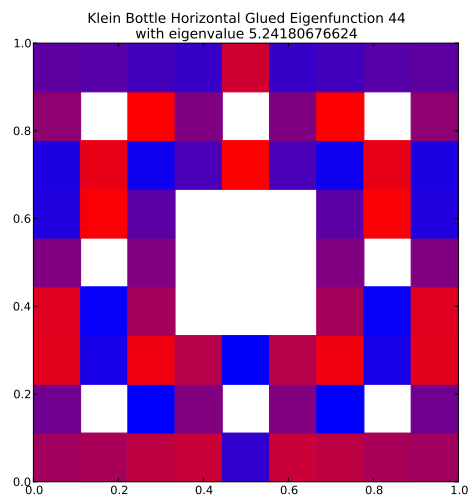
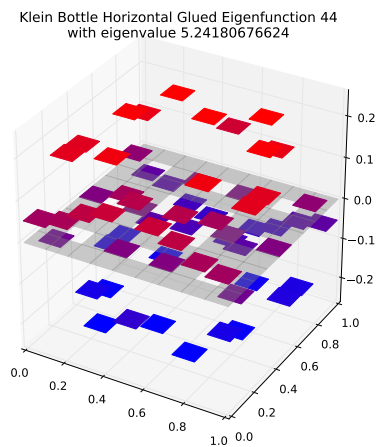
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.229172422487$   
Dot Value: 0.0541275918295551

## 52 $M = 3$ Eigenfunction 51

$M = 3$  Eigenfunction 51 has eigenvalue 1.29307032415



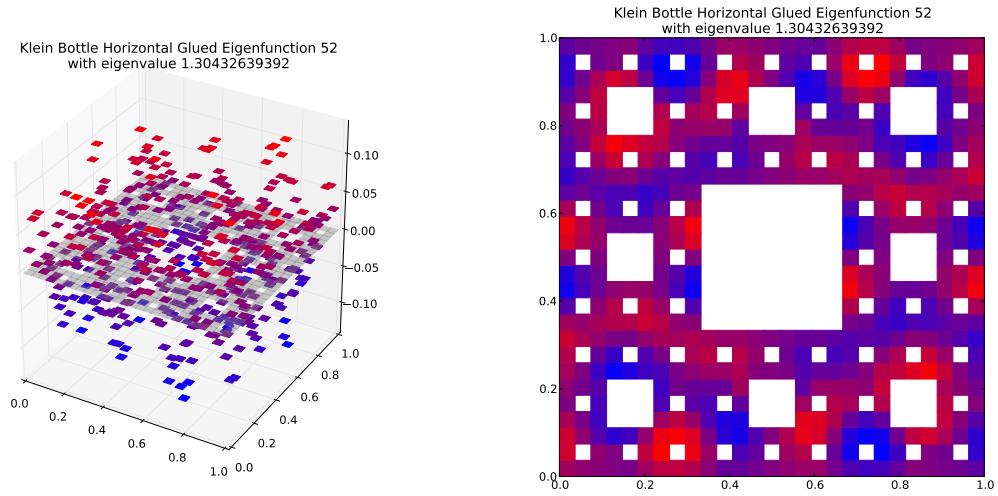
Compare to  $m = 2$  eigenspace with eigenvalue 5.24180676624



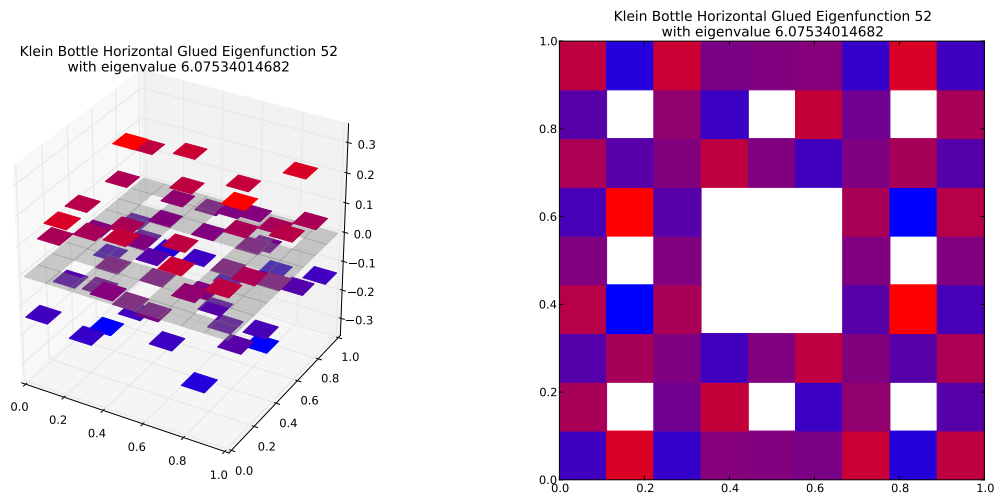
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.246684088486$   
Dot Value: 0.06431850210077272

### 53 $M = 3$ Eigenfunction 52

$M = 3$  Eigenfunction 52 has eigenvalue 1.30432639392



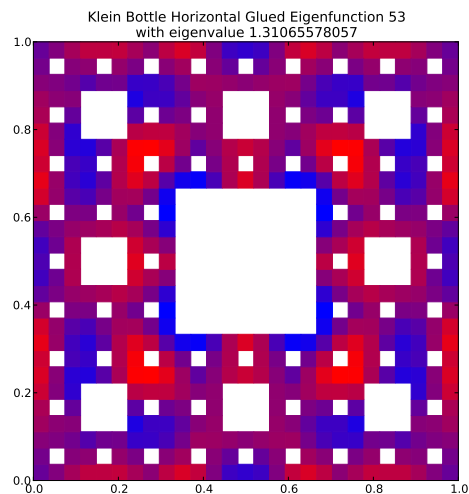
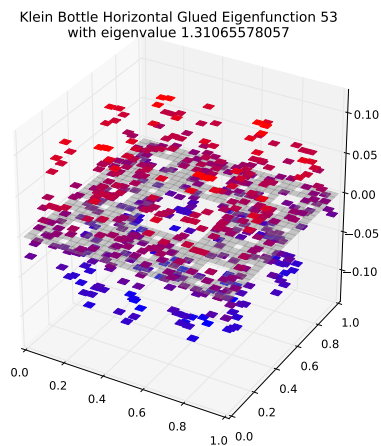
Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682



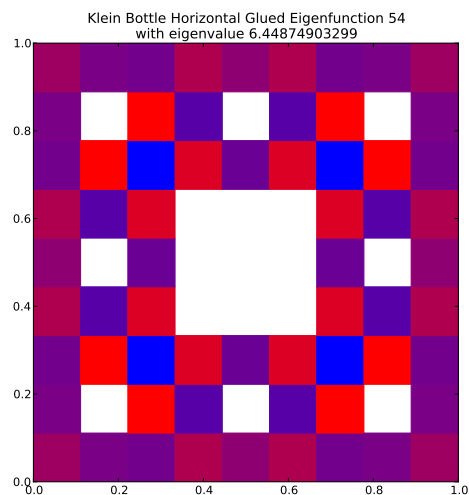
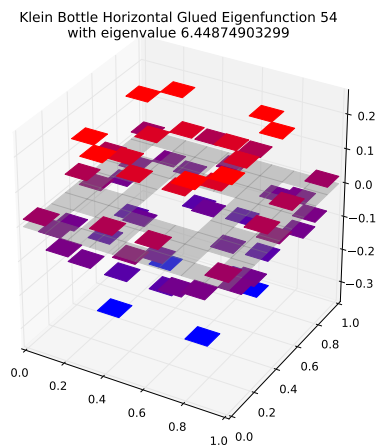
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.214691912288$   
Dot Value: 0.22552843633891662

## 54 $M = 3$ Eigenfunction 53

$M = 3$  Eigenfunction 53 has eigenvalue 1.31065578057



Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

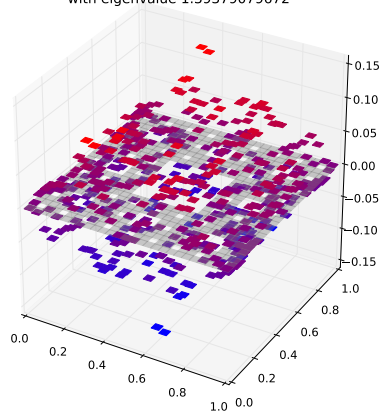


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.20324186503$   
Dot Value: 0.24335434177897874

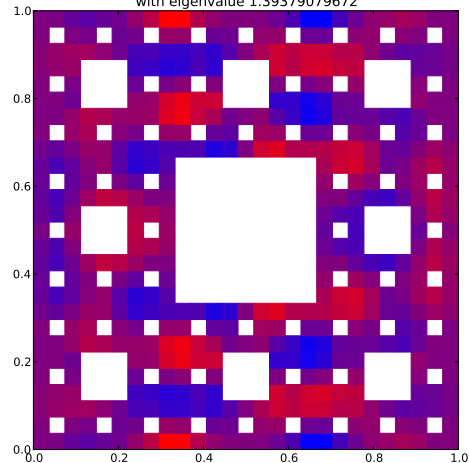
## 55 $M = 3$ Eigenfunction 54

$M = 3$  Eigenfunction 54 has eigenvalue 1.39379079672

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 1.39379079672

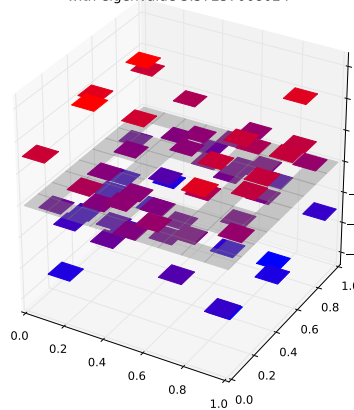


Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 1.39379079672

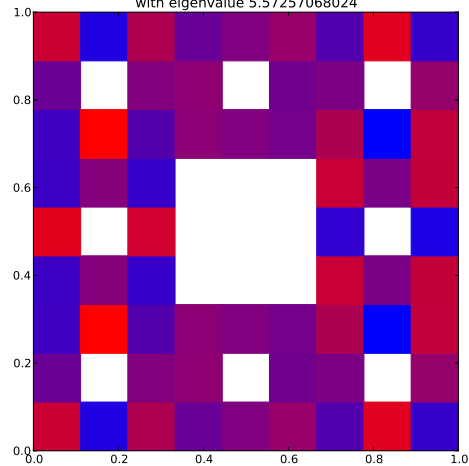


Compare to  $m = 2$  eigenspace with eigenvalue 5.57257068024

Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024



Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024

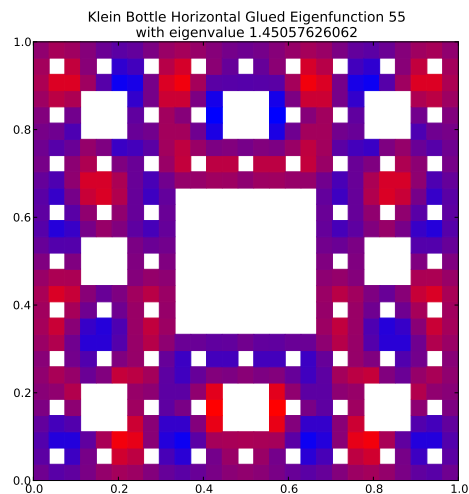
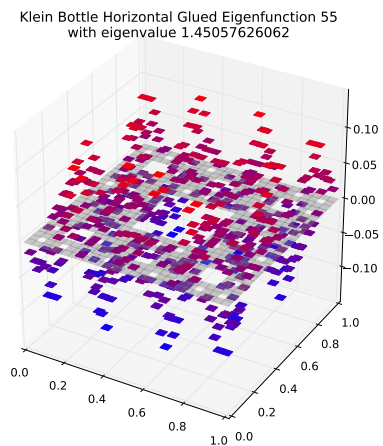


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.250116306583$

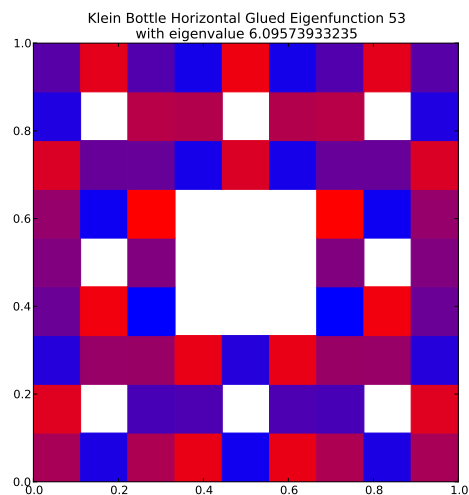
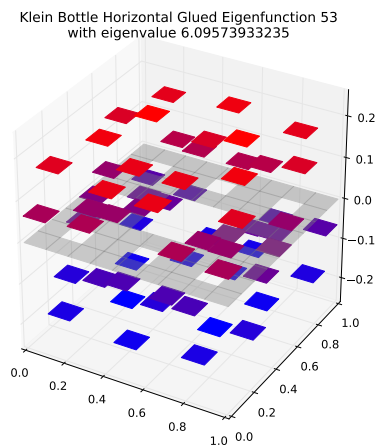
Dot Value: 0.36401385947399667

## 56 $M = 3$ Eigenfunction 55

$M = 3$  Eigenfunction 55 has eigenvalue 1.45057626062



Compare to  $m = 2$  eigenspace with eigenvalue 6.09573933235

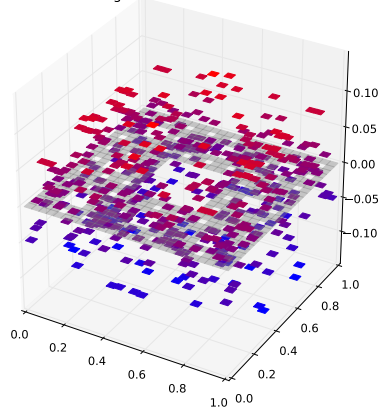


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.237965598844$   
Dot Value: 0.3020706740344563

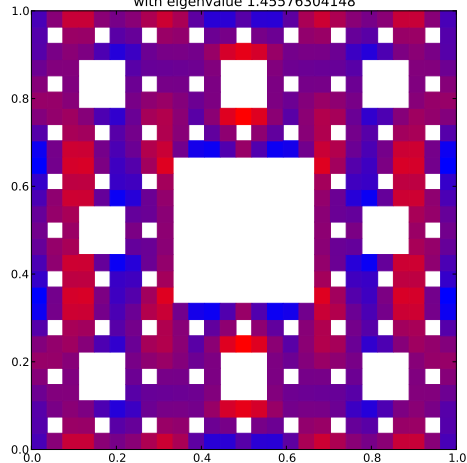
## 57 $M = 3$ Eigenfunction 56

$M = 3$  Eigenfunction 56 has eigenvalue 1.45576304148

Klein Bottle Horizontal Glued Eigenfunction 56  
with eigenvalue 1.45576304148

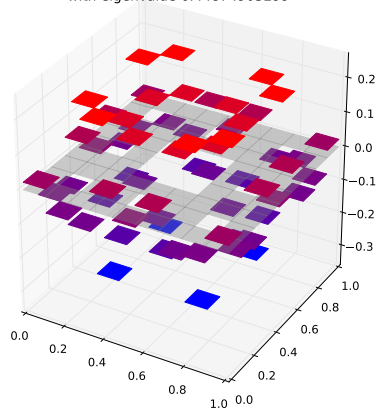


Klein Bottle Horizontal Glued Eigenfunction 56  
with eigenvalue 1.45576304148

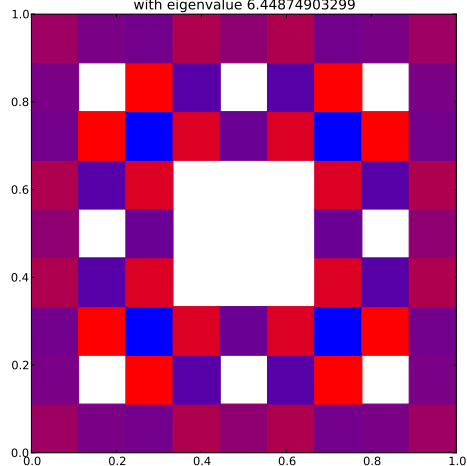


Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299



Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299

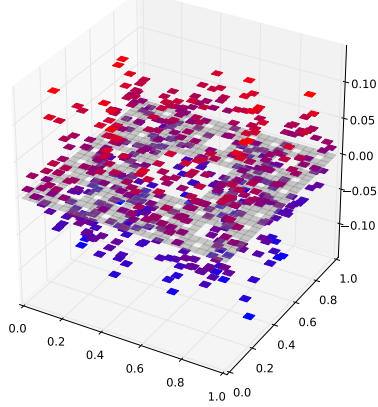


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.225743478934$   
Dot Value: 0.3699258451199551

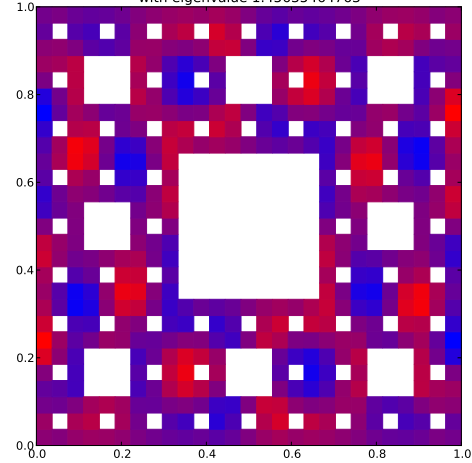
## 58 $M = 3$ Eigenfunction 57

$M = 3$  Eigenfunction 57 has eigenvalue 1.45635464763

Klein Bottle Horizontal Glued Eigenfunction 57  
with eigenvalue 1.45635464763

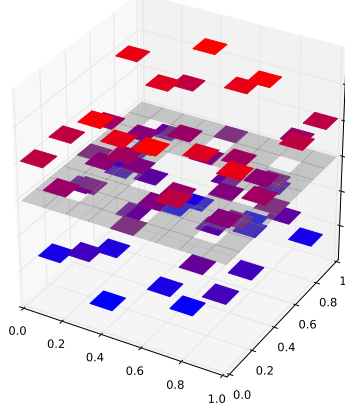


Klein Bottle Horizontal Glued Eigenfunction 57  
with eigenvalue 1.45635464763

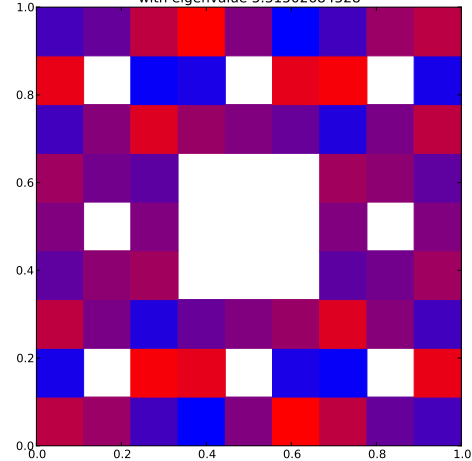


Compare to  $m = 2$  eigenspace with eigenvalue 5.31502684528

Klein Bottle Horizontal Glued Eigenfunction 46  
with eigenvalue 5.31502684528



Klein Bottle Horizontal Glued Eigenfunction 46  
with eigenvalue 5.31502684528

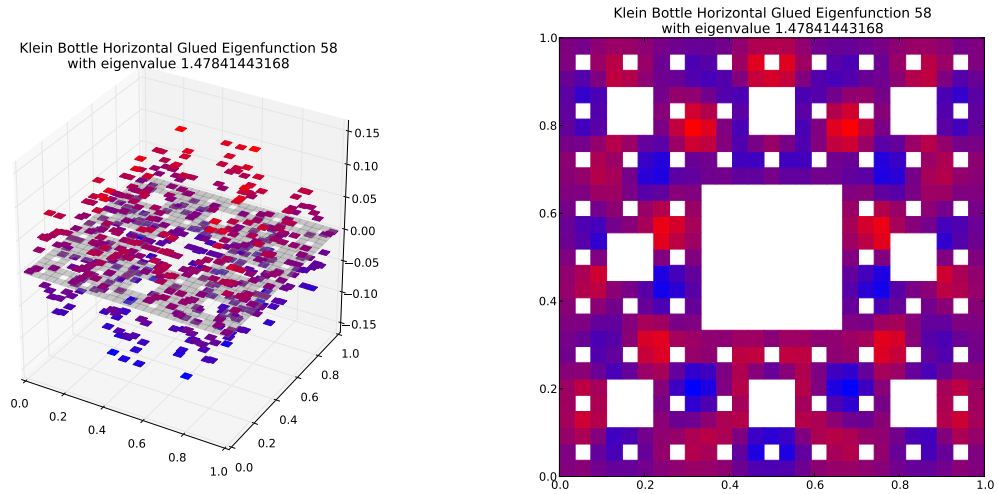


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.274007016338$

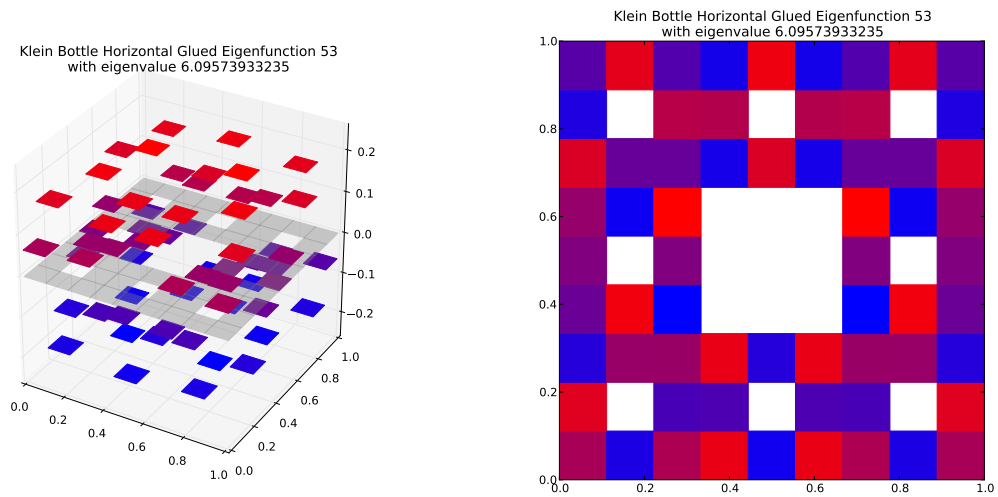
Dot Value: 0.29288977156531604

## 59 $M = 3$ Eigenfunction 58

$M = 3$  Eigenfunction 58 has eigenvalue 1.47841443168



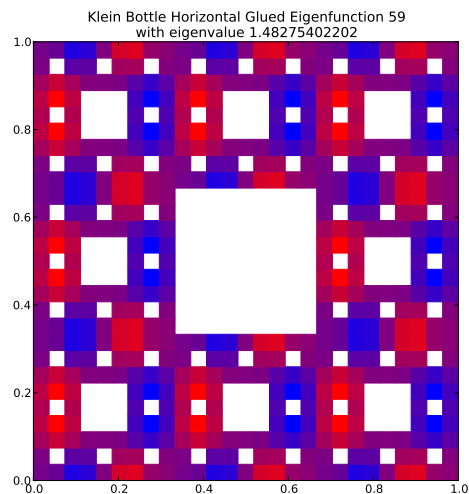
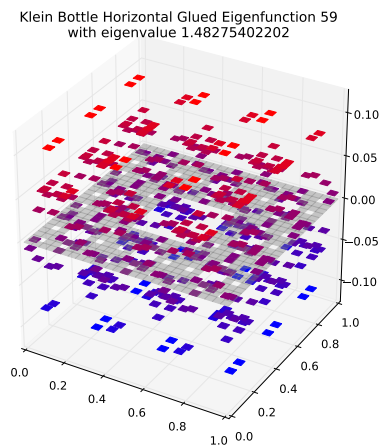
Compare to  $m = 2$  eigenspace with eigenvalue 6.09573933235



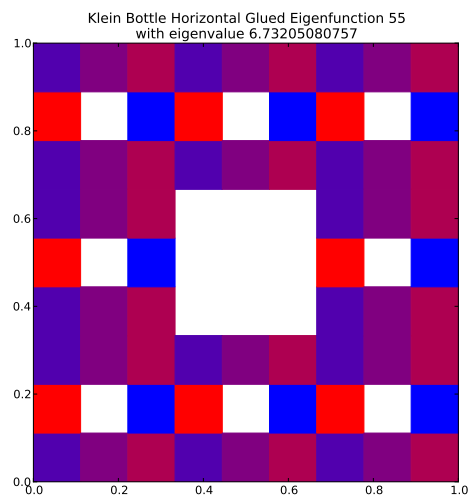
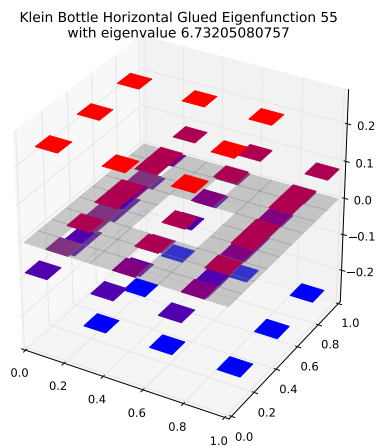
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.242532423235$   
Dot Value: 0.3180573866513199

## 60 $M = 3$ Eigenfunction 59

$M = 3$  Eigenfunction 59 has eigenvalue 1.48275402202



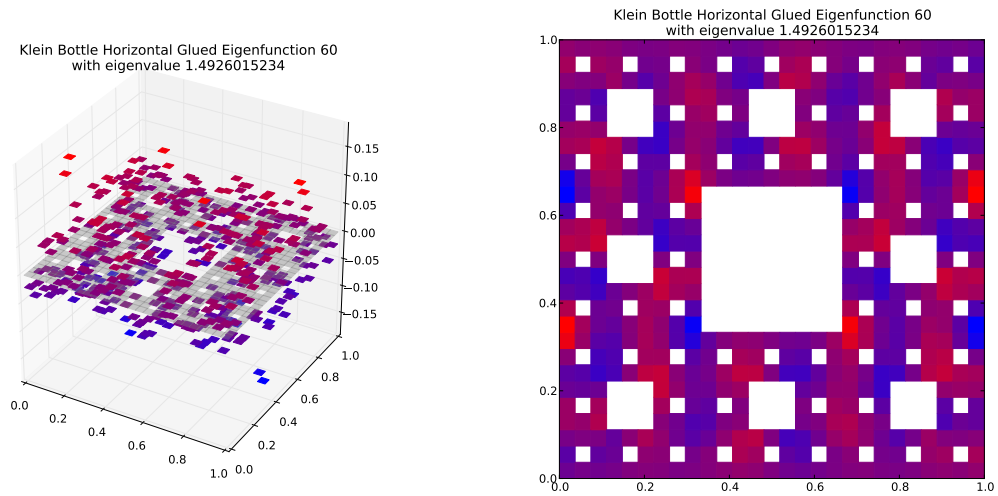
Compare to  $m = 2$  eigenspace with eigenvalue 6.73205080757



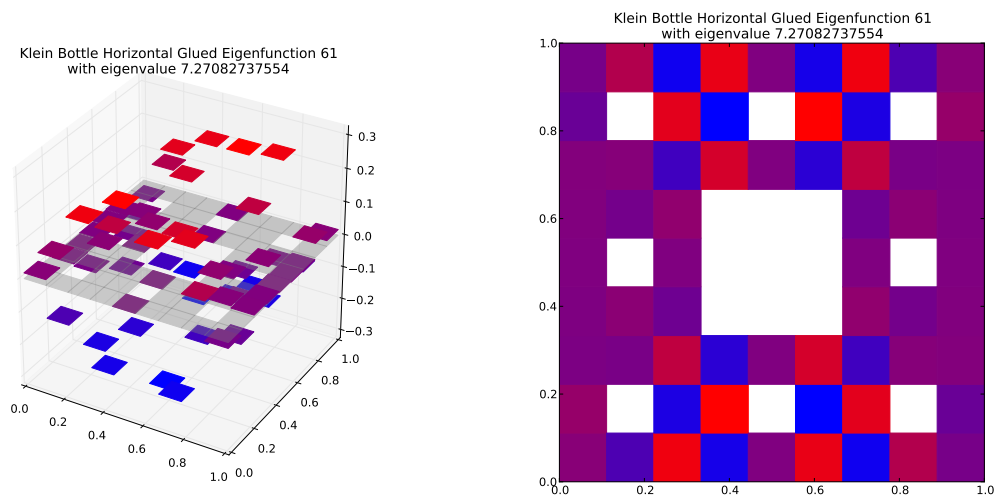
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.220252945856$   
Dot Value: 0.08916117818499214

# 61 $M = 3$ Eigenfunction 60

$M = 3$  Eigenfunction 60 has eigenvalue 1.4926015234



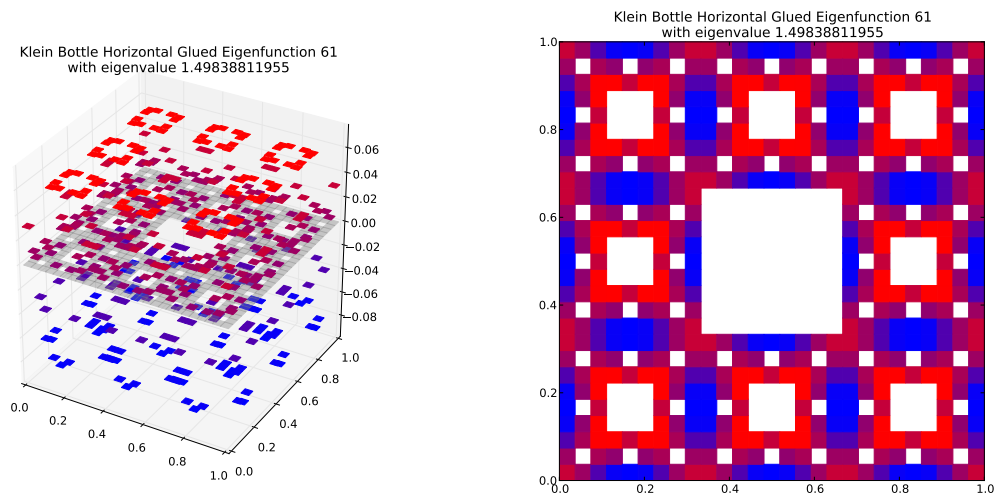
Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554



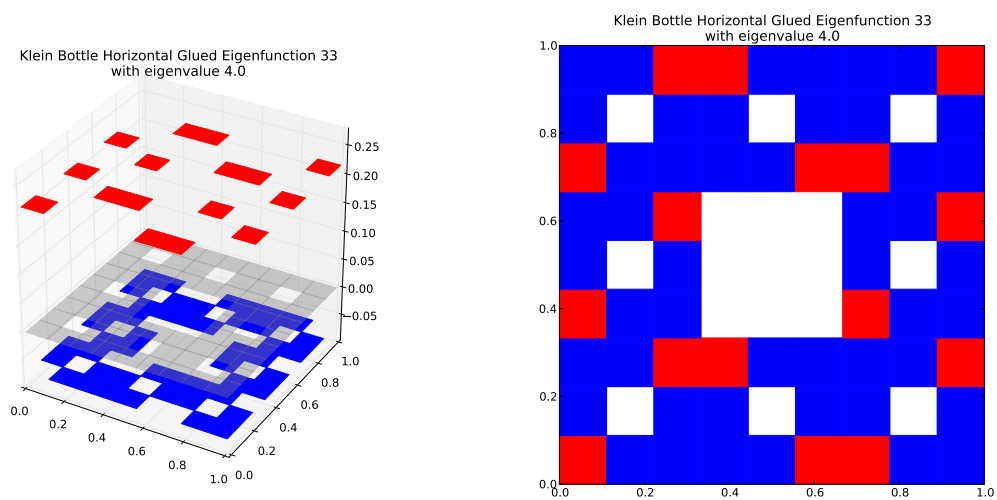
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.205286337621$   
Dot Value: 0.48233411110929336

## 62 $M = 3$ Eigenfunction 61

$M = 3$  Eigenfunction 61 has eigenvalue 1.49838811955



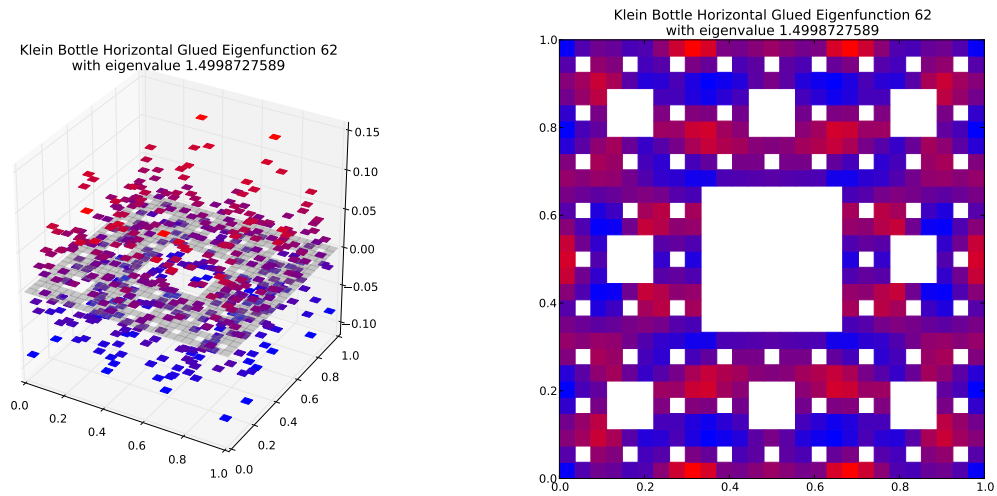
Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )



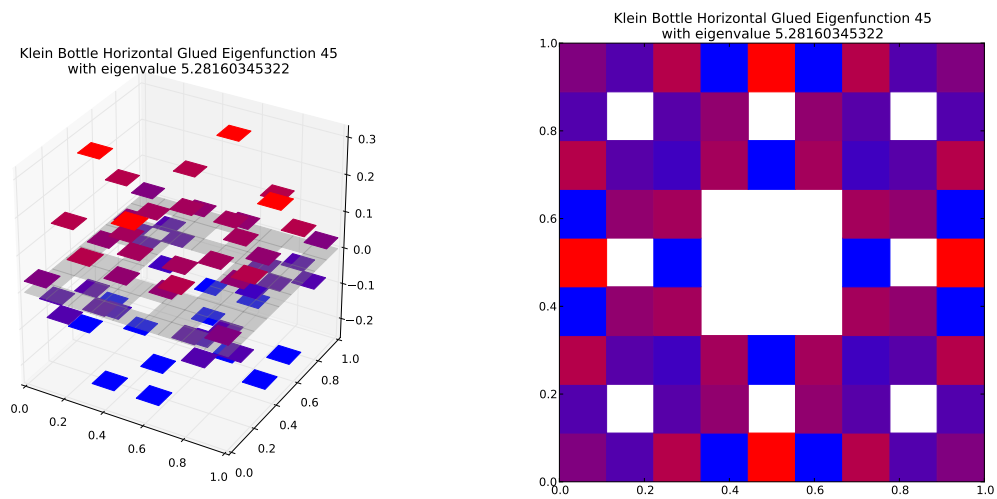
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.374597029888$   
Dot Value: 2.220446049250313e-16

## 63 $M = 3$ Eigenfunction 62

$M = 3$  Eigenfunction 62 has eigenvalue 1.4998727589



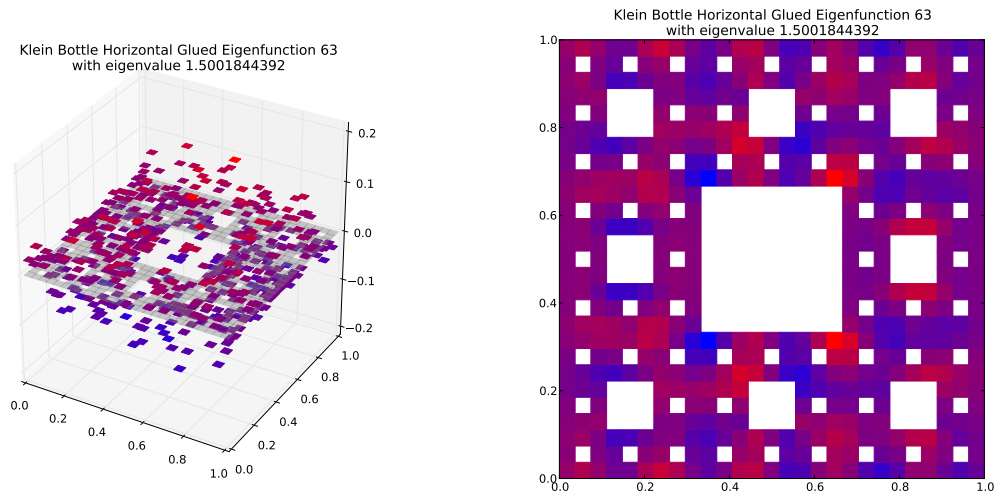
Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322



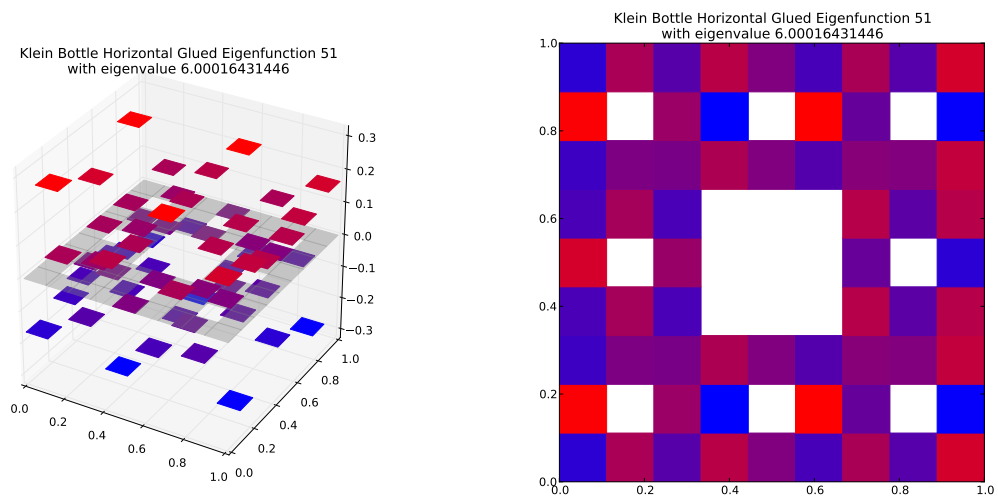
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.283980569952$   
Dot Value: 0.2369032207635885

## 64 $M = 3$ Eigenfunction 63

$M = 3$  Eigenfunction 63 has eigenvalue 1.5001844392



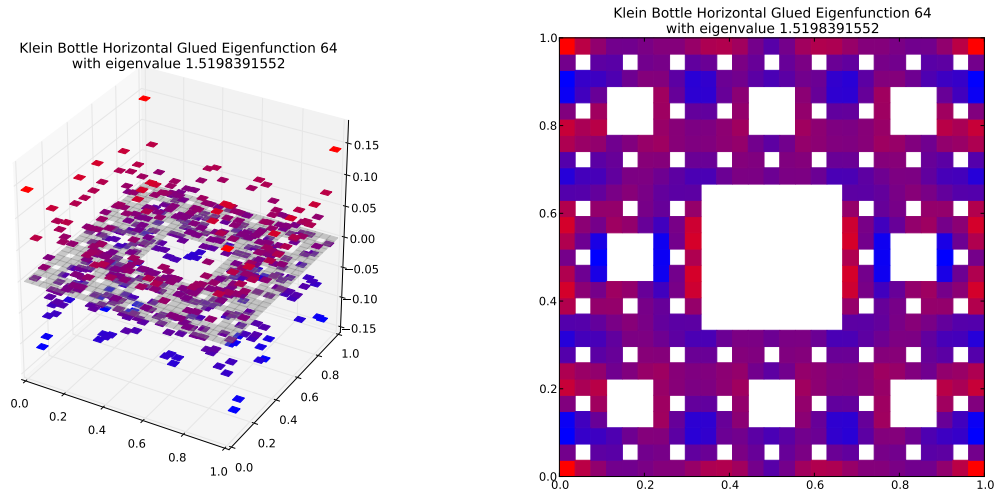
Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446



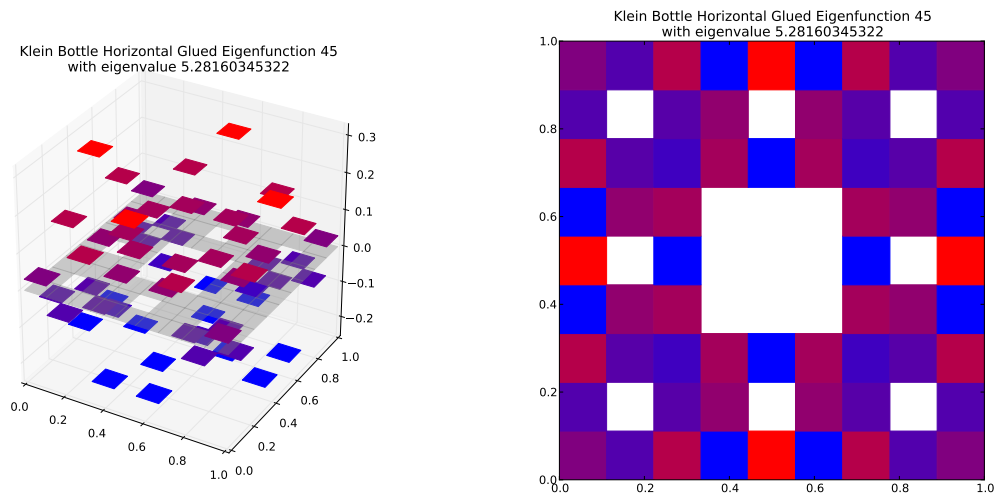
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.250023892776$   
Dot Value: 0.44297902501702946

## 65 $M = 3$ Eigenfunction 64

$M = 3$  Eigenfunction 64 has eigenvalue 1.5198391552



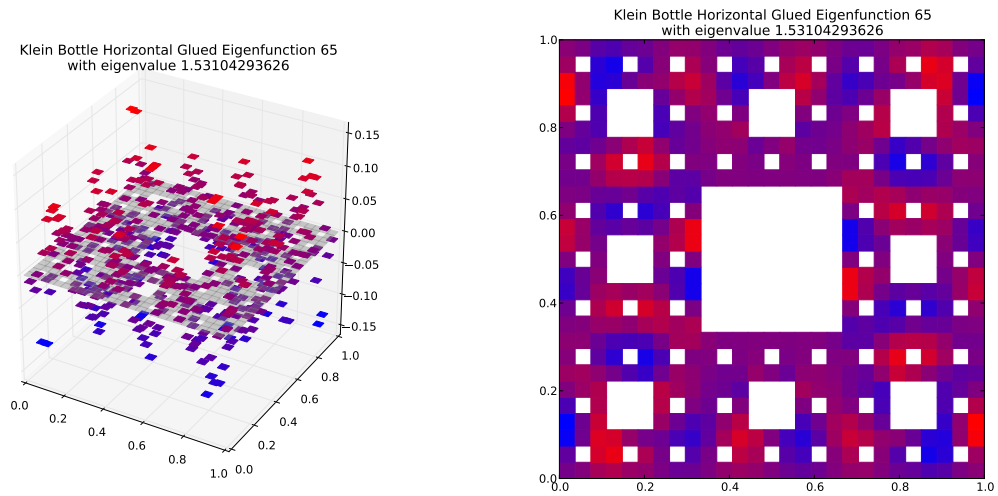
Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322



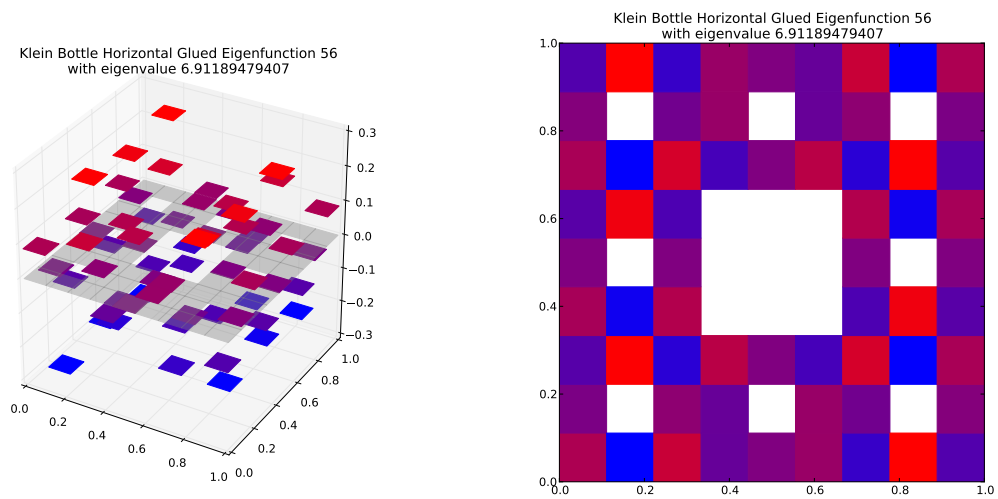
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.287760936364$   
Dot Value: 0.41711841336066924

## 66 $M = 3$ Eigenfunction 65

$M = 3$  Eigenfunction 65 has eigenvalue 1.53104293626



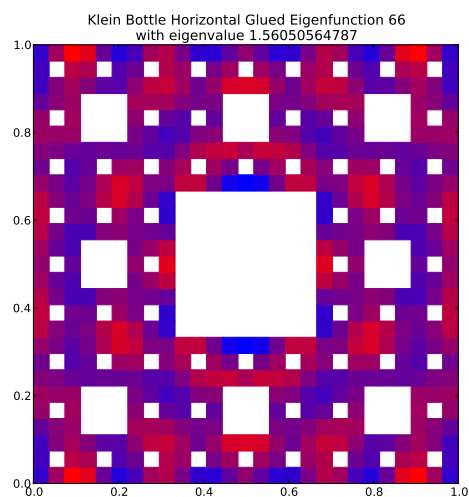
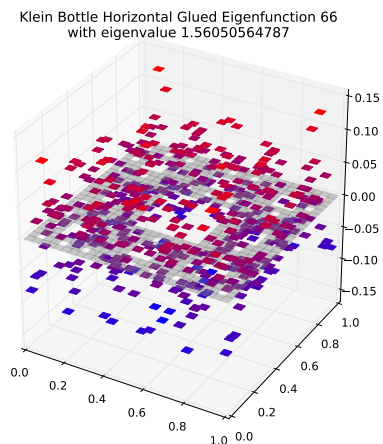
Compare to  $m = 2$  eigenspace with eigenvalue 6.91189479407



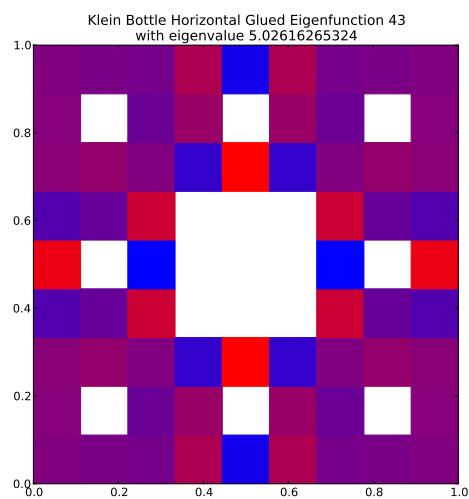
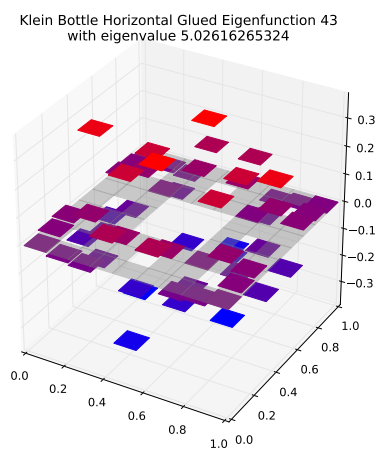
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.221508425963$   
Dot Value: 0.17795721330945014

## 67 $M = 3$ Eigenfunction 66

$M = 3$  Eigenfunction 66 has eigenvalue 1.56050564787



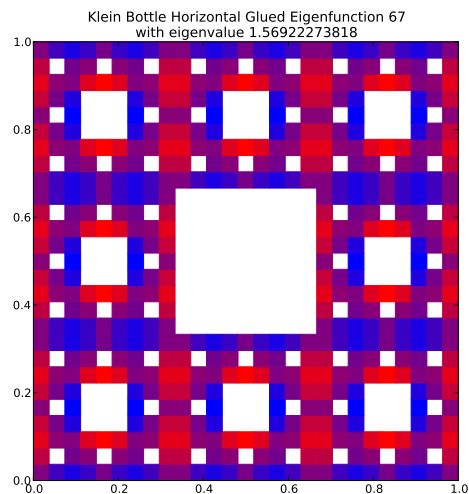
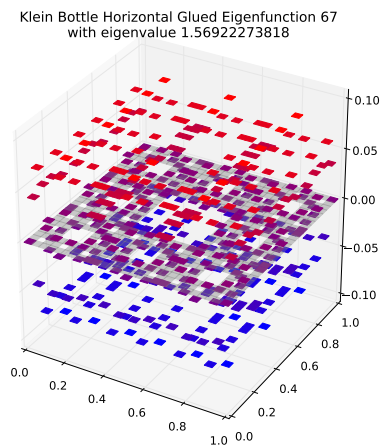
Compare to  $m = 2$  eigenspace with eigenvalue 5.02616265324



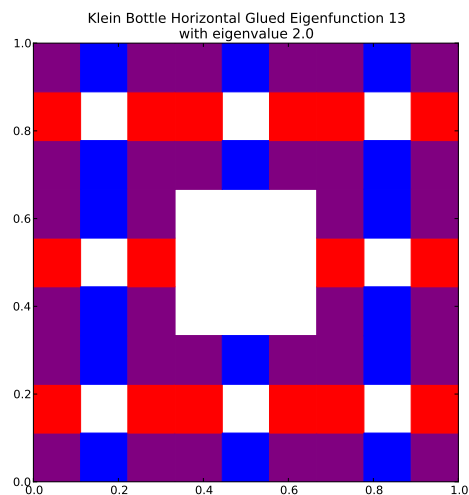
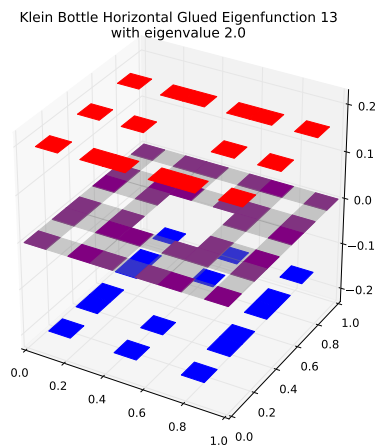
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.310476551503$   
Dot Value: 0.1921782256821204

# 68 $M = 3$ Eigenfunction 67

$M = 3$  Eigenfunction 67 has eigenvalue 1.56922273818



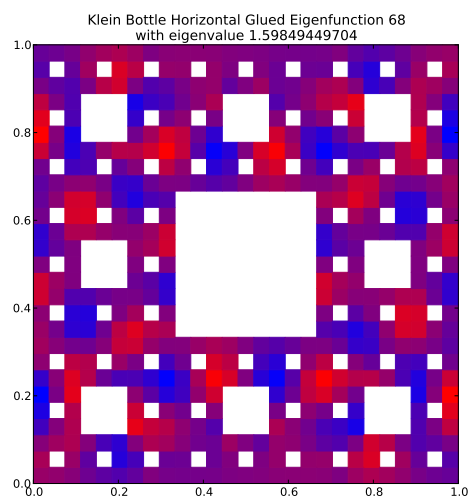
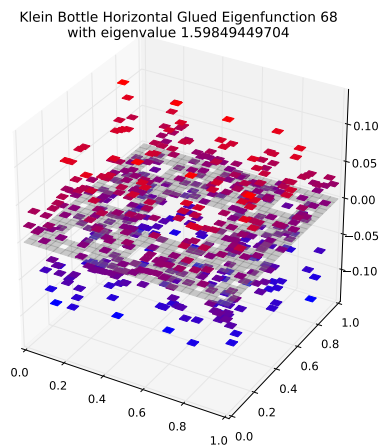
Compare to  $m = 2$  eigenspace with eigenvalue 2.0



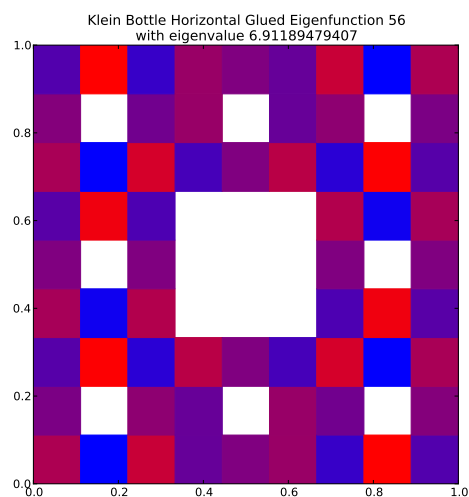
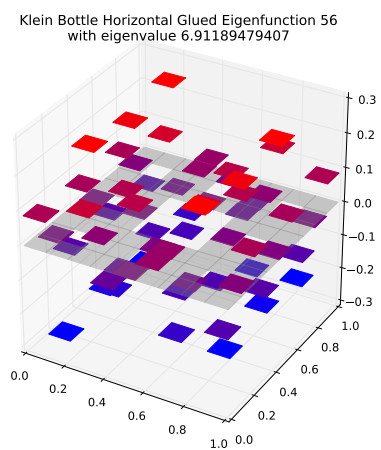
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.78461136909$   
Dot Value: 0.0

## 69 $M = 3$ Eigenfunction 68

$M = 3$  Eigenfunction 68 has eigenvalue 1.59849449704



Compare to  $m = 2$  eigenspace with eigenvalue 6.91189479407

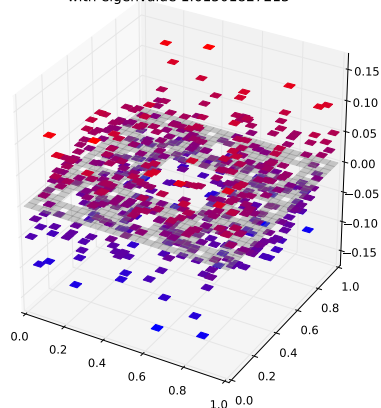


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.23126719151$   
Dot Value: 0.27133128611520296

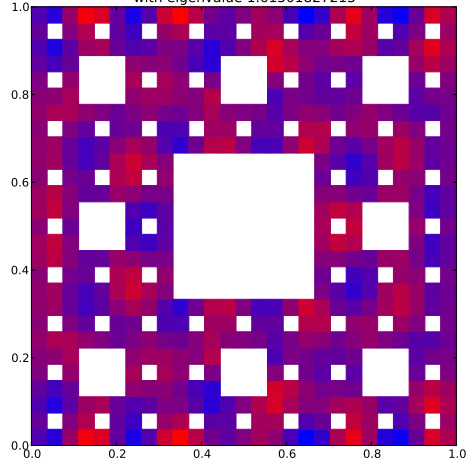
## 70 $M = 3$ Eigenfunction 69

$M = 3$  Eigenfunction 69 has eigenvalue 1.61501827213

Klein Bottle Horizontal Glued Eigenfunction 69  
with eigenvalue 1.61501827213

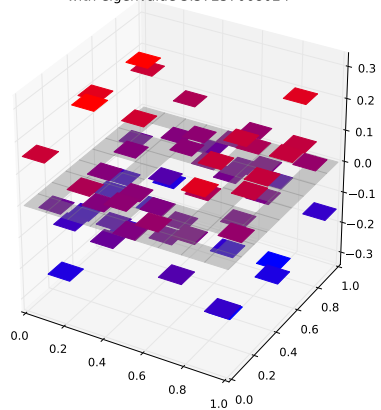


Klein Bottle Horizontal Glued Eigenfunction 69  
with eigenvalue 1.61501827213

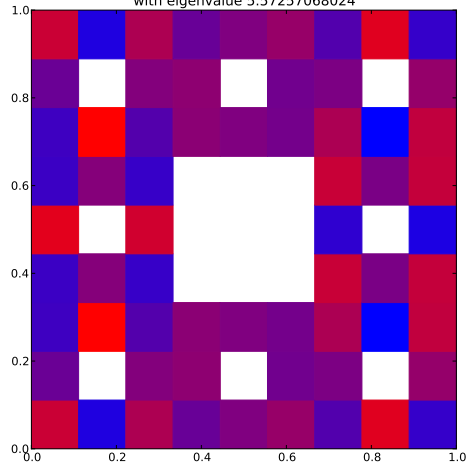


Compare to  $m = 2$  eigenspace with eigenvalue 5.57257068024

Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024



Klein Bottle Horizontal Glued Eigenfunction 48  
with eigenvalue 5.57257068024

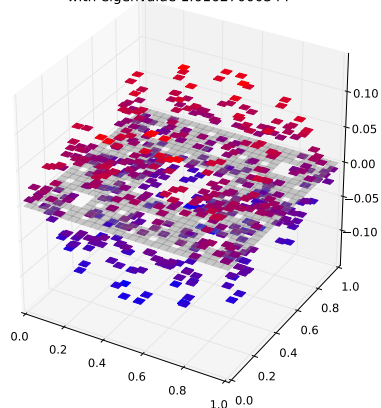


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.289815664044$   
Dot Value: 0.3099140151454426

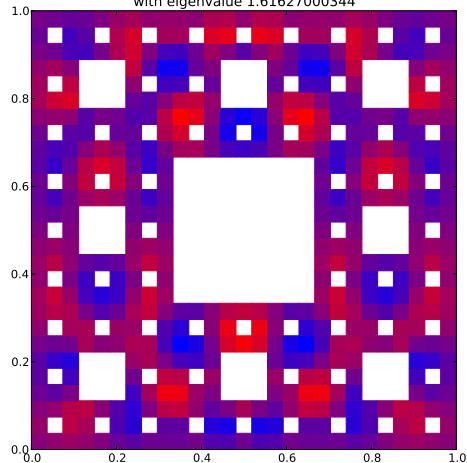
# 71 $M = 3$ Eigenfunction 70

$M = 3$  Eigenfunction 70 has eigenvalue 1.61627000344

Klein Bottle Horizontal Glued Eigenfunction 70  
with eigenvalue 1.61627000344

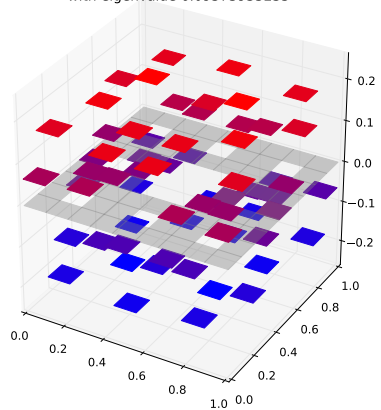


Klein Bottle Horizontal Glued Eigenfunction 70  
with eigenvalue 1.61627000344

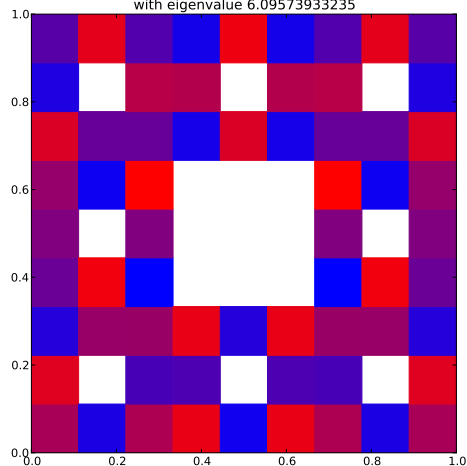


Compare to  $m = 2$  eigenspace with eigenvalue 6.09573933235

Klein Bottle Horizontal Glued Eigenfunction 53  
with eigenvalue 6.09573933235



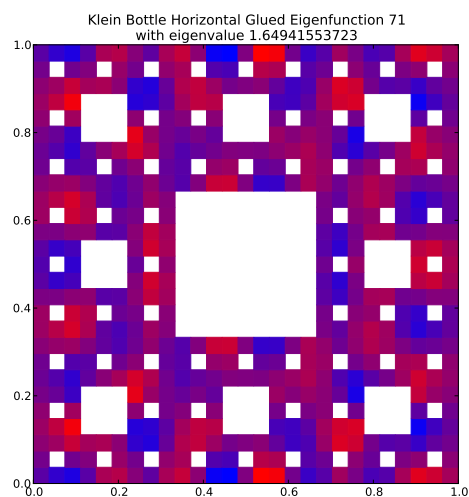
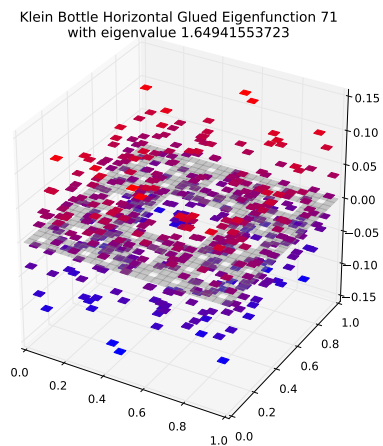
Klein Bottle Horizontal Glued Eigenfunction 53  
with eigenvalue 6.09573933235



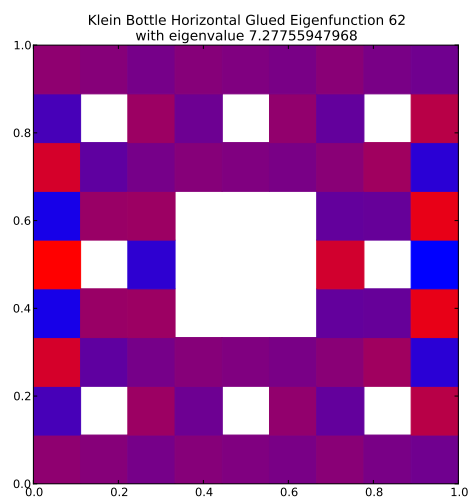
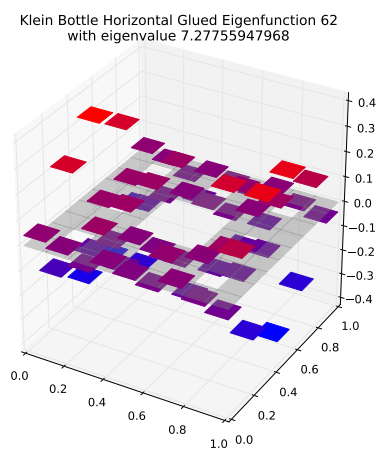
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.265147493243$   
Dot Value: 0.33104198111943617

## 72 $M = 3$ Eigenfunction 71

$M = 3$  Eigenfunction 71 has eigenvalue 1.64941553723



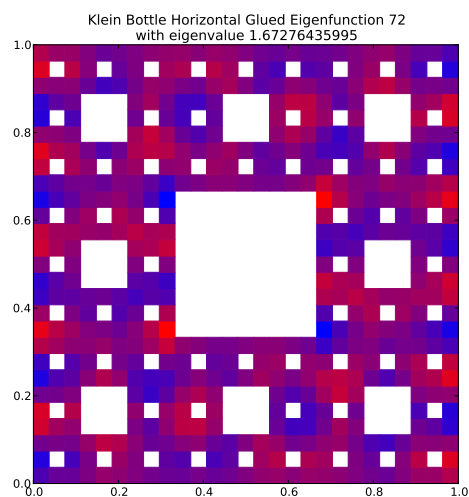
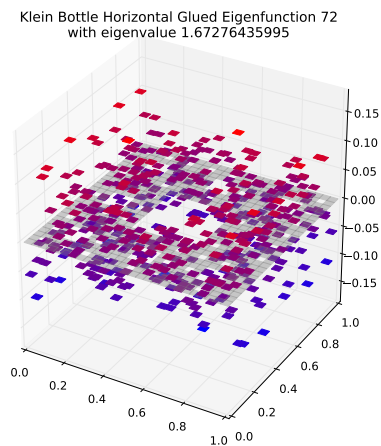
Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968



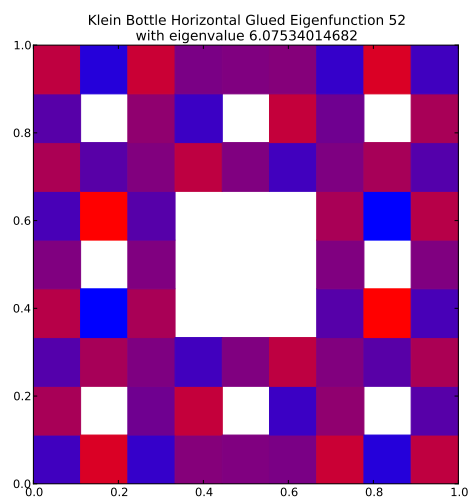
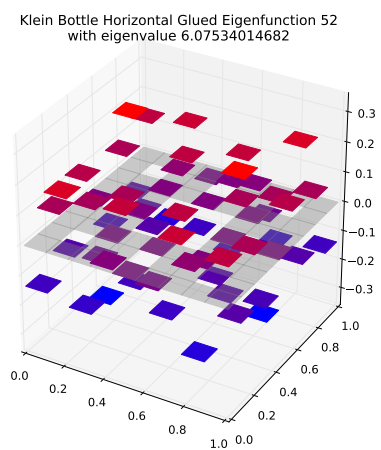
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.226644047615$   
Dot Value: 0.28260724888583166

### 73 $M = 3$ Eigenfunction 72

$M = 3$  Eigenfunction 72 has eigenvalue 1.67276435995



Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682

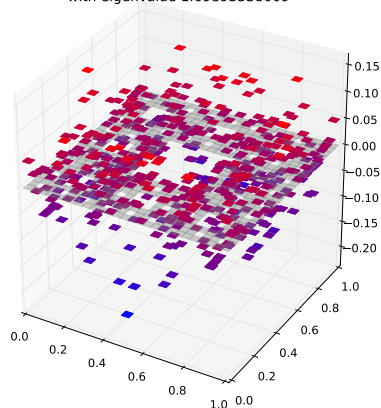


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.275336741569$   
Dot Value: 0.19711271874641978

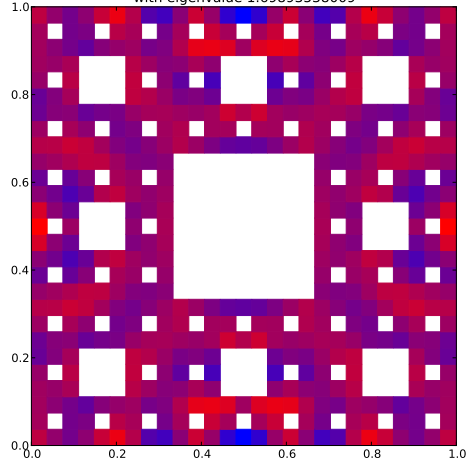
## 74 $M = 3$ Eigenfunction 73

$M = 3$  Eigenfunction 73 has eigenvalue 1.69893538009

Klein Bottle Horizontal Glued Eigenfunction 73  
with eigenvalue 1.69893538009

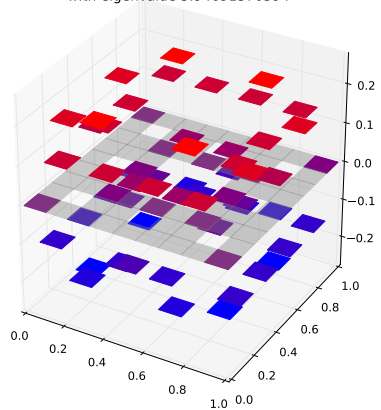


Klein Bottle Horizontal Glued Eigenfunction 73  
with eigenvalue 1.69893538009

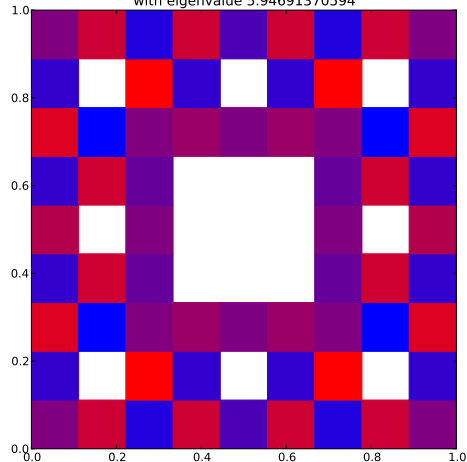


Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594

Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594



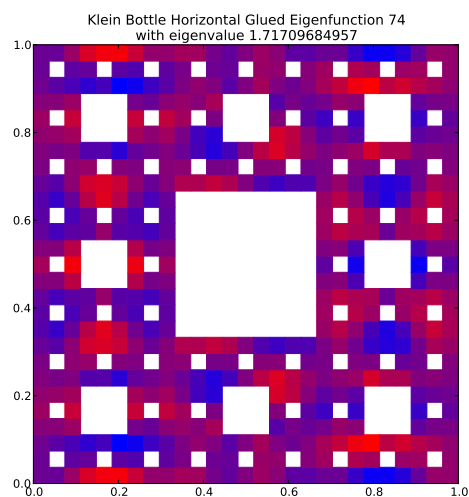
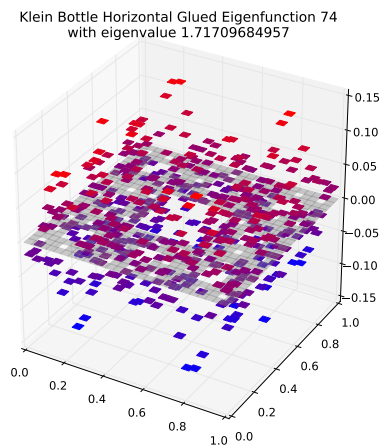
Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594



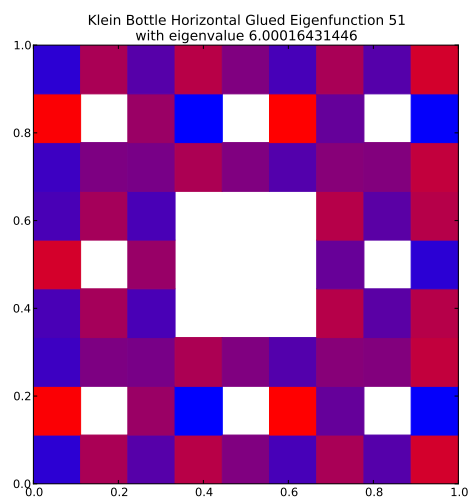
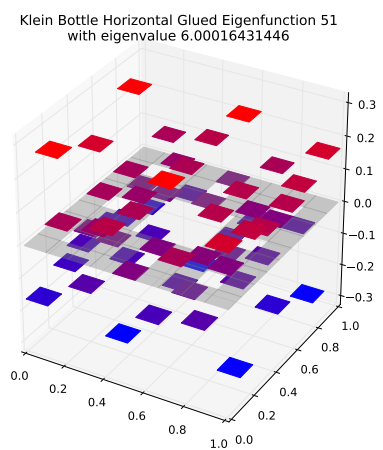
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.285683543448$   
Dot Value: 0.17720432525860041

## 75 $M = 3$ Eigenfunction 74

$M = 3$  Eigenfunction 74 has eigenvalue 1.71709684957



Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446

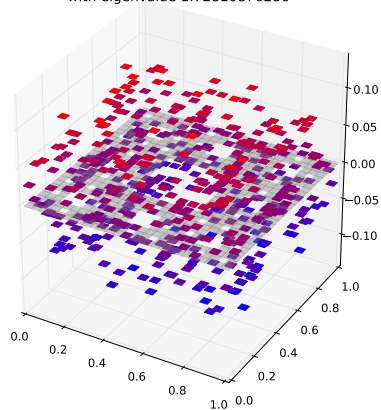


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.286174971148$   
Dot Value: 0.29656722301699656

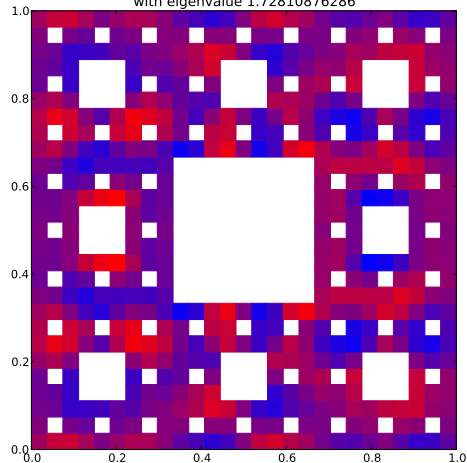
## 76 $M = 3$ Eigenfunction 75

$M = 3$  Eigenfunction 75 has eigenvalue 1.72810876286

Klein Bottle Horizontal Glued Eigenfunction 75  
with eigenvalue 1.72810876286

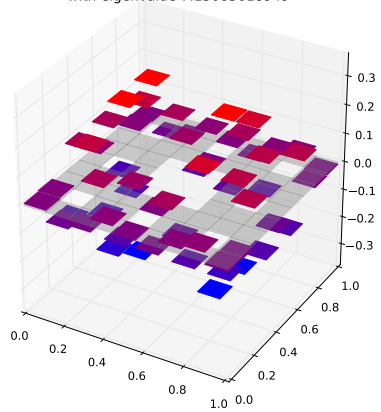


Klein Bottle Horizontal Glued Eigenfunction 75  
with eigenvalue 1.72810876286

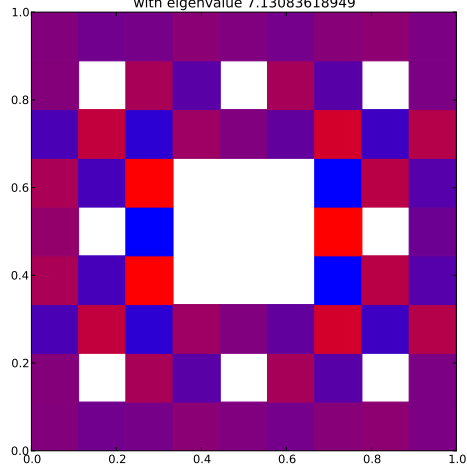


Compare to  $m = 2$  eigenspace with eigenvalue 7.13083618949

Klein Bottle Horizontal Glued Eigenfunction 58  
with eigenvalue 7.13083618949



Klein Bottle Horizontal Glued Eigenfunction 58  
with eigenvalue 7.13083618949

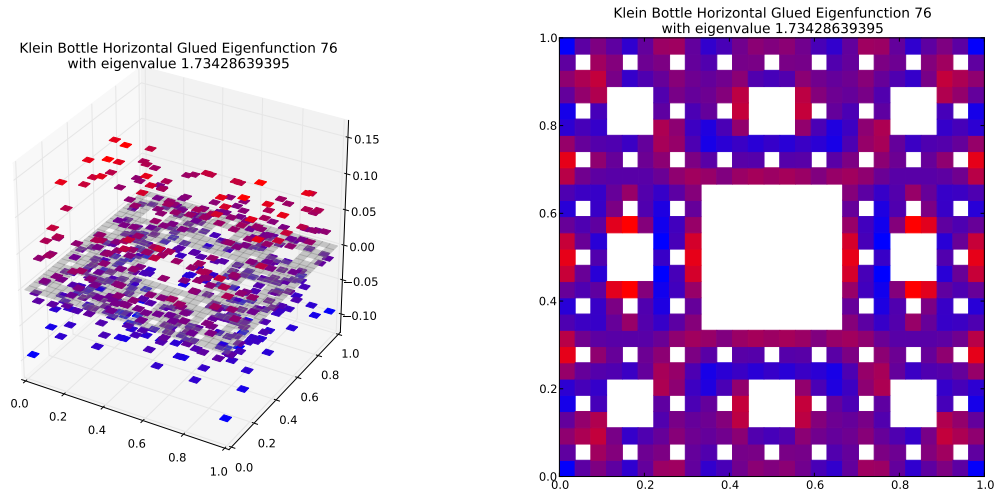


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.242343074072$

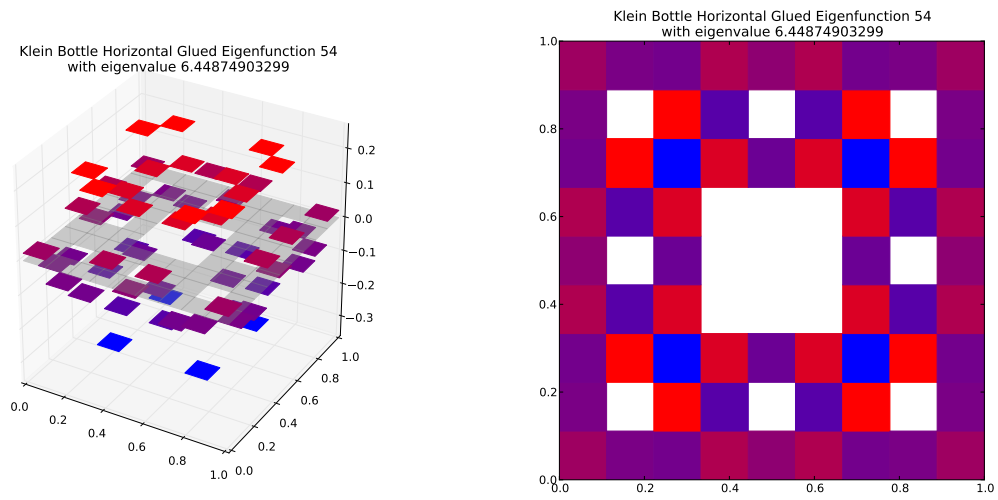
Dot Value: 0.35617904239986686

## 77 $M = 3$ Eigenfunction 76

$M = 3$  Eigenfunction 76 has eigenvalue 1.73428639395



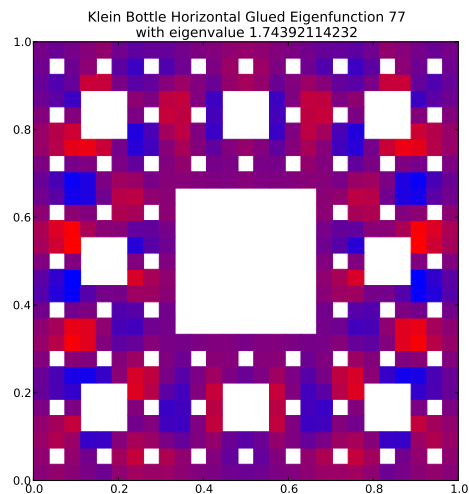
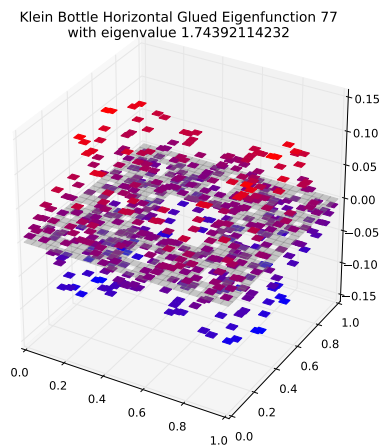
Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299



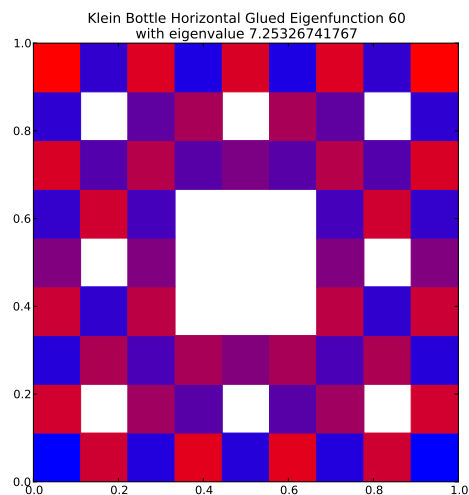
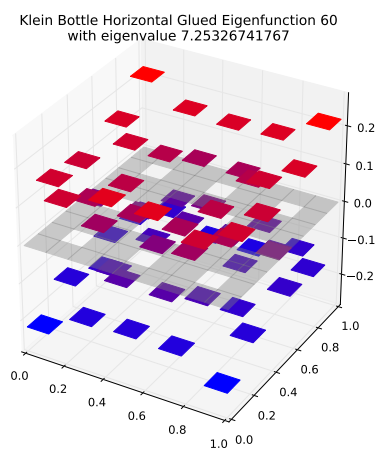
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.268933770733$   
Dot Value: 0.3946114640238394

## 78 $M = 3$ Eigenfunction 77

$M = 3$  Eigenfunction 77 has eigenvalue 1.74392114232



Compare to  $m = 2$  eigenspace with eigenvalue 7.25326741767

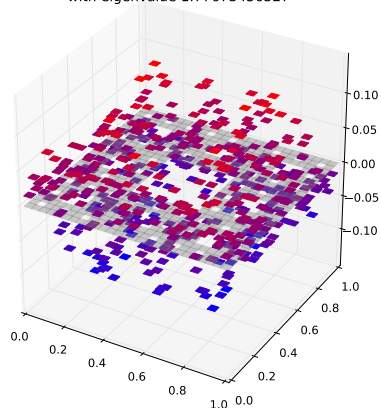


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.240432489511$   
Dot Value: 0.40590363943881425

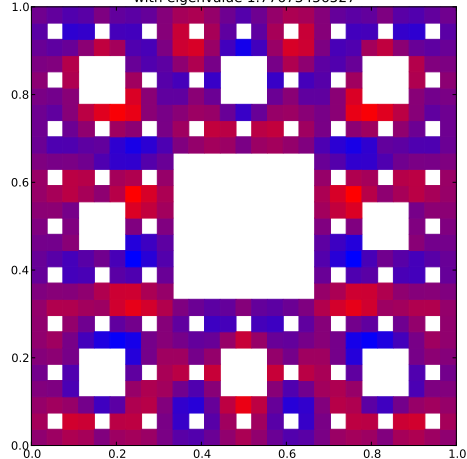
## 79 $M = 3$ Eigenfunction 78

$M = 3$  Eigenfunction 78 has eigenvalue 1.77673456527

Klein Bottle Horizontal Glued Eigenfunction 78  
with eigenvalue 1.77673456527

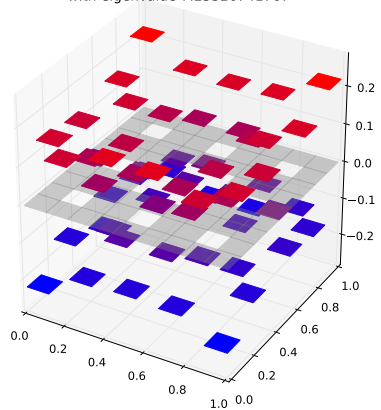


Klein Bottle Horizontal Glued Eigenfunction 78  
with eigenvalue 1.77673456527

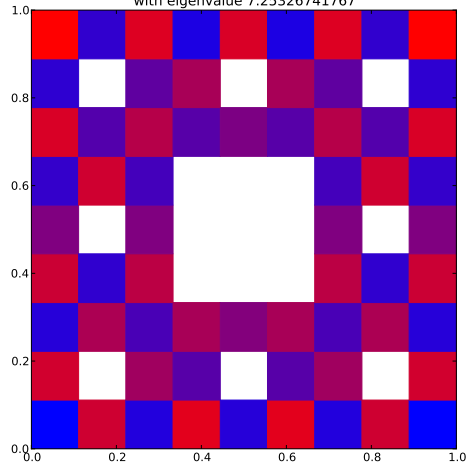


Compare to  $m = 2$  eigenspace with eigenvalue 7.25326741767

Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767

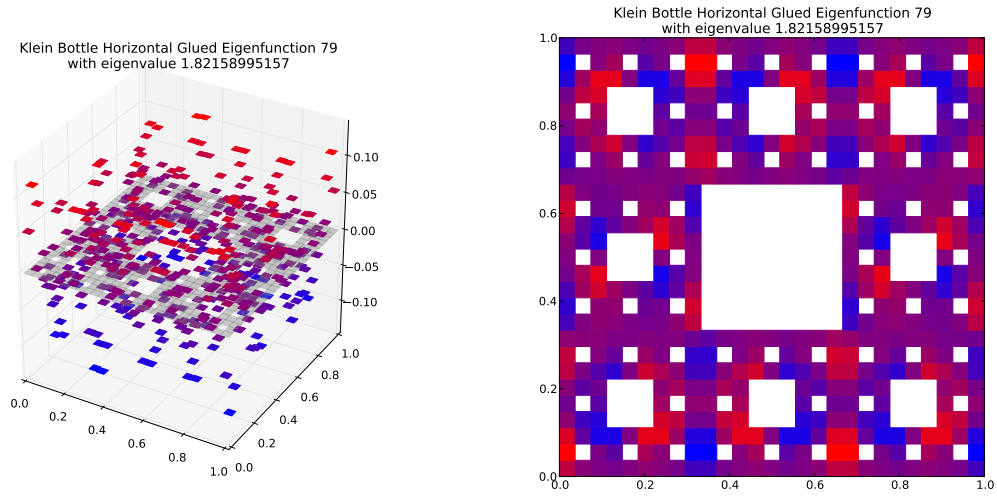


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.244956440037$

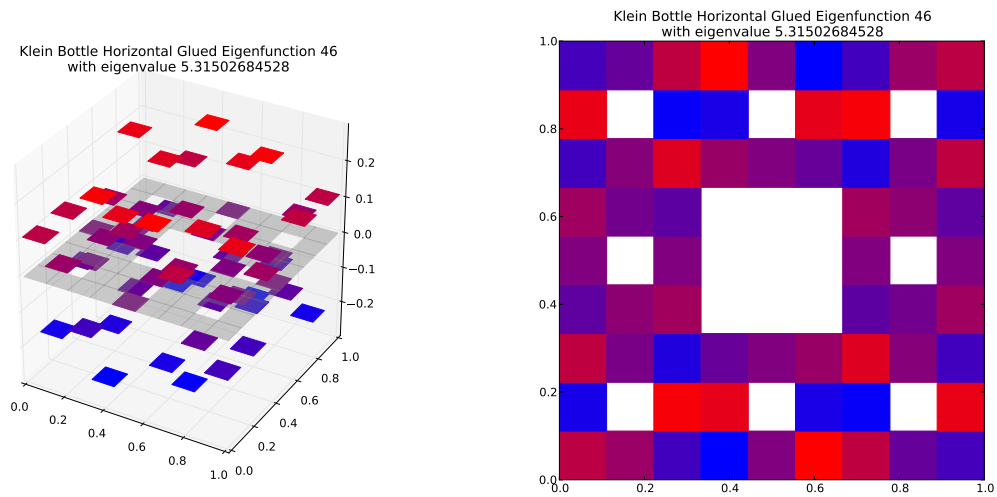
Dot Value: 0.20365202952588635

## 80 $M = 3$ Eigenfunction 79

$M = 3$  Eigenfunction 79 has eigenvalue 1.82158995157



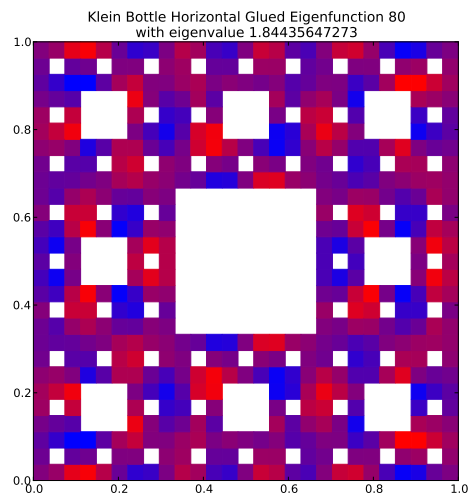
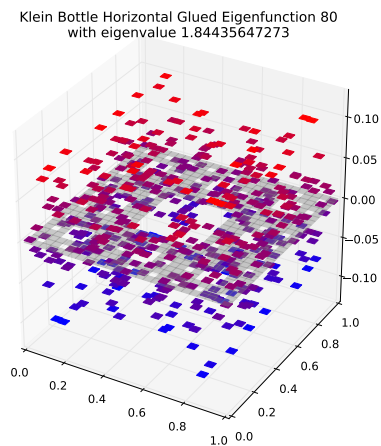
Compare to  $m = 2$  eigenspace with eigenvalue 5.31502684528



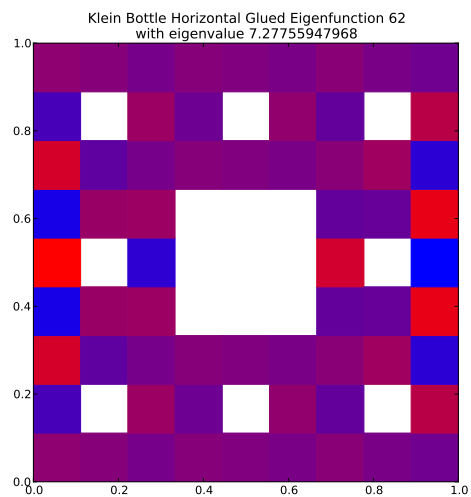
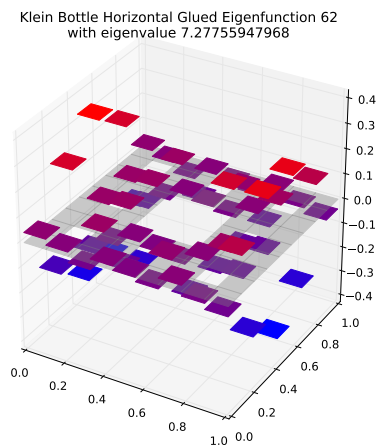
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.34272450631$   
Dot Value: 0.05471357365990914

# 81 $M = 3$ Eigenfunction 80

$M = 3$  Eigenfunction 80 has eigenvalue 1.84435647273



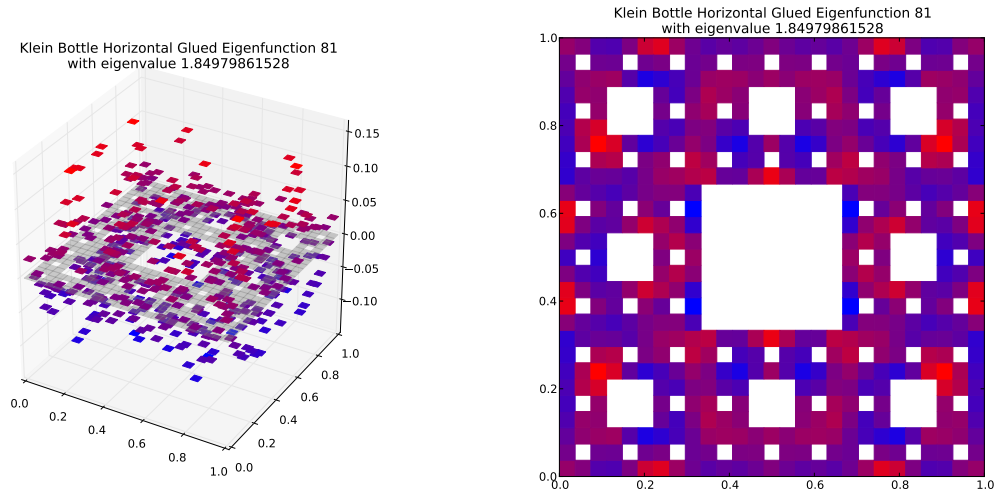
Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968



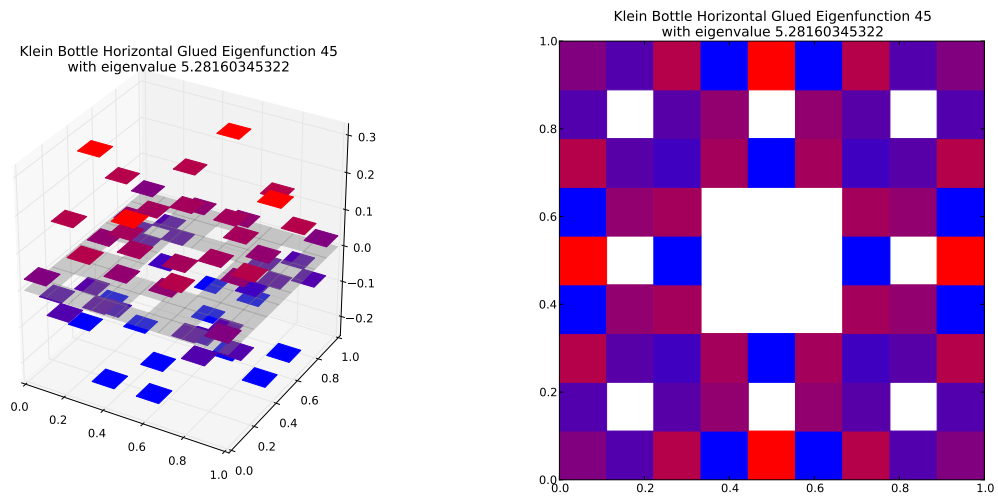
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.253430628479$   
Dot Value: 0.36963905145376375

## 82 $M = 3$ Eigenfunction 81

$M = 3$  Eigenfunction 81 has eigenvalue 1.84979861528



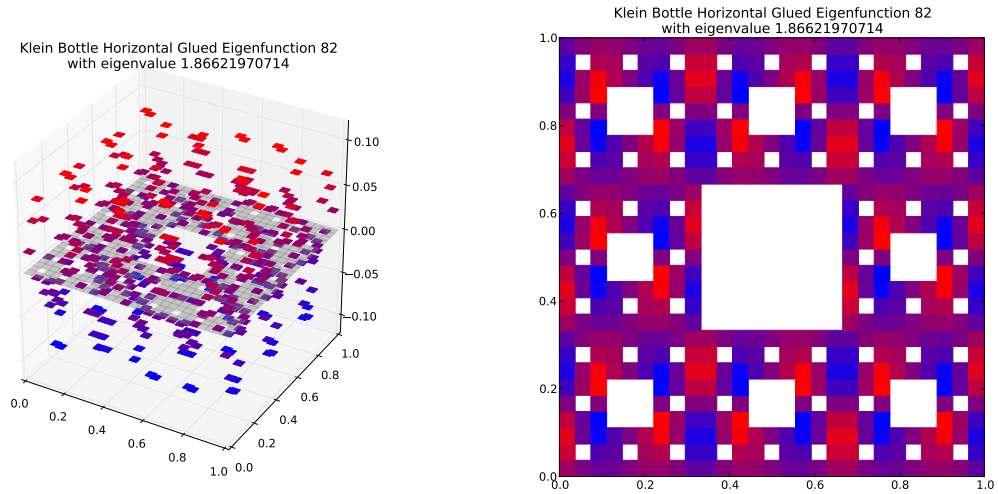
Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322



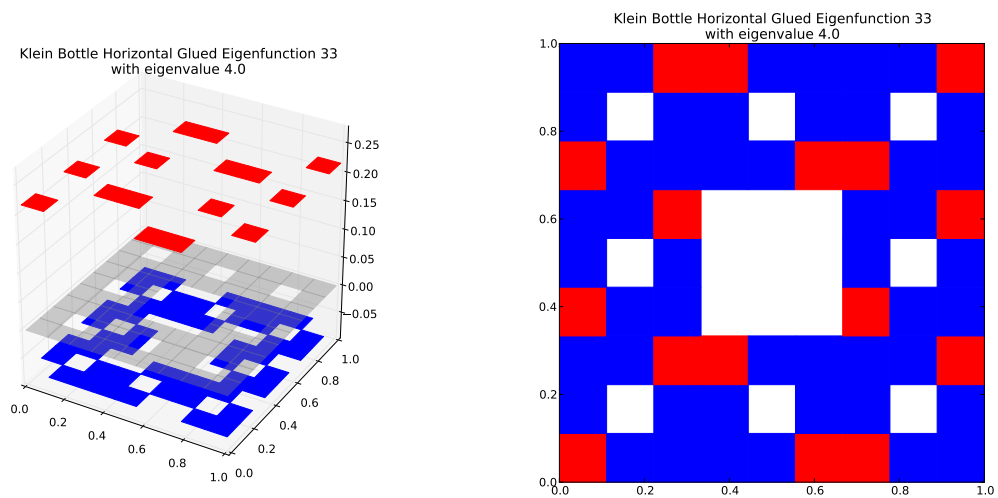
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.350234286172$   
Dot Value: 0.42837500067982714

### 83 $M = 3$ Eigenfunction 82

$M = 3$  Eigenfunction 82 has eigenvalue 1.86621970714



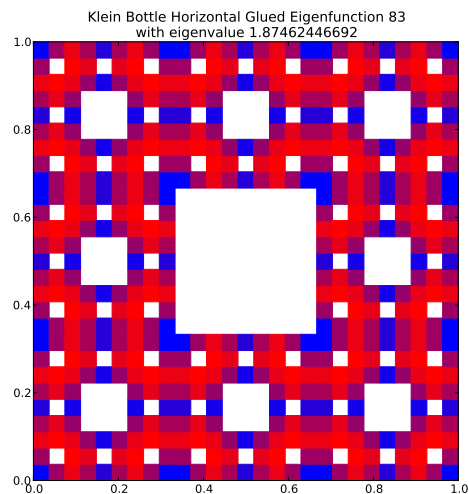
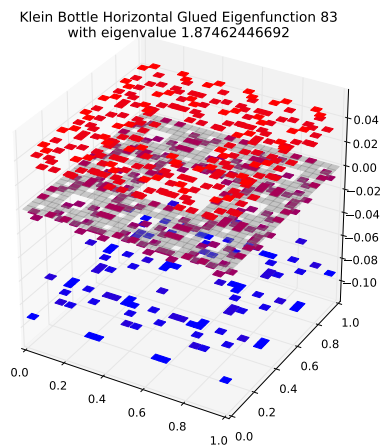
Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )



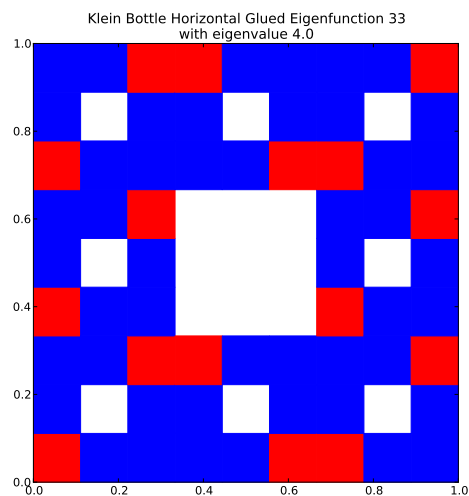
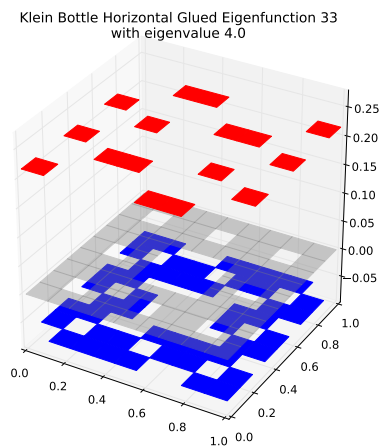
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.466554926786$   
Dot Value: 0.0

# 84 $M = 3$ Eigenfunction 83

$M = 3$  Eigenfunction 83 has eigenvalue 1.87462446692



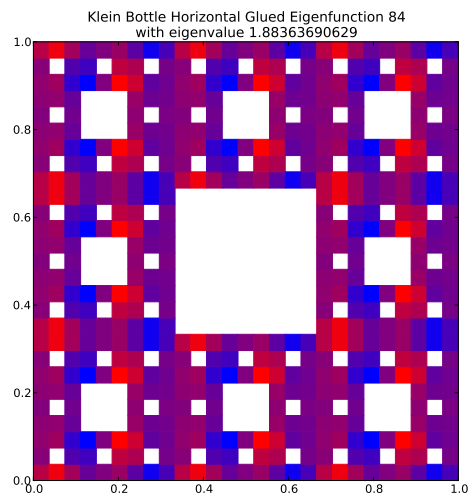
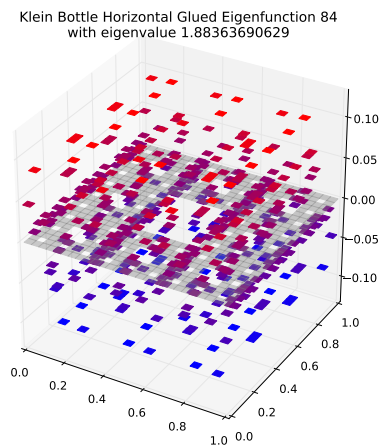
Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )



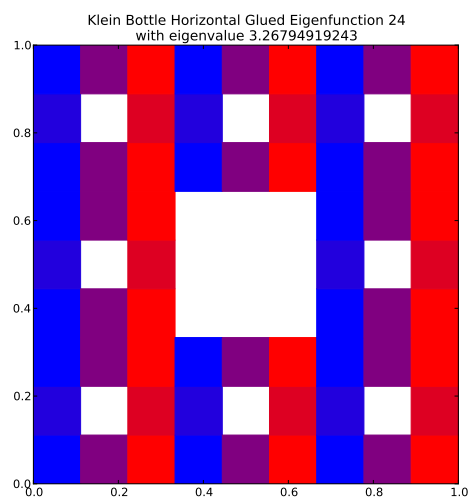
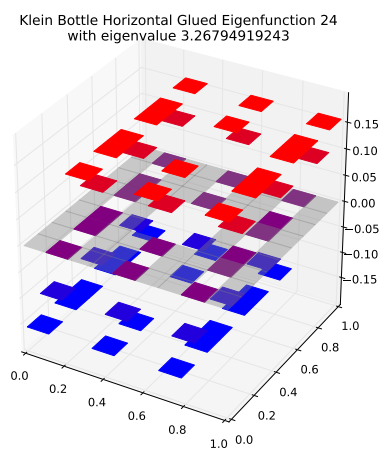
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.468656116731$   
Dot Value: 2.220446049250313e-16

## 85 $M = 3$ Eigenfunction 84

$M = 3$  Eigenfunction 84 has eigenvalue 1.88363690629



Compare to  $m = 2$  eigenspace with eigenvalue 3.26794919243

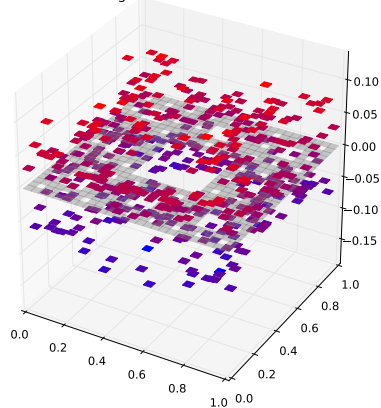


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.576397243462$   
Dot Value: 0.1131236807071877

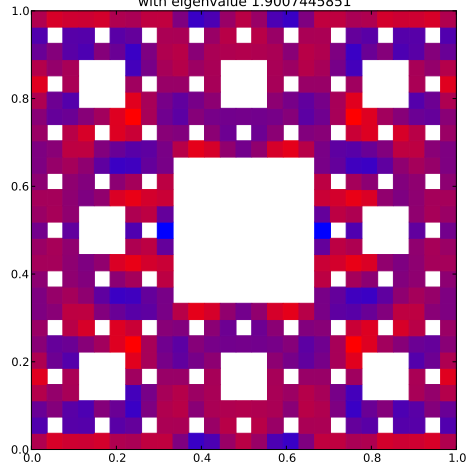
## 86 $M = 3$ Eigenfunction 85

$M = 3$  Eigenfunction 85 has eigenvalue 1.9007445851

Klein Bottle Horizontal Glued Eigenfunction 85  
with eigenvalue 1.9007445851

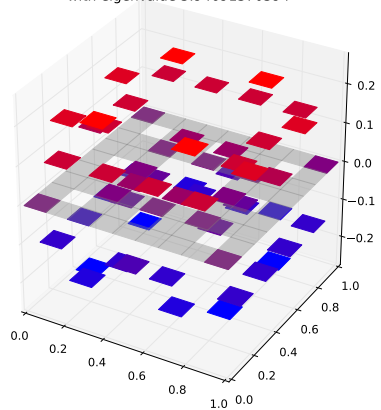


Klein Bottle Horizontal Glued Eigenfunction 85  
with eigenvalue 1.9007445851

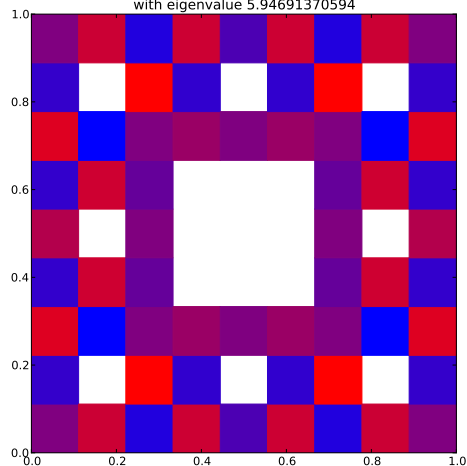


Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594

Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594



Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594



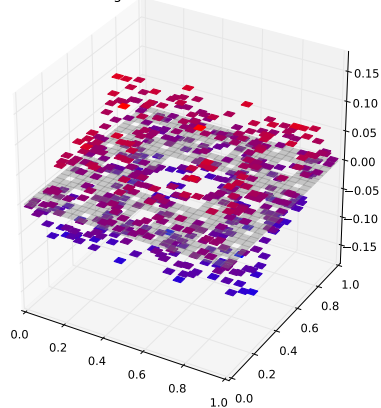
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.319618659205$

Dot Value: 0.42316907141149374

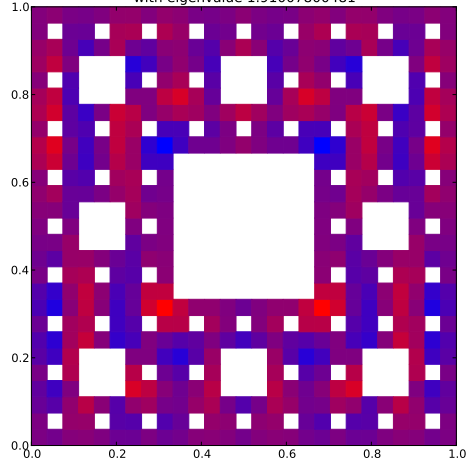
## 87 $M = 3$ Eigenfunction 86

$M = 3$  Eigenfunction 86 has eigenvalue 1.91007800481

Klein Bottle Horizontal Glued Eigenfunction 86  
with eigenvalue 1.91007800481

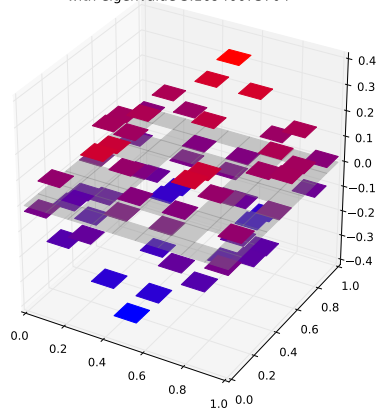


Klein Bottle Horizontal Glued Eigenfunction 86  
with eigenvalue 1.91007800481

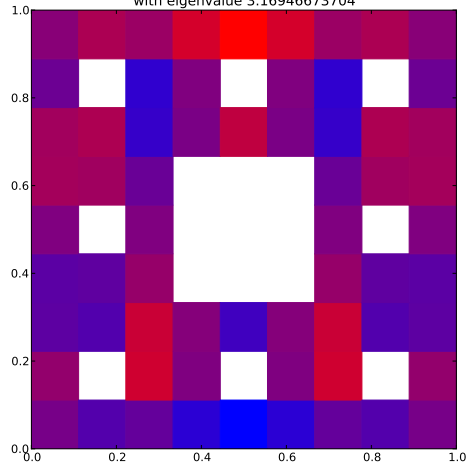


Compare to  $m = 2$  eigenspace with eigenvalue 3.16946673704

Klein Bottle Horizontal Glued Eigenfunction 22  
with eigenvalue 3.16946673704



Klein Bottle Horizontal Glued Eigenfunction 22  
with eigenvalue 3.16946673704

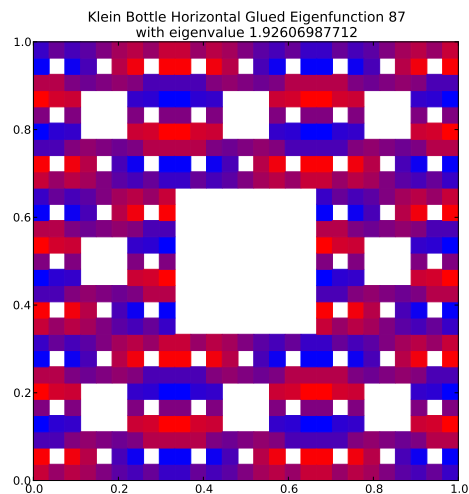
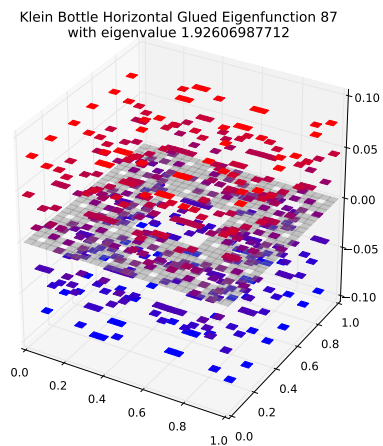


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.602649645282$

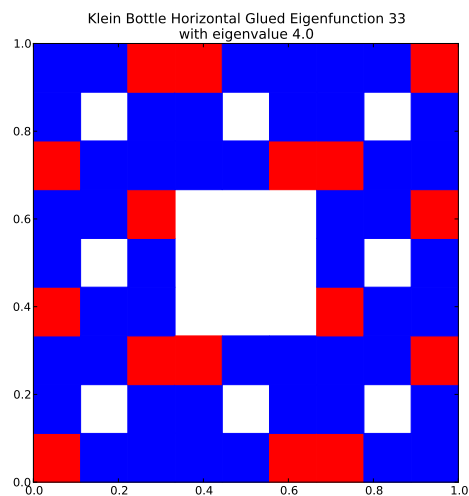
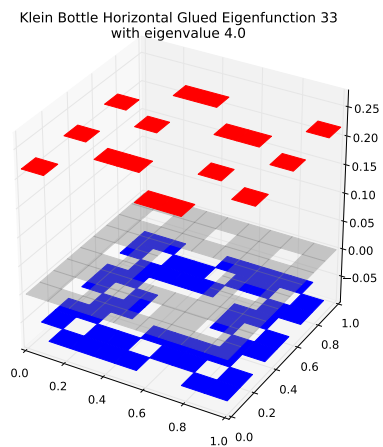
Dot Value: 0.5388353506197976

# 88 $M = 3$ Eigenfunction 87

$M = 3$  Eigenfunction 87 has eigenvalue 1.92606987712



Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )

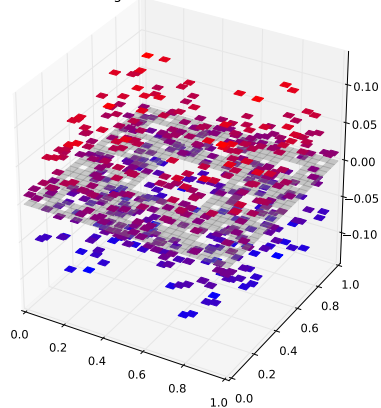


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.481517469281$   
Dot Value: 0.0

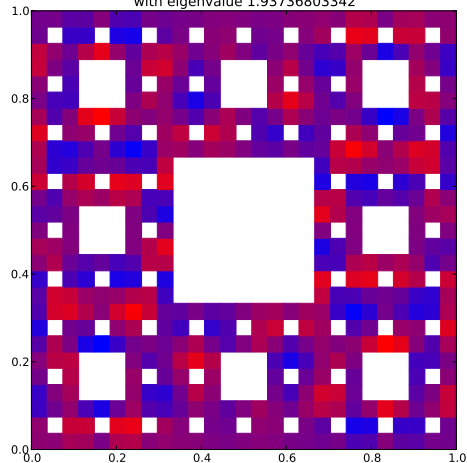
## 89 $M = 3$ Eigenfunction 88

$M = 3$  Eigenfunction 88 has eigenvalue 1.93736803342

Klein Bottle Horizontal Glued Eigenfunction 88  
with eigenvalue 1.93736803342

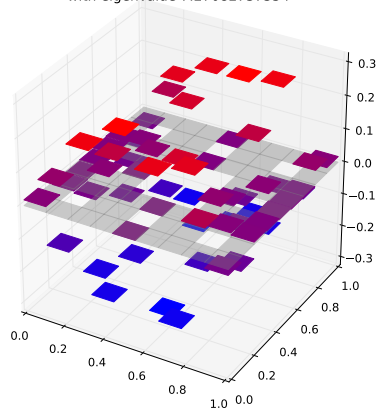


Klein Bottle Horizontal Glued Eigenfunction 88  
with eigenvalue 1.93736803342

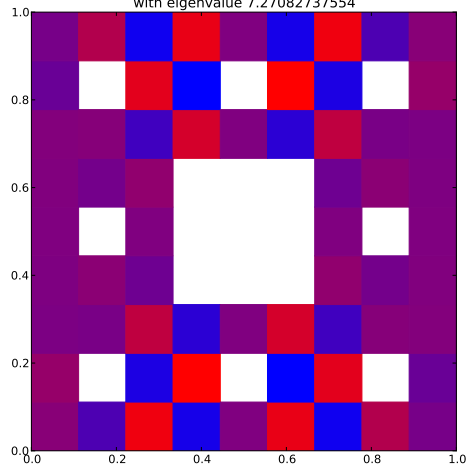


Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554

Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554



Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554

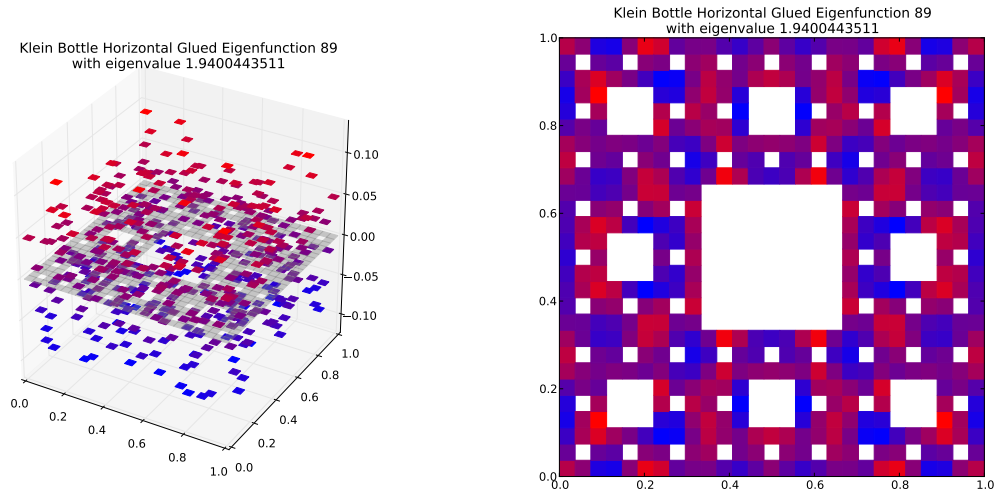


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.266457712906$

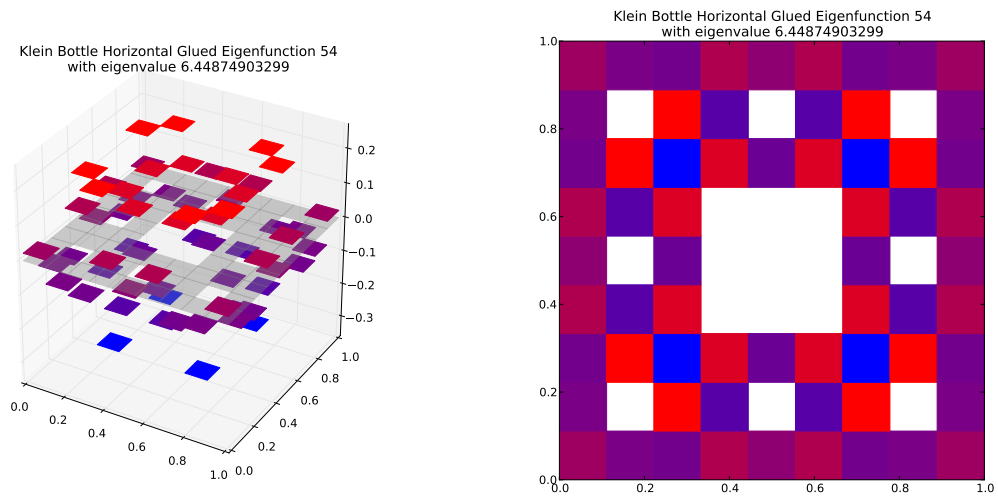
Dot Value: 0.32747259870684253

## 90 $M = 3$ Eigenfunction 89

$M = 3$  Eigenfunction 89 has eigenvalue 1.9400443511



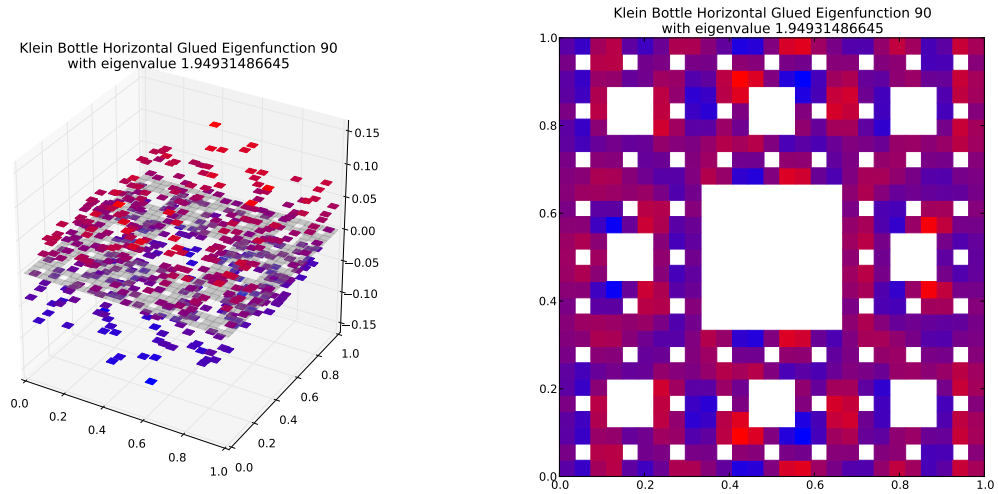
Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299



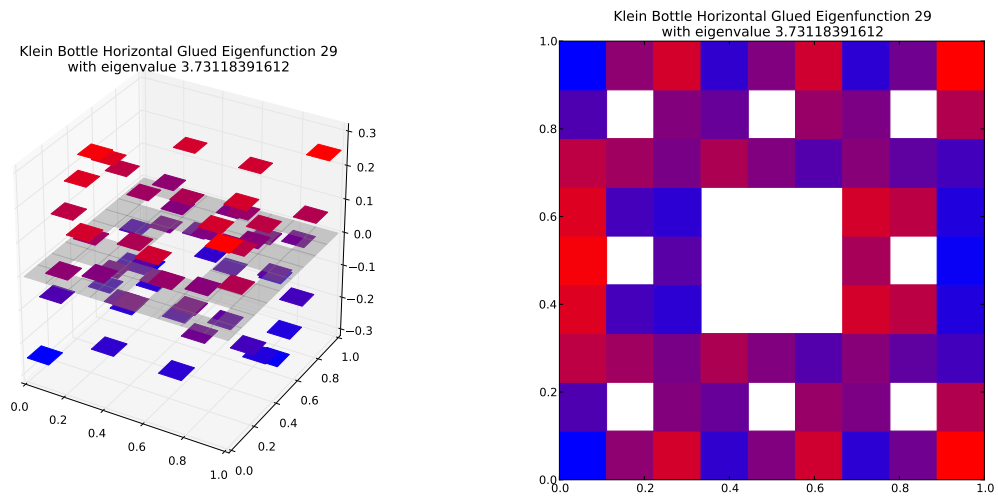
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.300840417448$   
Dot Value: 0.2847592280148472

# 91 $M = 3$ Eigenfunction 90

$M = 3$  Eigenfunction 90 has eigenvalue 1.94931486645



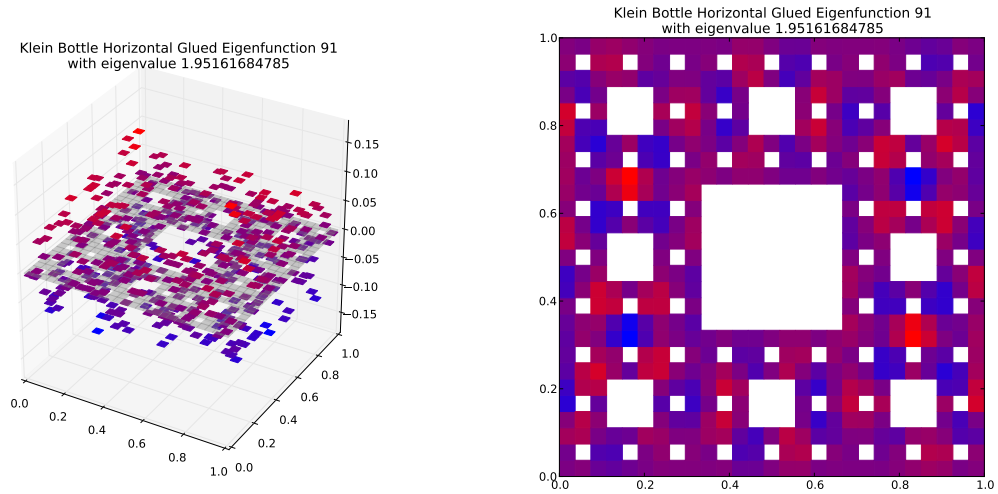
Compare to  $m = 2$  eigenspace with eigenvalue 3.73118391612



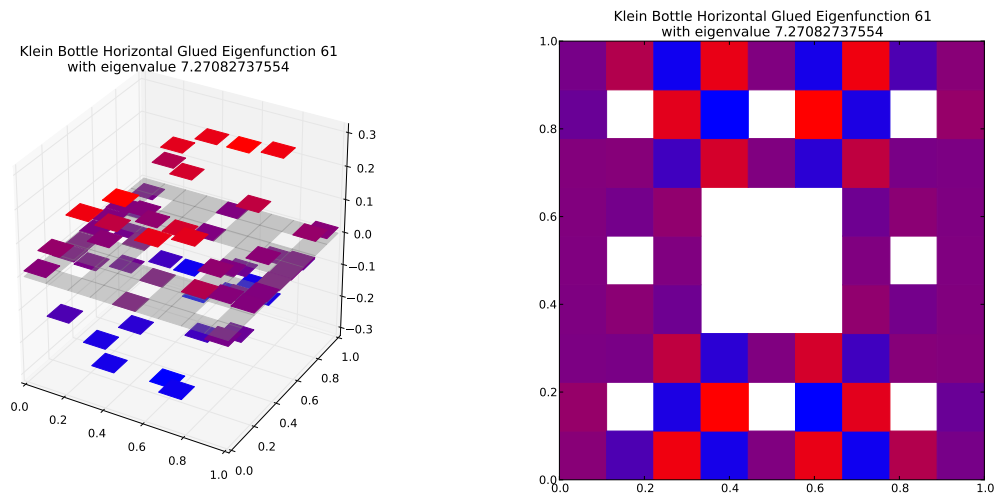
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.522438697817$   
Dot Value: 0.5268369081847544

## 92 $M = 3$ Eigenfunction 91

$M = 3$  Eigenfunction 91 has eigenvalue 1.95161684785



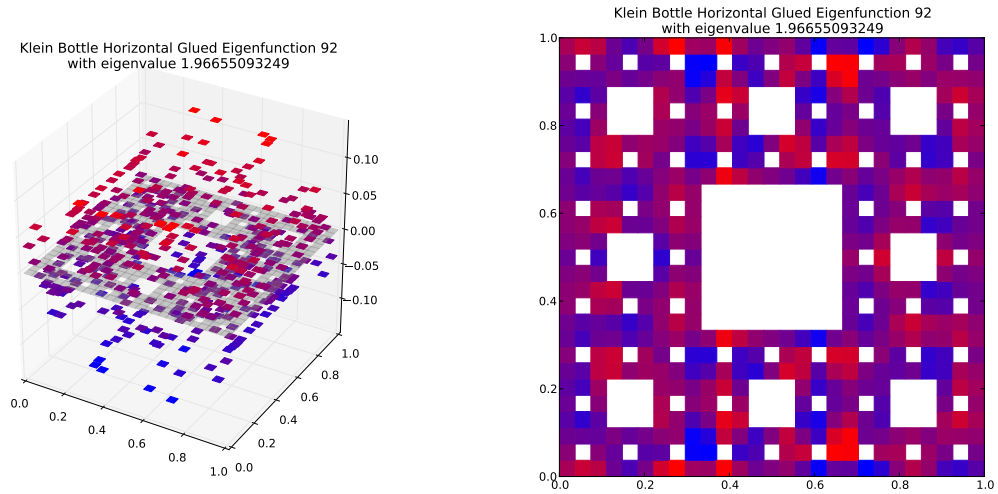
Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554



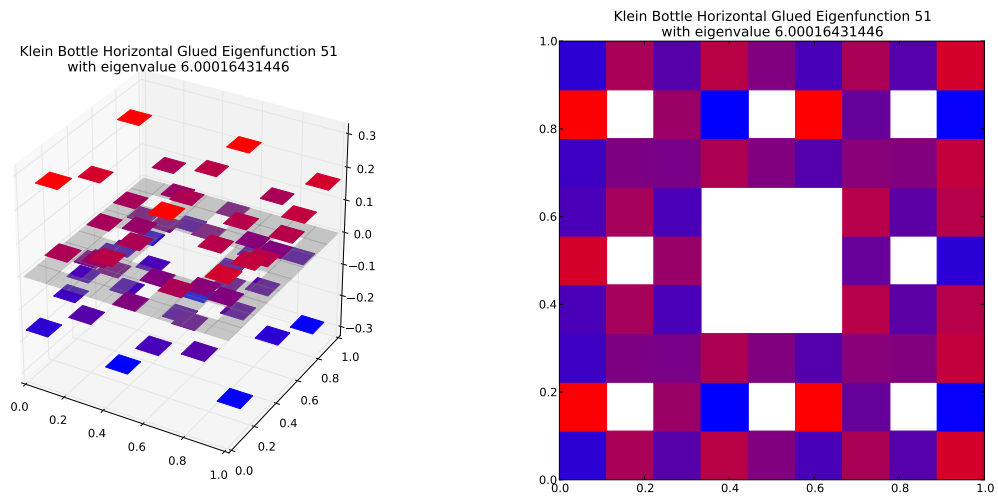
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.26841743684$   
Dot Value: 0.269598320889046

### 93 $M = 3$ Eigenfunction 92

$M = 3$  Eigenfunction 92 has eigenvalue 1.96655093249



Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446

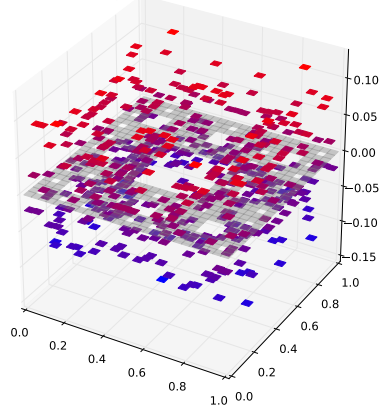


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.327749513084$   
Dot Value: 0.23863900621499734

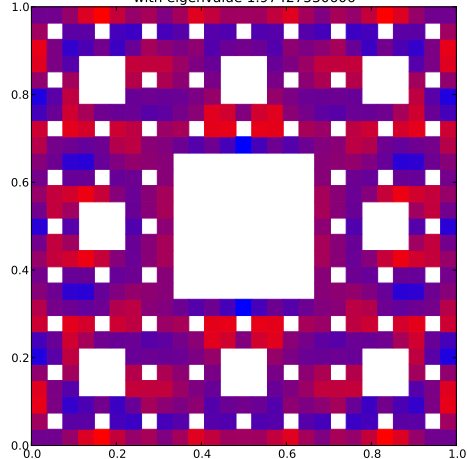
## 94 $M = 3$ Eigenfunction 93

$M = 3$  Eigenfunction 93 has eigenvalue 1.97427530606

Klein Bottle Horizontal Glued Eigenfunction 93  
with eigenvalue 1.97427530606

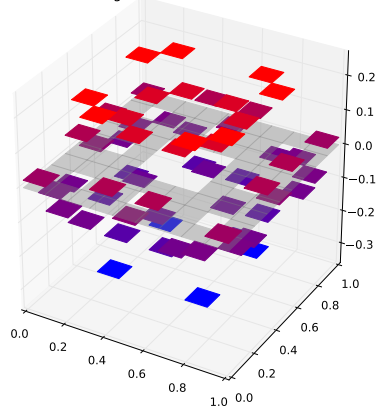


Klein Bottle Horizontal Glued Eigenfunction 93  
with eigenvalue 1.97427530606

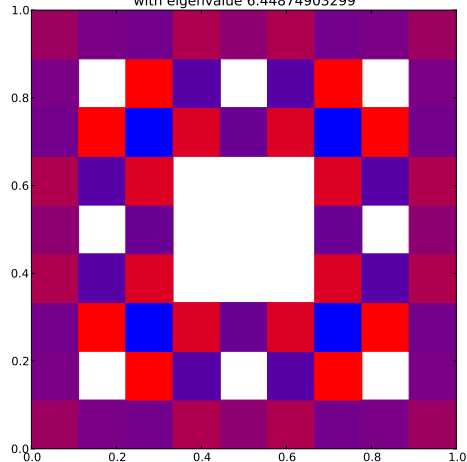


Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299



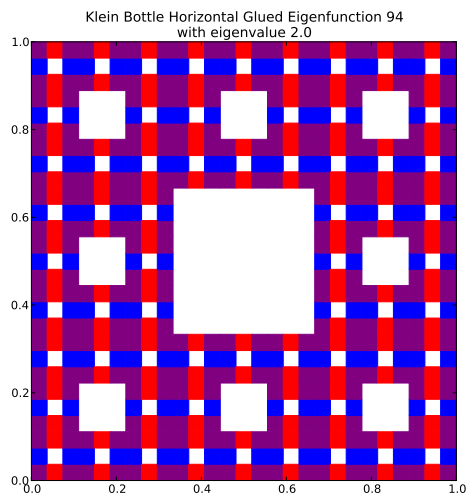
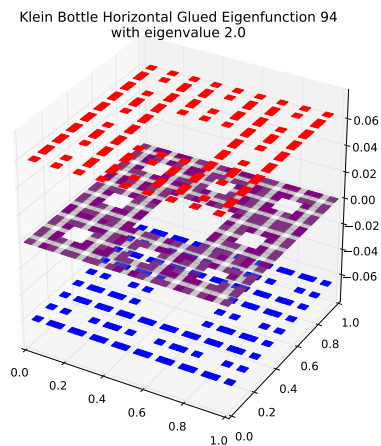
Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299



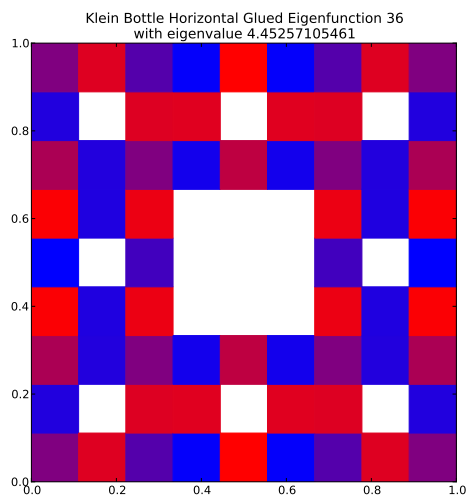
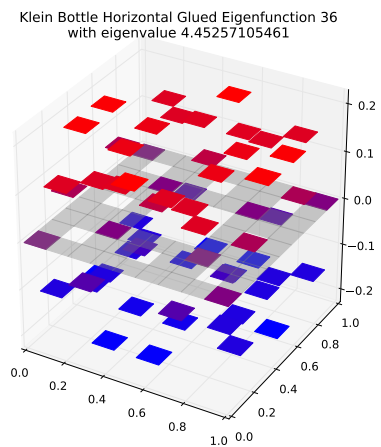
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.306148571756$   
Dot Value: 0.5104284160935966

# 95 $M = 3$ Eigenfunction 94

$M = 3$  Eigenfunction 94 has eigenvalue 2.0



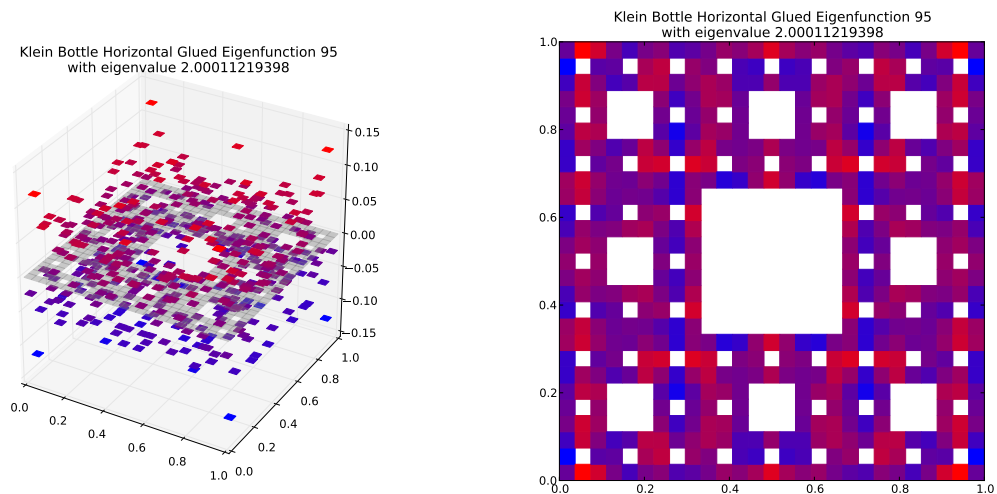
Compare to  $m = 2$  eigenspace with eigenvalue 4.45257105461



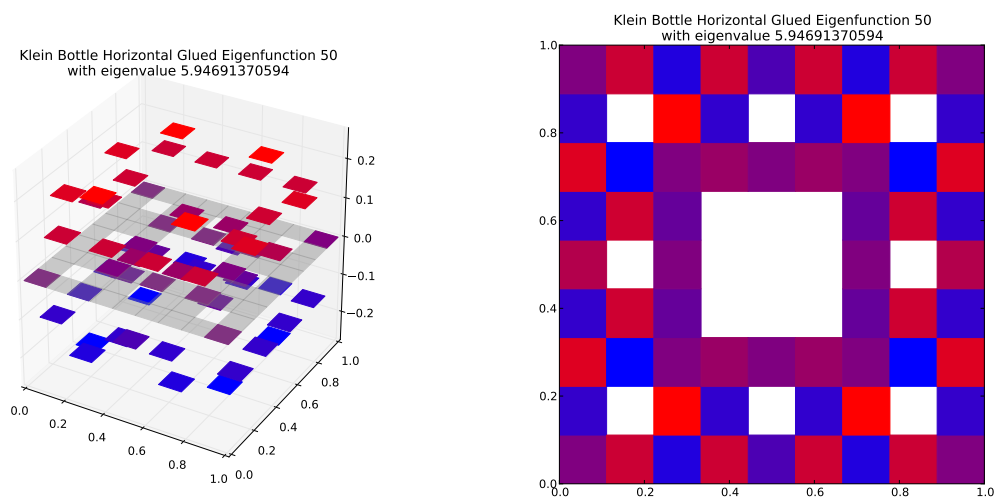
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.44917868249$   
 Dot Value: 2

## 96 $M = 3$ Eigenfunction 95

$M = 3$  Eigenfunction 95 has eigenvalue 2.00011219398



Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594

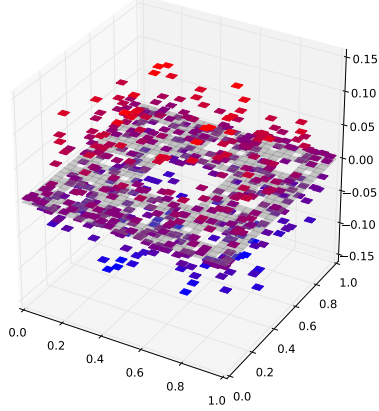


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.336327764768$   
Dot Value: 0.4525207479979455

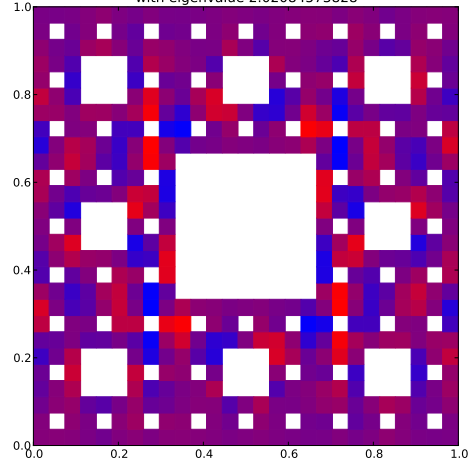
# 97 $M = 3$ Eigenfunction 96

$M = 3$  Eigenfunction 96 has eigenvalue 2.02084573828

Klein Bottle Horizontal Glued Eigenfunction 96  
with eigenvalue 2.02084573828

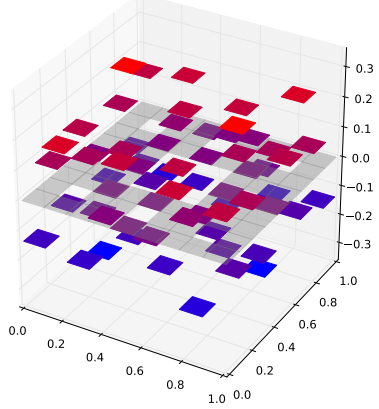


Klein Bottle Horizontal Glued Eigenfunction 96  
with eigenvalue 2.02084573828

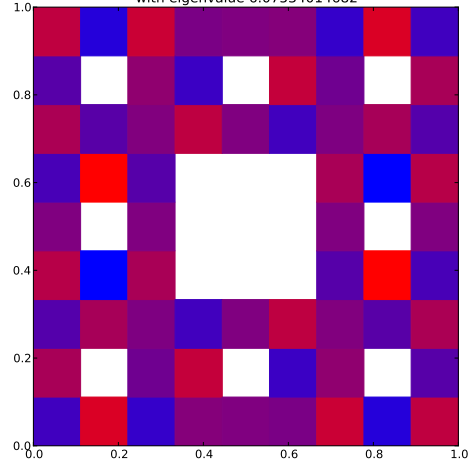


Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682

Klein Bottle Horizontal Glued Eigenfunction 52  
with eigenvalue 6.07534014682



Klein Bottle Horizontal Glued Eigenfunction 52  
with eigenvalue 6.07534014682



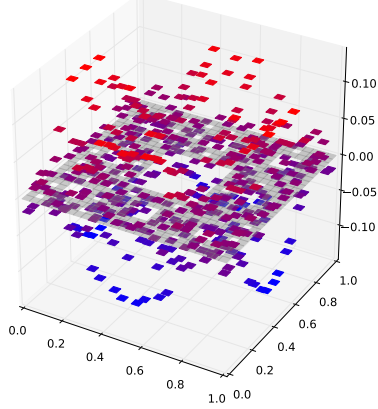
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.332630879827$

Dot Value: 0.2786193239792307

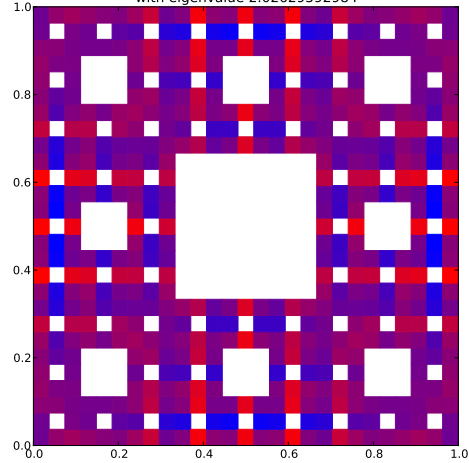
# 98 $M = 3$ Eigenfunction 97

$M = 3$  Eigenfunction 97 has eigenvalue 2.02629592984

Klein Bottle Horizontal Glued Eigenfunction 97  
with eigenvalue 2.02629592984

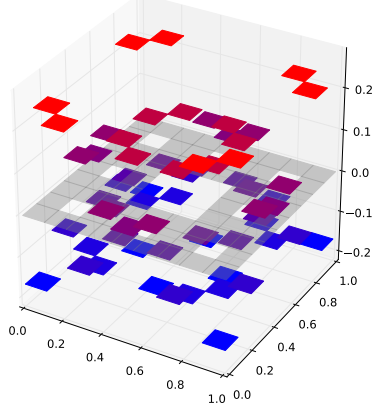


Klein Bottle Horizontal Glued Eigenfunction 97  
with eigenvalue 2.02629592984

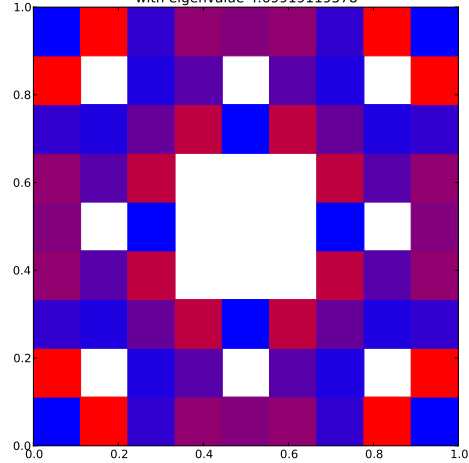


Compare to  $m = 2$  eigenspace with eigenvalue 4.69919119578

Klein Bottle Horizontal Glued Eigenfunction 39  
with eigenvalue 4.69919119578



Klein Bottle Horizontal Glued Eigenfunction 39  
with eigenvalue 4.69919119578



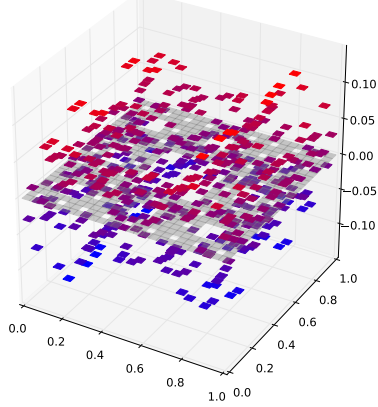
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.431200997238$

Dot Value: 0.359409009233972

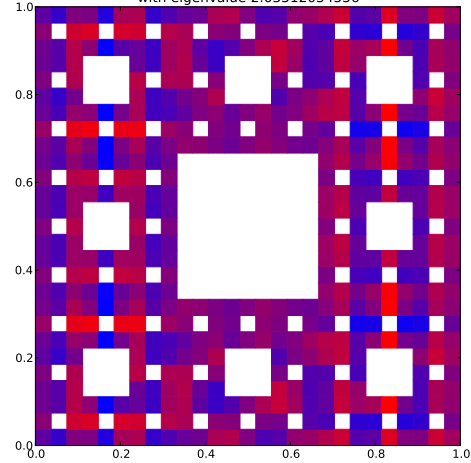
# 99 $M = 3$ Eigenfunction 98

$M = 3$  Eigenfunction 98 has eigenvalue 2.03512054556

Klein Bottle Horizontal Glued Eigenfunction 98  
with eigenvalue 2.03512054556

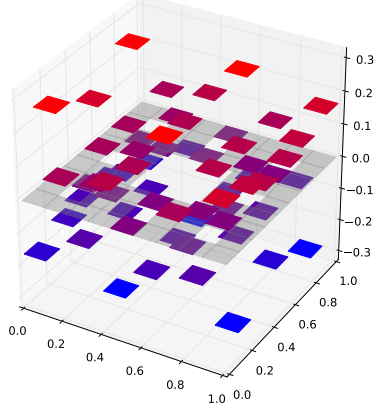


Klein Bottle Horizontal Glued Eigenfunction 98  
with eigenvalue 2.03512054556

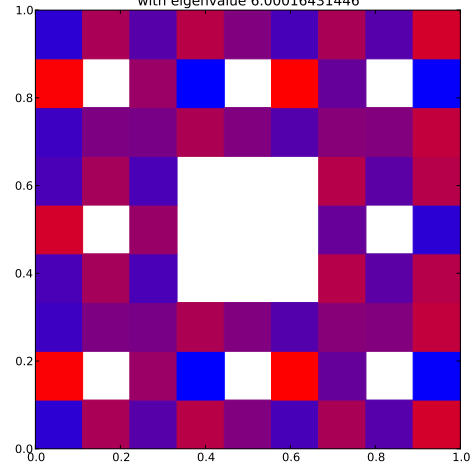


Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446

Klein Bottle Horizontal Glued Eigenfunction 51  
with eigenvalue 6.00016431446



Klein Bottle Horizontal Glued Eigenfunction 51  
with eigenvalue 6.00016431446

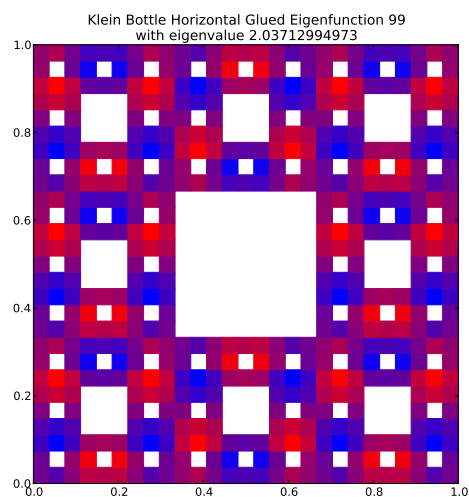
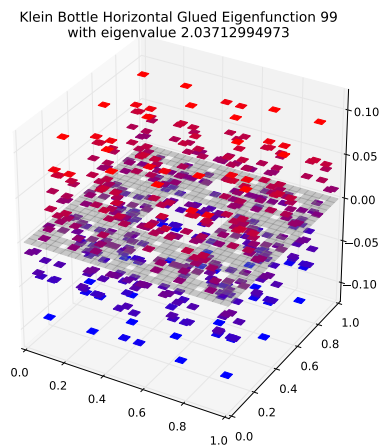


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.339177468966$

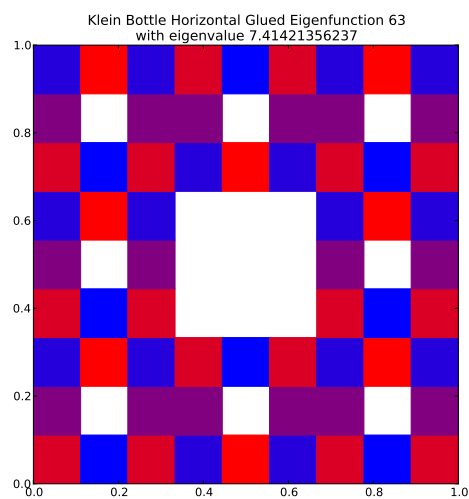
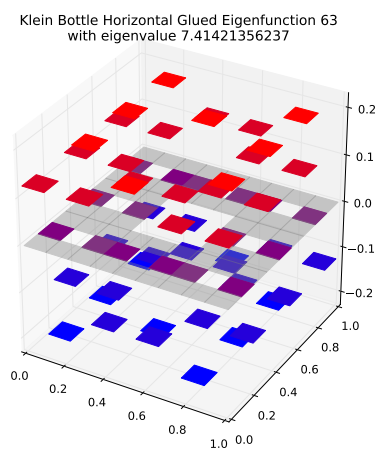
Dot Value: 0.5493487157983998

# 100 $M = 3$ Eigenfunction 99

$M = 3$  Eigenfunction 99 has eigenvalue 2.03712994973



Compare to  $m = 2$  eigenspace with eigenvalue 7.41421356237

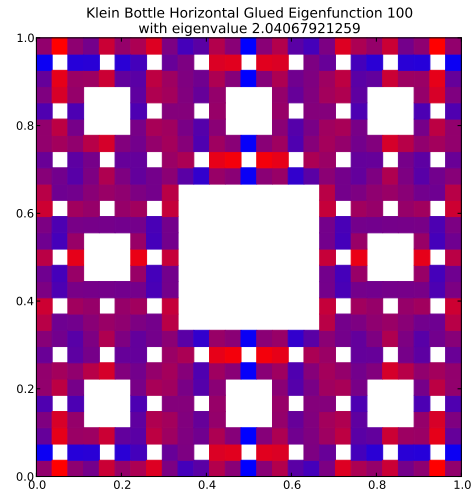
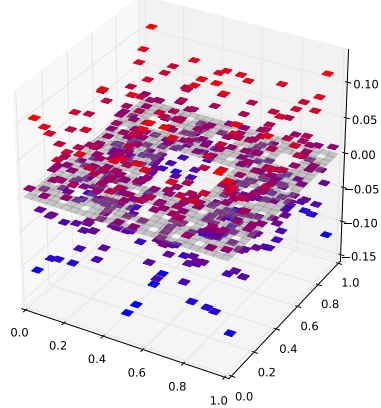


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.274760085152$   
Dot Value: 0.0017108198585333856

# 101 $M = 3$ Eigenfunction 100

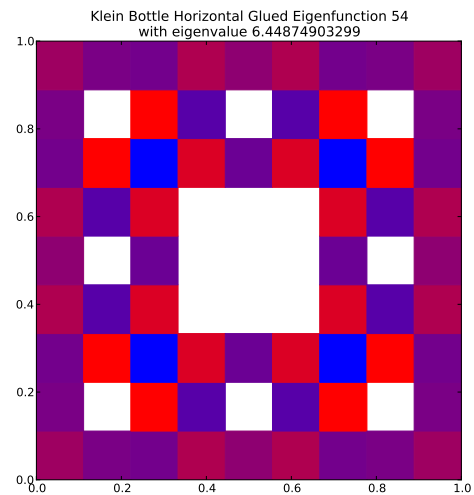
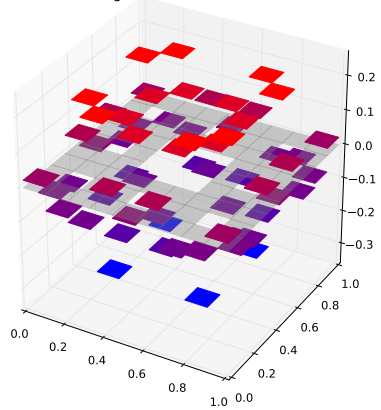
$M = 3$  Eigenfunction 100 has eigenvalue 2.04067921259

Klein Bottle Horizontal Glued Eigenfunction 100  
with eigenvalue 2.04067921259



Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299

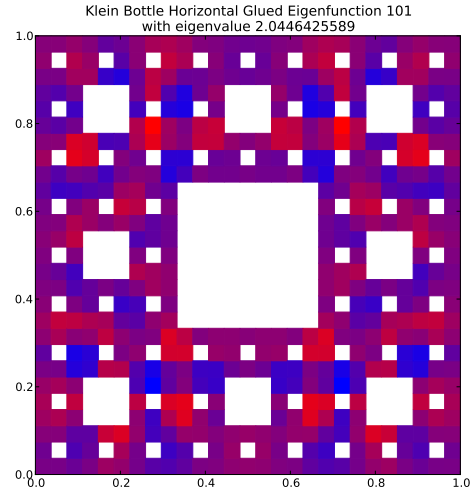
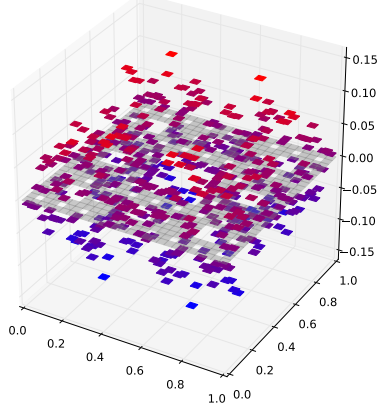


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.316445748183$   
Dot Value: 0.4911859568560668

# 102 $M = 3$ Eigenfunction 101

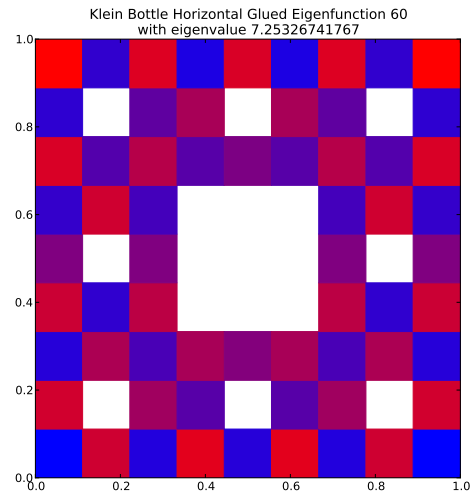
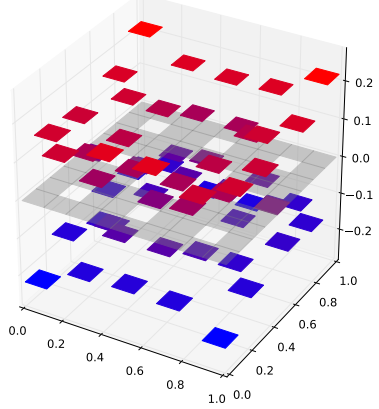
$M = 3$  Eigenfunction 101 has eigenvalue 2.0446425589

Klein Bottle Horizontal Glued Eigenfunction 101  
with eigenvalue 2.0446425589



Compare to  $m = 2$  eigenspace with eigenvalue 7.25326741767

Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



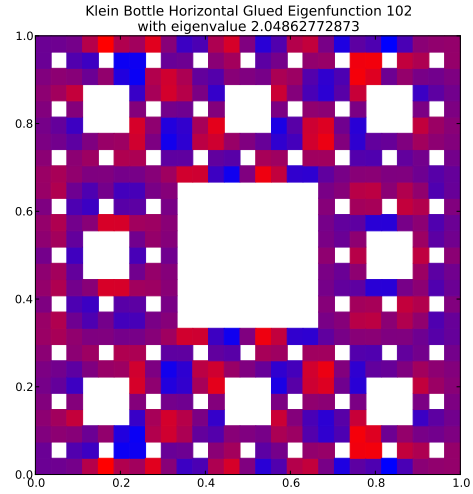
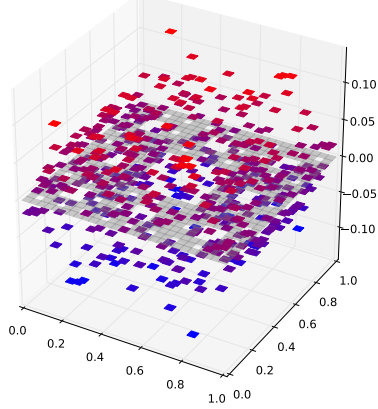
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.281892620409$

Dot Value: 0.3700175288632037

# 103 $M = 3$ Eigenfunction 102

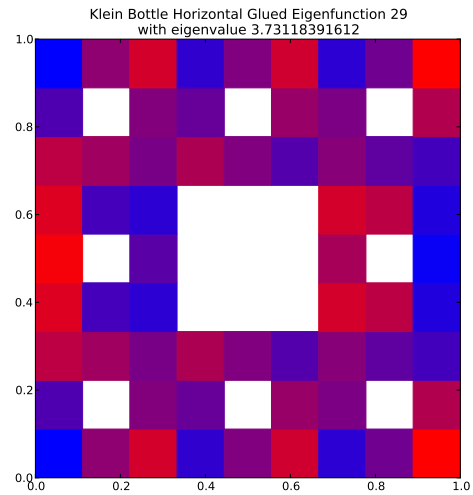
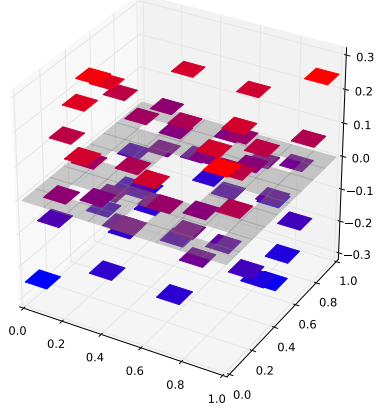
$M = 3$  Eigenfunction 102 has eigenvalue 2.04862772873

Klein Bottle Horizontal Glued Eigenfunction 102  
with eigenvalue 2.04862772873



Compare to  $m = 2$  eigenspace with eigenvalue 3.73118391612

Klein Bottle Horizontal Glued Eigenfunction 29  
with eigenvalue 3.73118391612



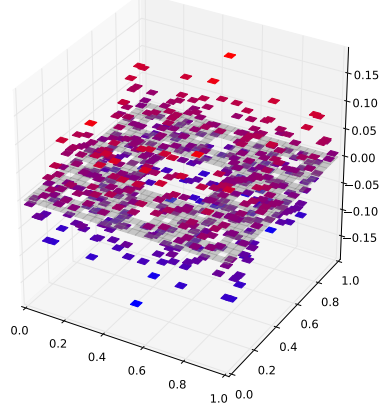
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.54905568173$

Dot Value: 0.3651437694767886

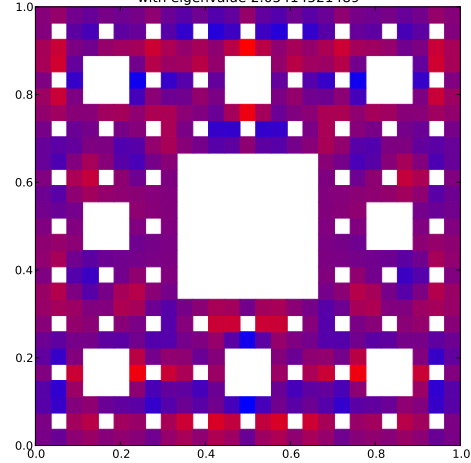
# 104 $M = 3$ Eigenfunction 103

$M = 3$  Eigenfunction 103 has eigenvalue 2.05414521489

Klein Bottle Horizontal Glued Eigenfunction 103  
with eigenvalue 2.05414521489

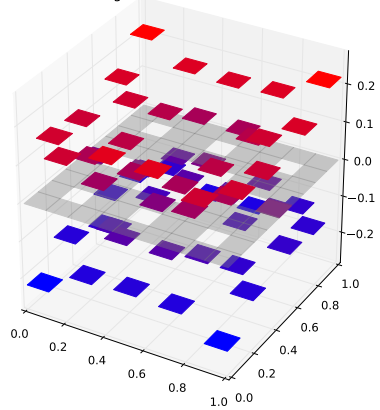


Klein Bottle Horizontal Glued Eigenfunction 103  
with eigenvalue 2.05414521489

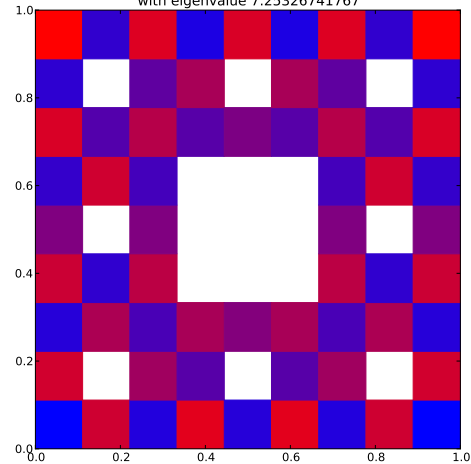


Compare to  $m = 2$  eigenspace with eigenvalue 7.25326741767

Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



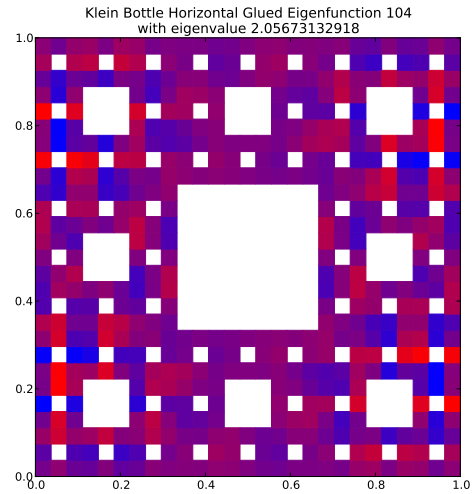
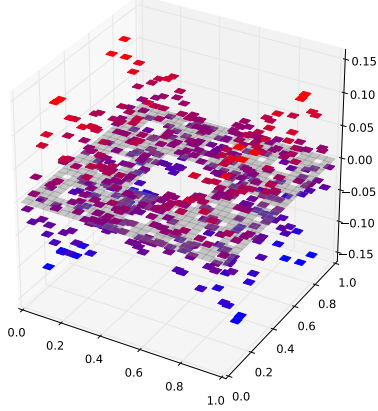
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.283202741137$

Dot Value: 0.3964938006953268

# 105 $M = 3$ Eigenfunction 104

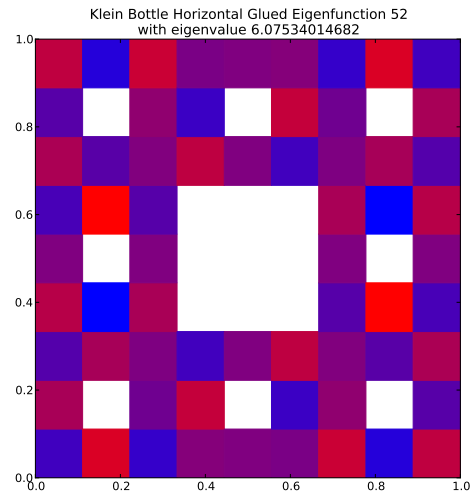
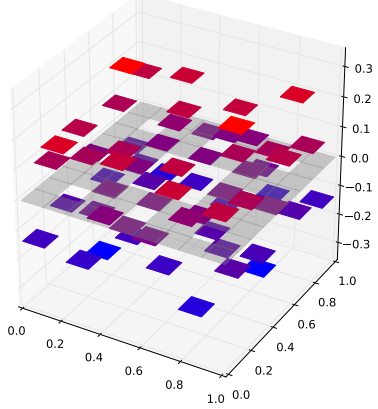
$M = 3$  Eigenfunction 104 has eigenvalue 2.05673132918

Klein Bottle Horizontal Glued Eigenfunction 104  
with eigenvalue 2.05673132918



Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682

Klein Bottle Horizontal Glued Eigenfunction 52  
with eigenvalue 6.07534014682

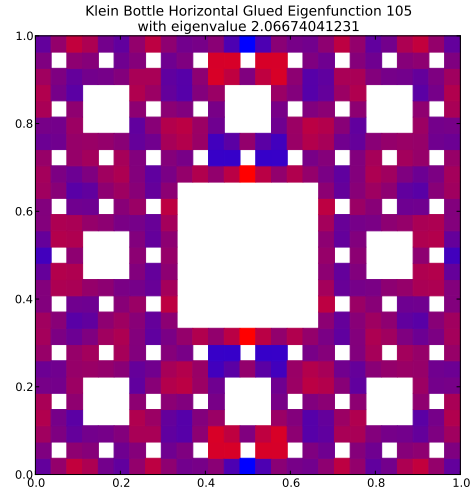
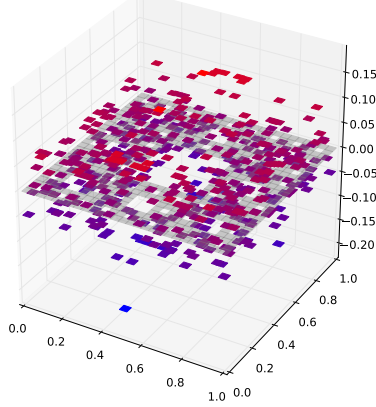


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.338537642252$   
Dot Value: 0.5825305796459691

# 106 $M = 3$ Eigenfunction 105

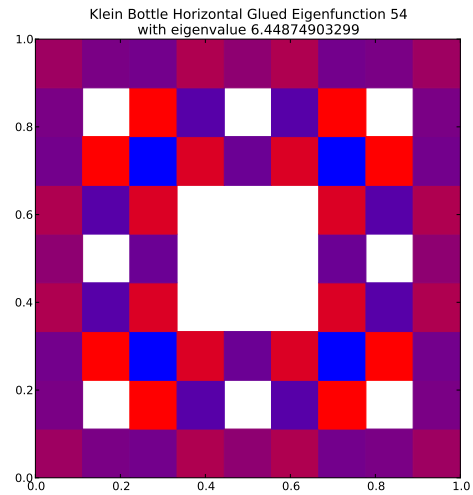
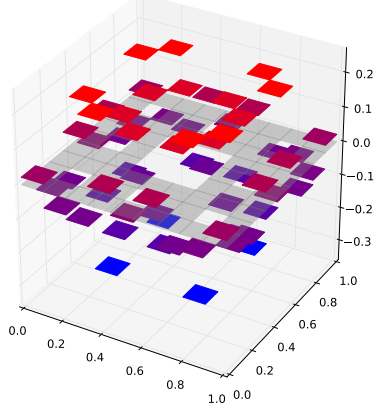
$M = 3$  Eigenfunction 105 has eigenvalue 2.06674041231

Klein Bottle Horizontal Glued Eigenfunction 105  
with eigenvalue 2.06674041231



Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299



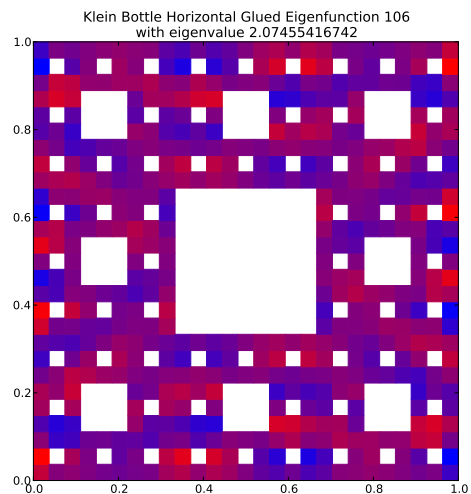
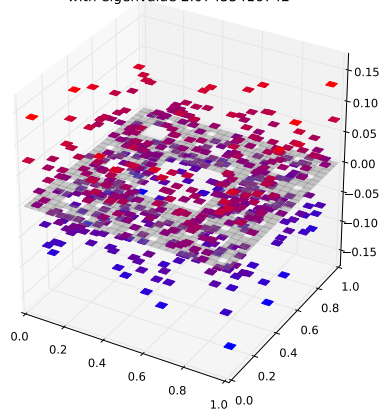
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.320487028063$

Dot Value: 0.3735691922308576

# 107 $M = 3$ Eigenfunction 106

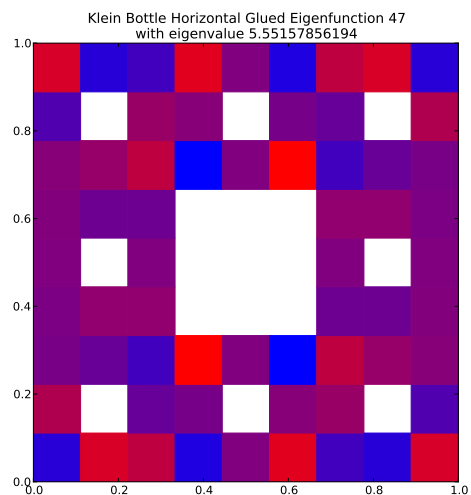
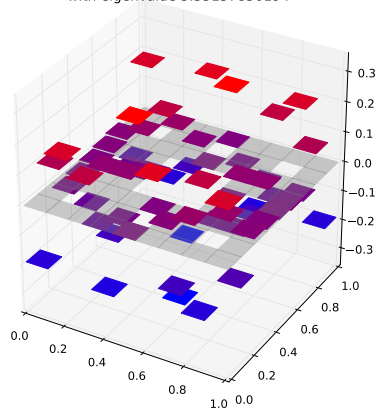
$M = 3$  Eigenfunction 106 has eigenvalue 2.07455416742

Klein Bottle Horizontal Glued Eigenfunction 106  
with eigenvalue 2.07455416742



Compare to  $m = 2$  eigenspace with eigenvalue 5.55157856194

Klein Bottle Horizontal Glued Eigenfunction 47  
with eigenvalue 5.55157856194

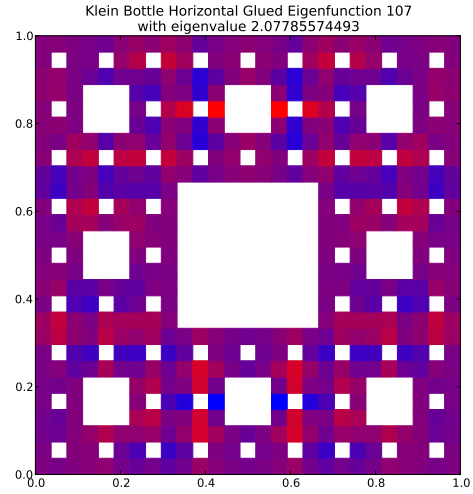
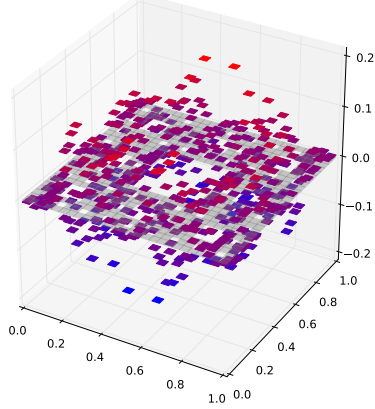


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.373687257467$   
Dot Value: 0.32135406431464786

# 108 $M = 3$ Eigenfunction 107

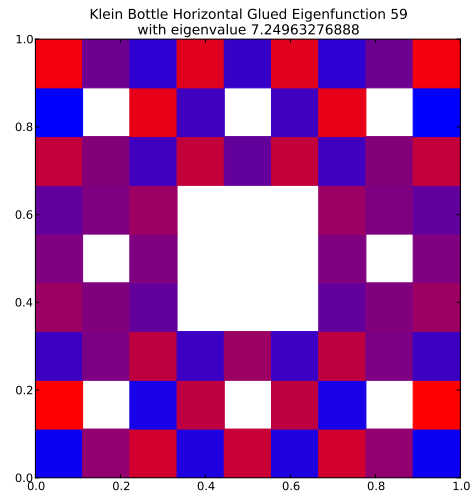
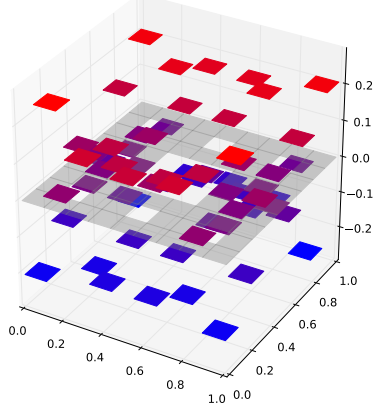
$M = 3$  Eigenfunction 107 has eigenvalue 2.07785574493

Klein Bottle Horizontal Glued Eigenfunction 107  
with eigenvalue 2.07785574493



Compare to  $m = 2$  eigenspace with eigenvalue 7.24963276888

Klein Bottle Horizontal Glued Eigenfunction 59  
with eigenvalue 7.24963276888



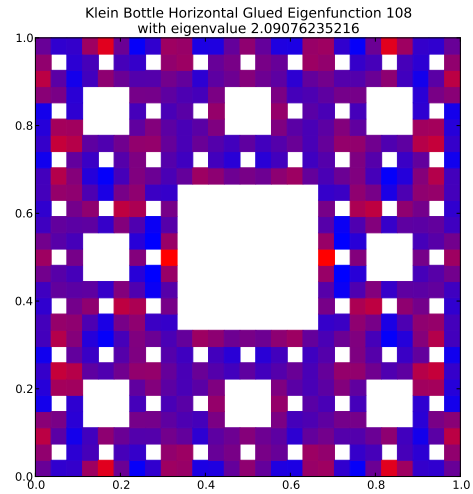
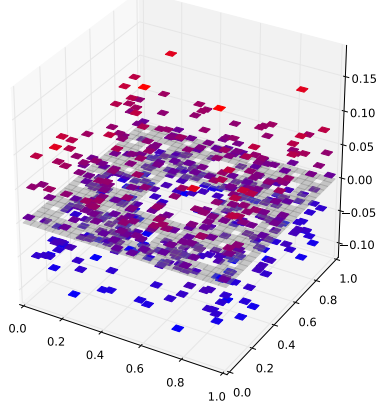
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.286615310206$

Dot Value: 0.348299905695193

# 109 $M = 3$ Eigenfunction 108

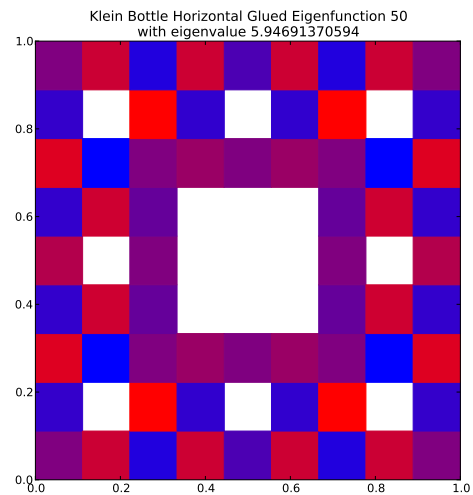
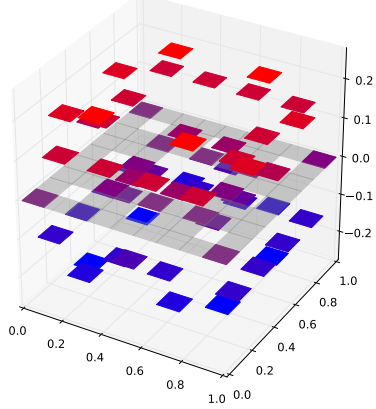
$M = 3$  Eigenfunction 108 has eigenvalue 2.09076235216

Klein Bottle Horizontal Glued Eigenfunction 108  
with eigenvalue 2.09076235216



Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594

Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594

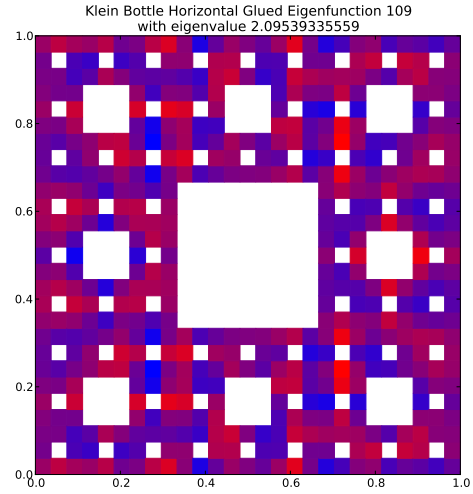
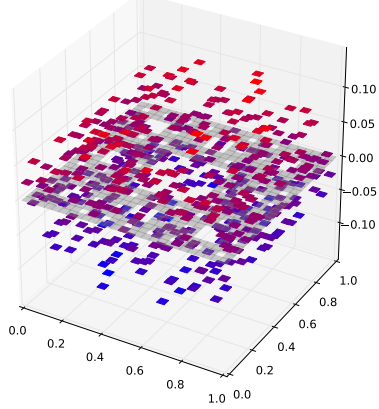


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.351570992206$   
Dot Value: 0.47041846106467444

# 110 $M = 3$ Eigenfunction 109

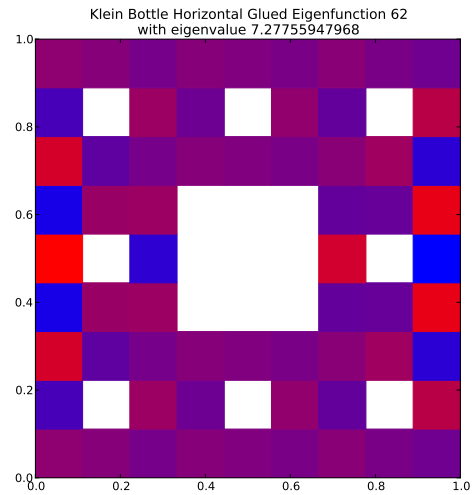
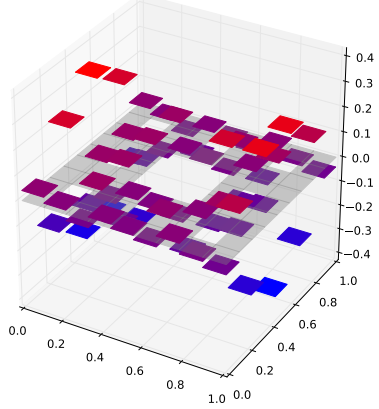
$M = 3$  Eigenfunction 109 has eigenvalue 2.09539335559

Klein Bottle Horizontal Glued Eigenfunction 109  
with eigenvalue 2.09539335559



Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

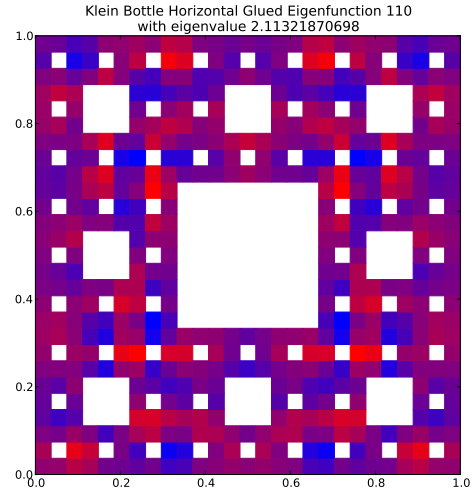
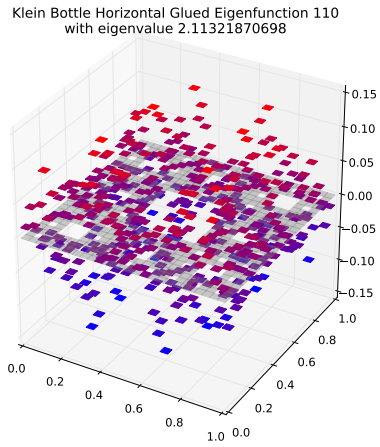
Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



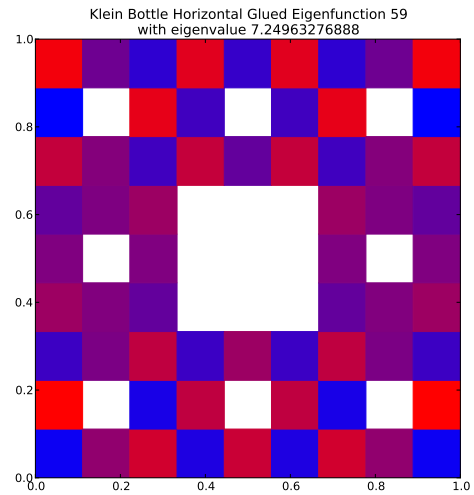
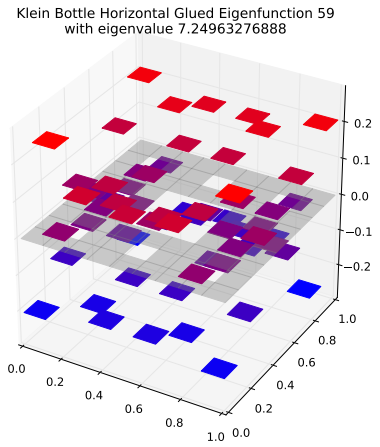
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.287925280644$   
Dot Value: 0.2717676960570785

# 111 $M = 3$ Eigenfunction 110

$M = 3$  Eigenfunction 110 has eigenvalue 2.11321870698



Compare to  $m = 2$  eigenspace with eigenvalue 7.24963276888

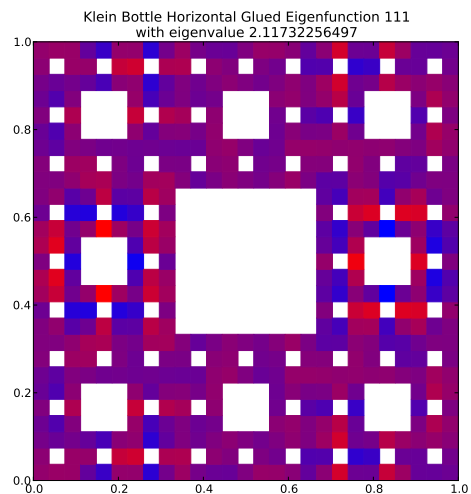
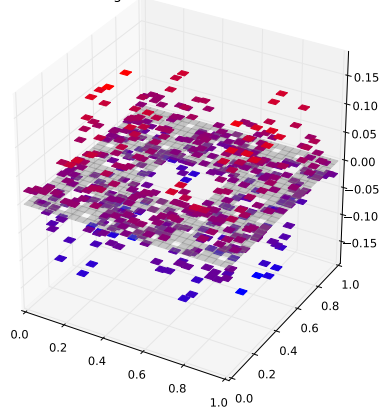


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.291493207221$   
Dot Value: 0.41179307630053885

# 112 $M = 3$ Eigenfunction 111

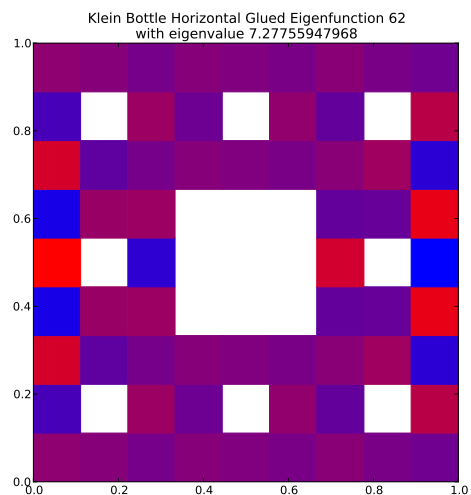
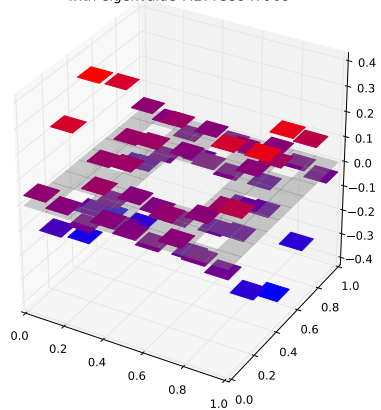
$M = 3$  Eigenfunction 111 has eigenvalue 2.11732256497

Klein Bottle Horizontal Glued Eigenfunction 111  
with eigenvalue 2.11732256497



Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

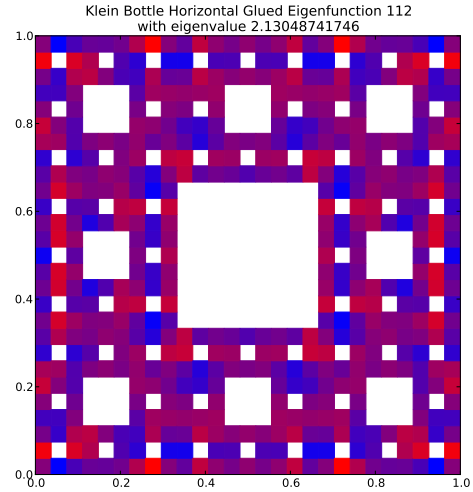
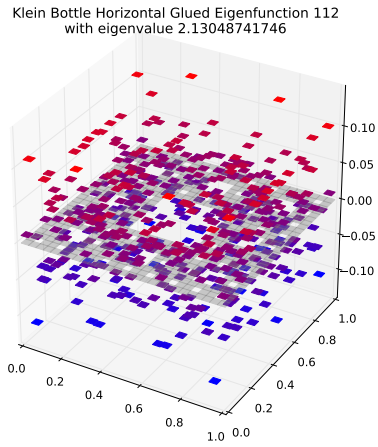
Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



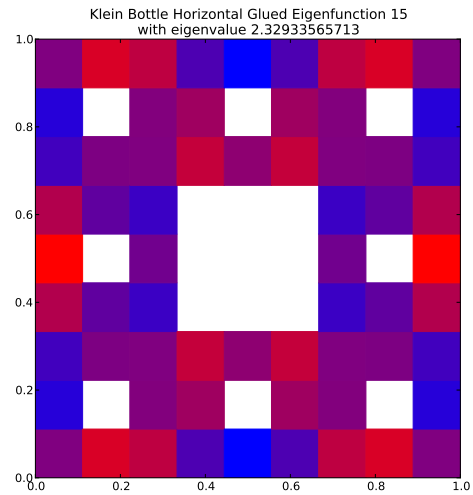
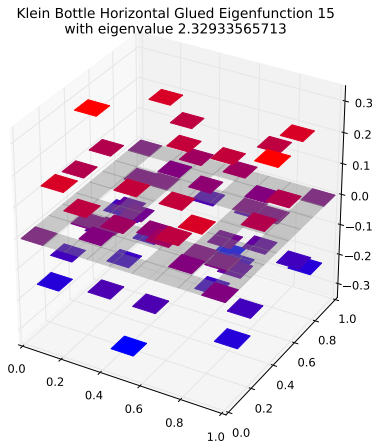
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.290938544835$   
Dot Value: 0.508164131162447

# 113 $M = 3$ Eigenfunction 112

$M = 3$  Eigenfunction 112 has eigenvalue 2.13048741746



Compare to  $m = 2$  eigenspace with eigenvalue 2.32933565713

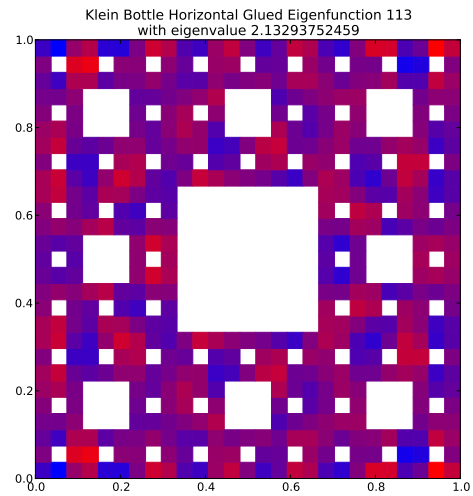
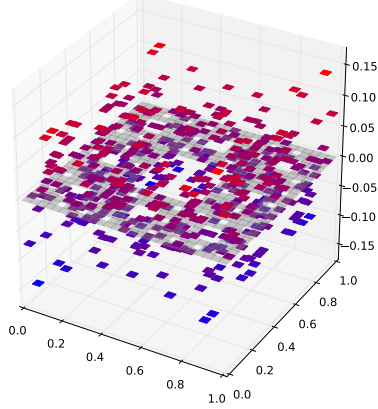


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.914633067559$   
Dot Value: 0.5060054879258207

# 114 $M = 3$ Eigenfunction 113

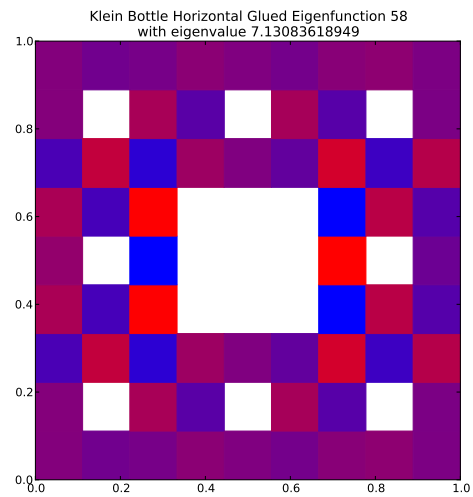
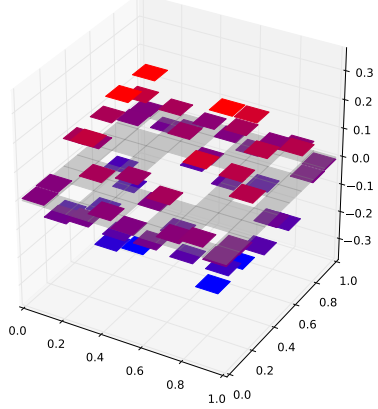
$M = 3$  Eigenfunction 113 has eigenvalue 2.13293752459

Klein Bottle Horizontal Glued Eigenfunction 113  
with eigenvalue 2.13293752459



Compare to  $m = 2$  eigenspace with eigenvalue 7.13083618949

Klein Bottle Horizontal Glued Eigenfunction 58  
with eigenvalue 7.13083618949

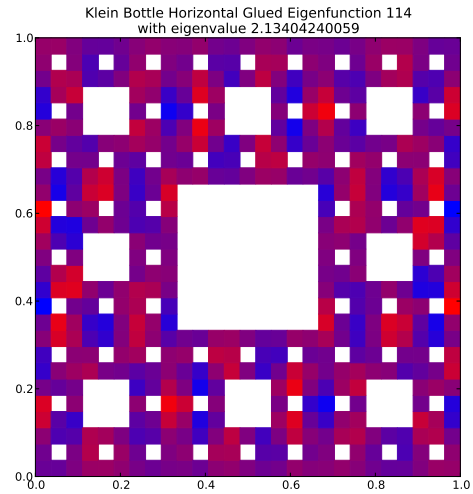
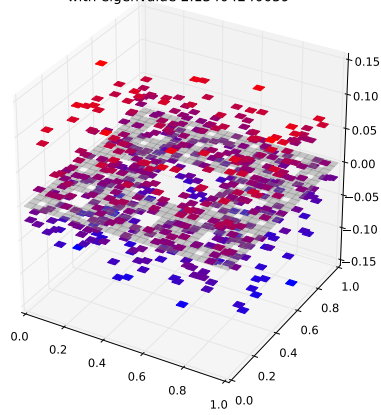


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.299114643487$   
Dot Value: 0.41795312925718586

# 115 $M = 3$ Eigenfunction 114

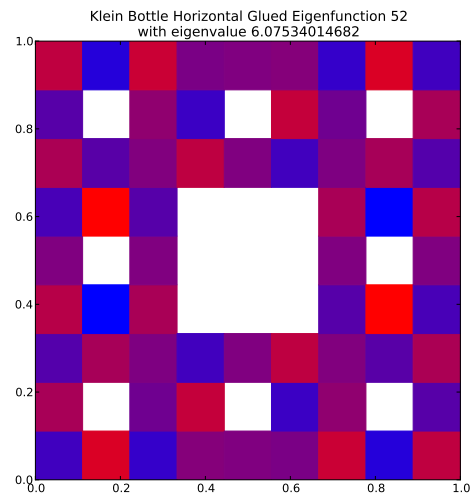
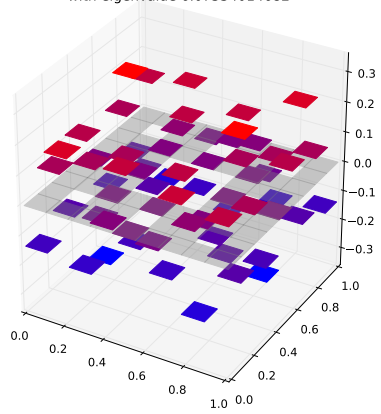
$M = 3$  Eigenfunction 114 has eigenvalue 2.13404240059

Klein Bottle Horizontal Glued Eigenfunction 114  
with eigenvalue 2.13404240059



Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682

Klein Bottle Horizontal Glued Eigenfunction 52  
with eigenvalue 6.07534014682

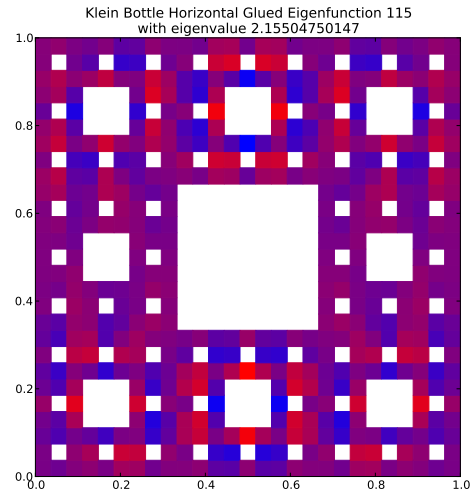
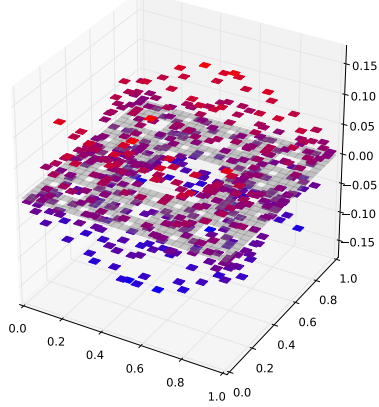


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.351263032031$   
Dot Value: 0.46124592015462784

# 116 $M = 3$ Eigenfunction 115

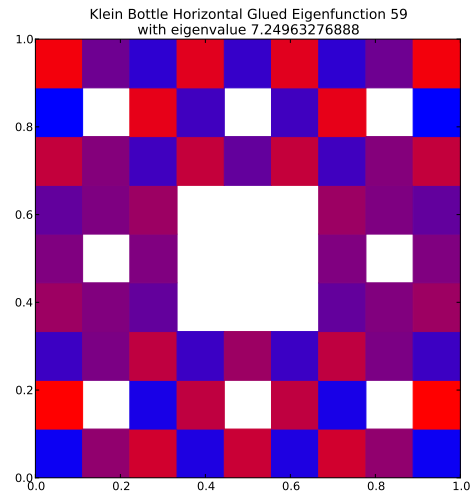
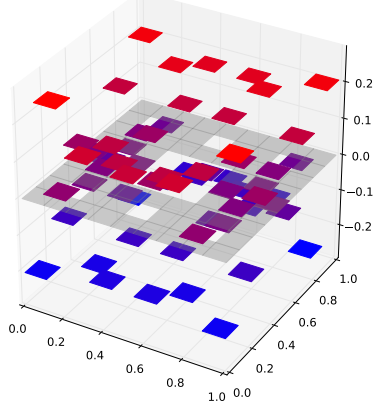
$M = 3$  Eigenfunction 115 has eigenvalue 2.15504750147

Klein Bottle Horizontal Glued Eigenfunction 115  
with eigenvalue 2.15504750147



Compare to  $m = 2$  eigenspace with eigenvalue 7.24963276888

Klein Bottle Horizontal Glued Eigenfunction 59  
with eigenvalue 7.24963276888



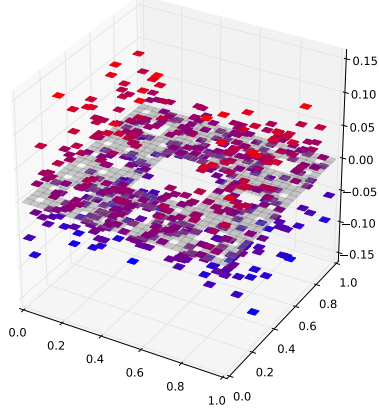
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.29726298837$

Dot Value: 0.42082876646795675

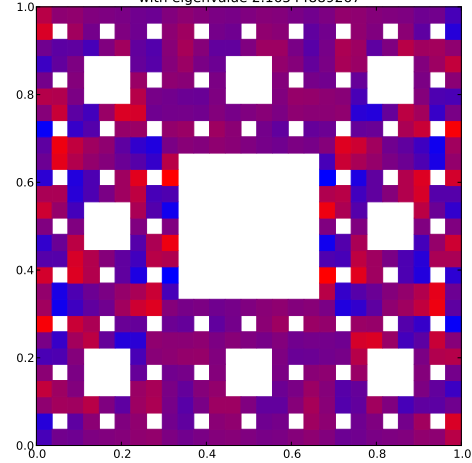
# 117 $M = 3$ Eigenfunction 116

$M = 3$  Eigenfunction 116 has eigenvalue 2.16544889267

Klein Bottle Horizontal Glued Eigenfunction 116  
with eigenvalue 2.16544889267

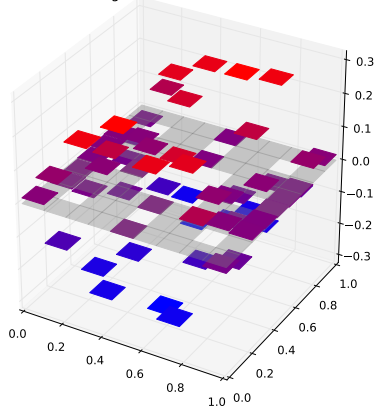


Klein Bottle Horizontal Glued Eigenfunction 116  
with eigenvalue 2.16544889267

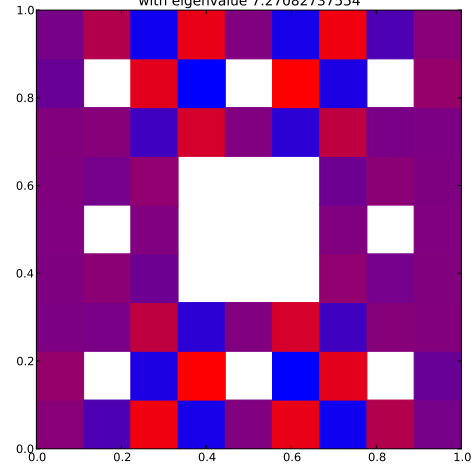


Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554

Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554



Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554



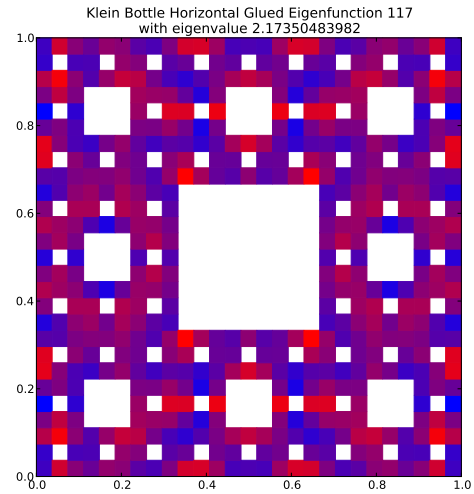
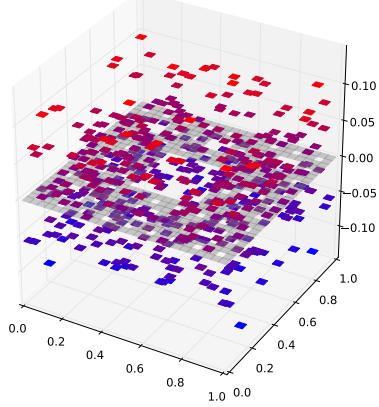
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.297827025842$

Dot Value: 0.49103019390917513

# 118 $M = 3$ Eigenfunction 117

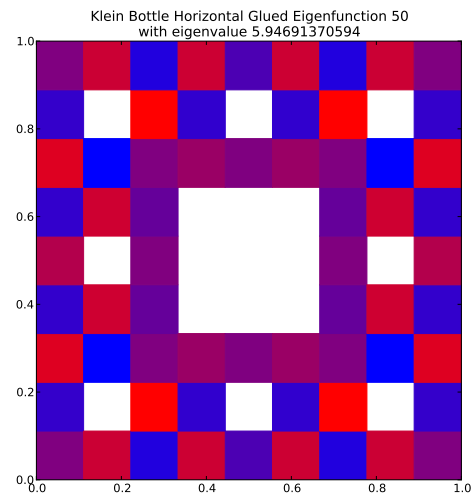
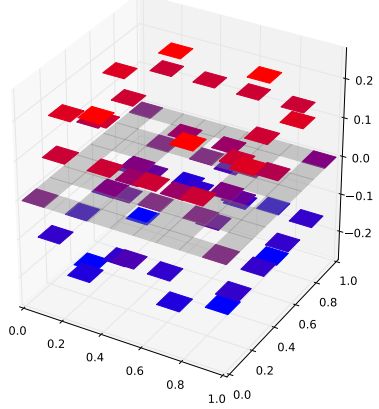
$M = 3$  Eigenfunction 117 has eigenvalue 2.17350483982

Klein Bottle Horizontal Glued Eigenfunction 117  
with eigenvalue 2.17350483982



Compare to  $m = 2$  eigenspace with eigenvalue 5.94691370594

Klein Bottle Horizontal Glued Eigenfunction 50  
with eigenvalue 5.94691370594

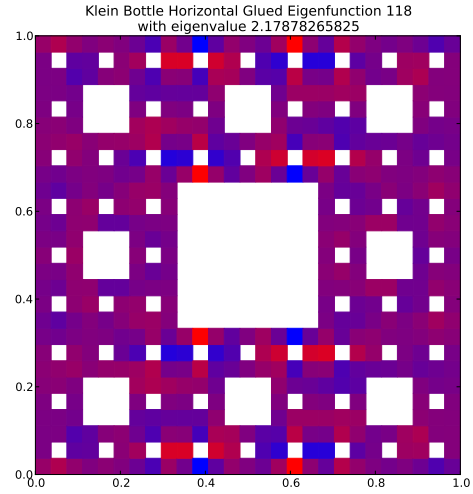
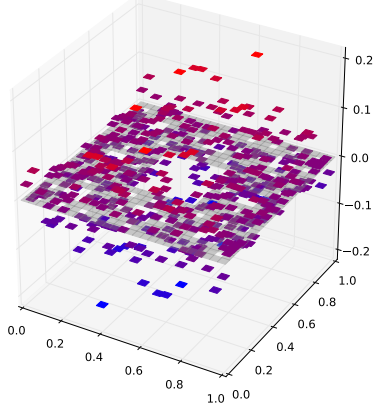


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.365484509662$   
Dot Value: 0.3883365675779792

# 119 $M = 3$ Eigenfunction 118

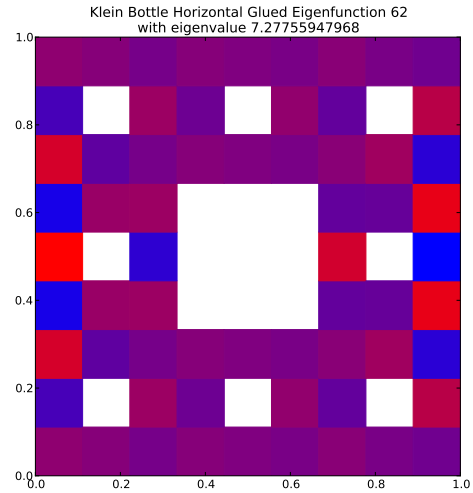
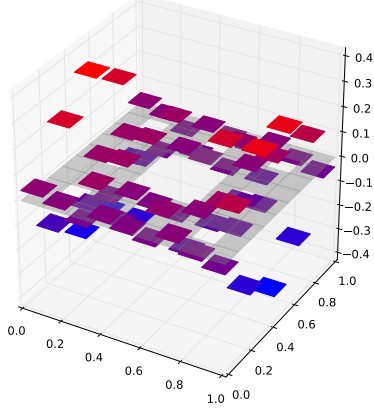
$M = 3$  Eigenfunction 118 has eigenvalue 2.17878265825

Klein Bottle Horizontal Glued Eigenfunction 118  
with eigenvalue 2.17878265825



Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



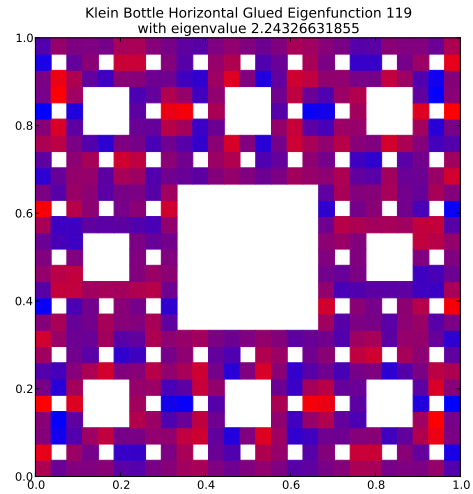
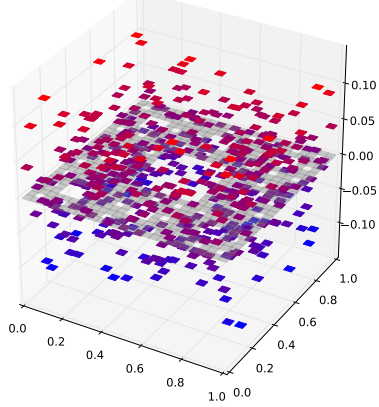
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.299383696462$

Dot Value: 0.31822354484543913

# 120 $M = 3$ Eigenfunction 119

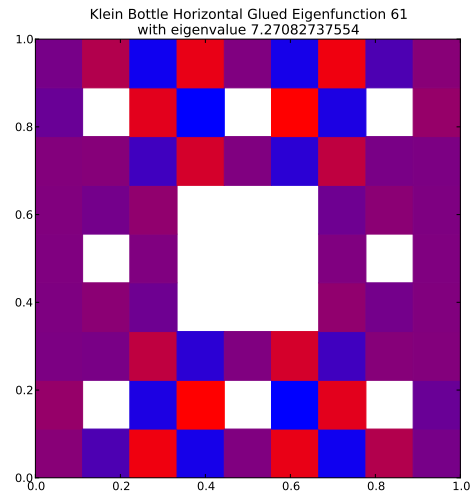
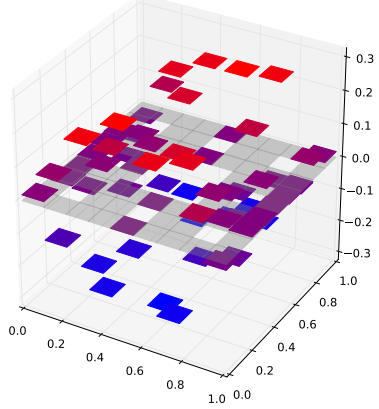
$M = 3$  Eigenfunction 119 has eigenvalue 2.24326631855

Klein Bottle Horizontal Glued Eigenfunction 119  
with eigenvalue 2.24326631855



Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554

Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554

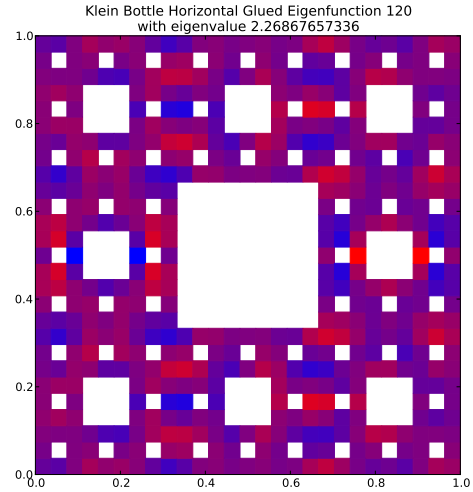
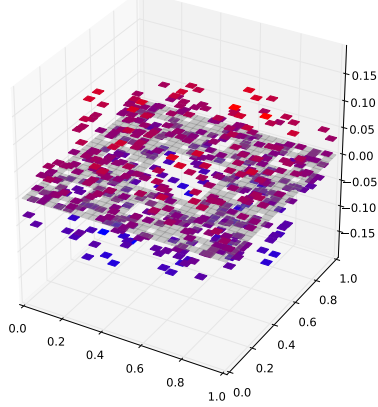


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.308529717828$   
Dot Value: 0.46226216059330993

# 121 $M = 3$ Eigenfunction 120

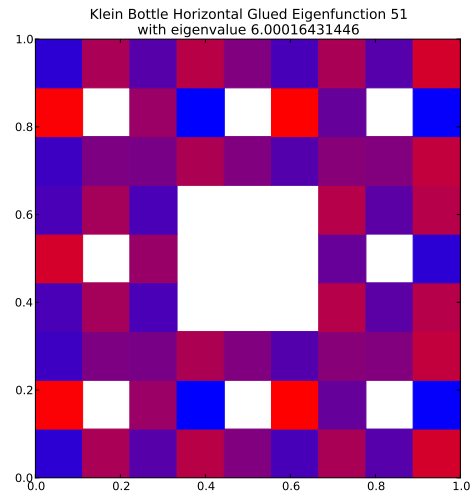
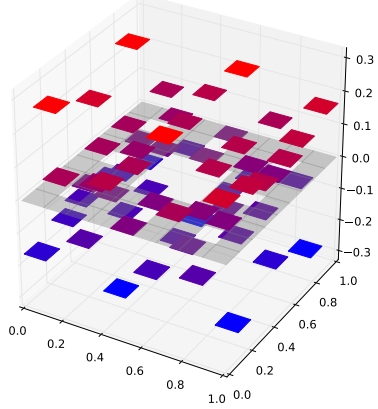
$M = 3$  Eigenfunction 120 has eigenvalue 2.26867657336

Klein Bottle Horizontal Glued Eigenfunction 120  
with eigenvalue 2.26867657336



Compare to  $m = 2$  eigenspace with eigenvalue 6.00016431446

Klein Bottle Horizontal Glued Eigenfunction 51  
with eigenvalue 6.00016431446



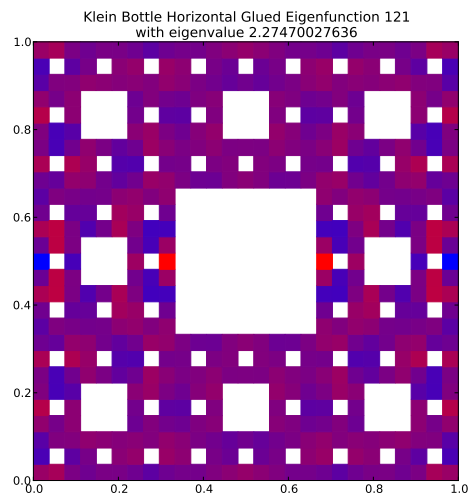
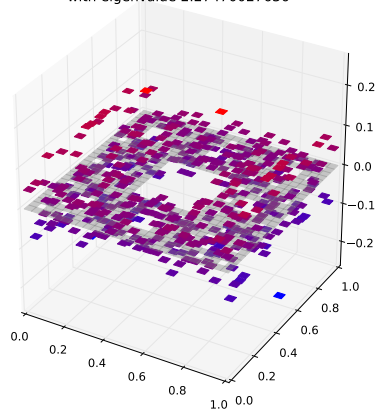
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.378102407612$

Dot Value: 0.3572913234255721

## 122 $M = 3$ Eigenfunction 121

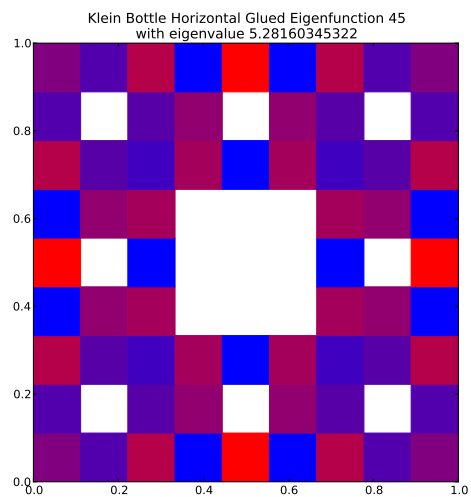
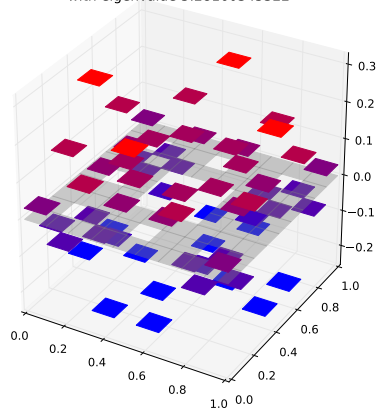
$M = 3$  Eigenfunction 121 has eigenvalue 2.27470027636

Klein Bottle Horizontal Glued Eigenfunction 121  
with eigenvalue 2.27470027636



Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322

Klein Bottle Horizontal Glued Eigenfunction 45  
with eigenvalue 5.28160345322



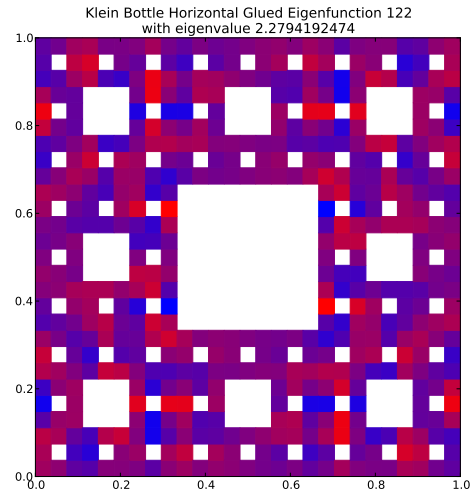
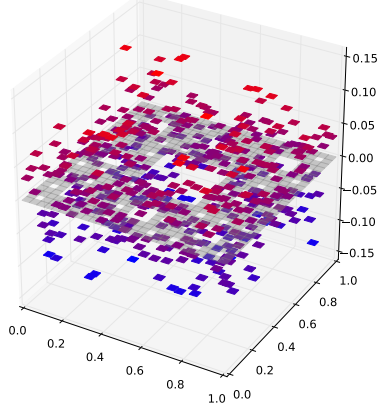
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.430683654407$

Dot Value: 0.4401670981515956

# 123 $M = 3$ Eigenfunction 122

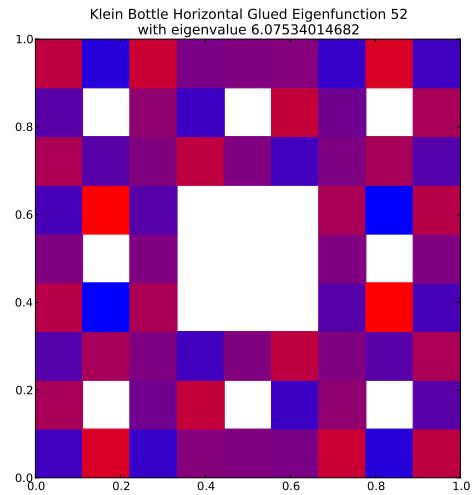
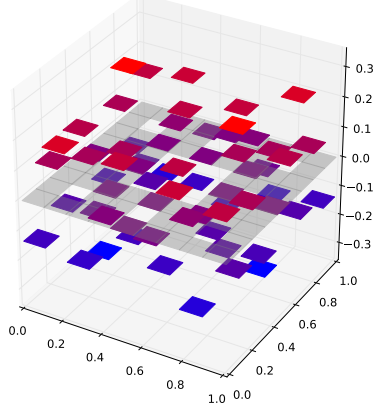
$M = 3$  Eigenfunction 122 has eigenvalue 2.2794192474

Klein Bottle Horizontal Glued Eigenfunction 122  
with eigenvalue 2.2794192474



Compare to  $m = 2$  eigenspace with eigenvalue 6.07534014682

Klein Bottle Horizontal Glued Eigenfunction 52  
with eigenvalue 6.07534014682

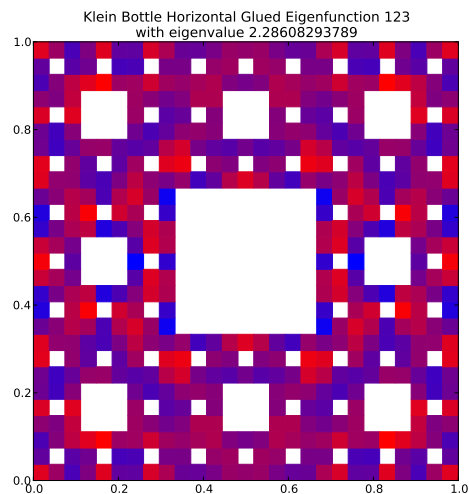
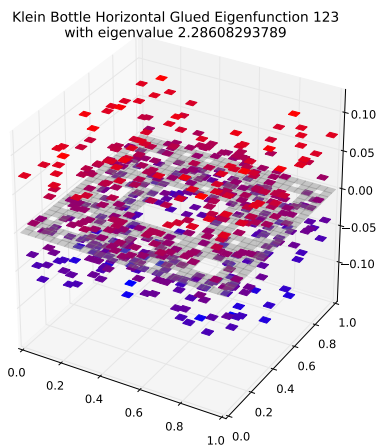


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.37519203737$

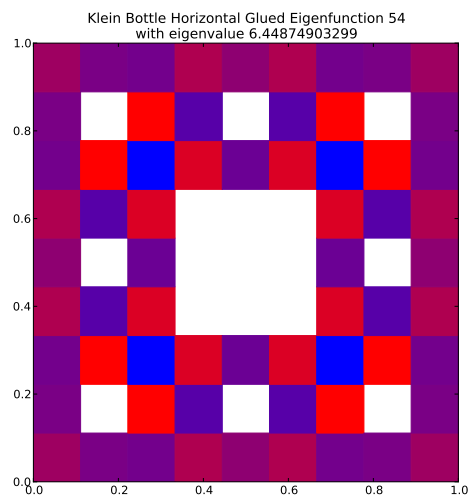
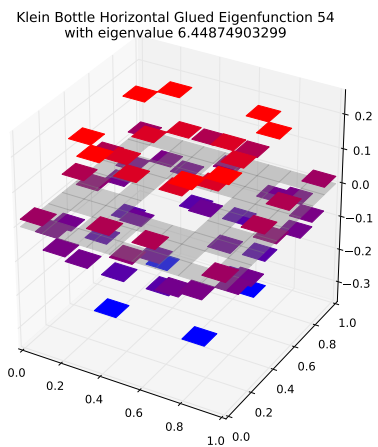
Dot Value: 0.5294219860945326

# 124 $M = 3$ Eigenfunction 123

$M = 3$  Eigenfunction 123 has eigenvalue 2.28608293789



Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

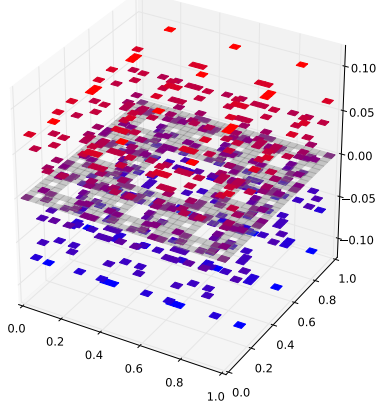


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.35450021798$   
Dot Value: 0.3322033675994438

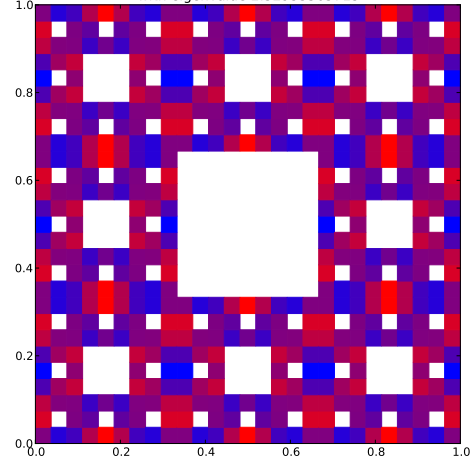
# 125 $M = 3$ Eigenfunction 124

$M = 3$  Eigenfunction 124 has eigenvalue 2.32933565713

Klein Bottle Horizontal Glued Eigenfunction 124  
with eigenvalue 2.32933565713

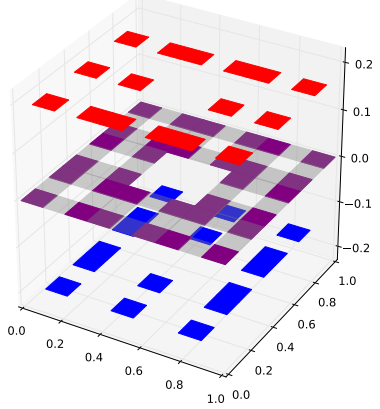


Klein Bottle Horizontal Glued Eigenfunction 124  
with eigenvalue 2.32933565713

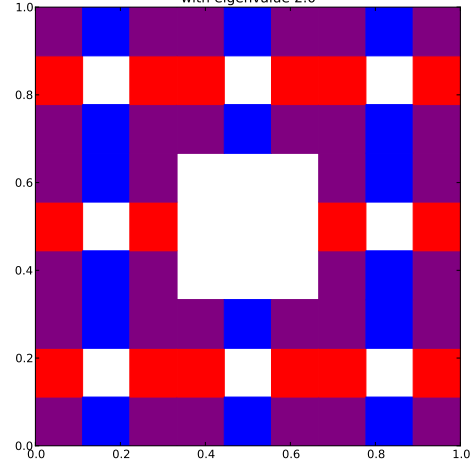


Compare to  $m = 2$  eigenspace with eigenvalue 2.0

Klein Bottle Horizontal Glued Eigenfunction 13  
with eigenvalue 2.0



Klein Bottle Horizontal Glued Eigenfunction 13  
with eigenvalue 2.0



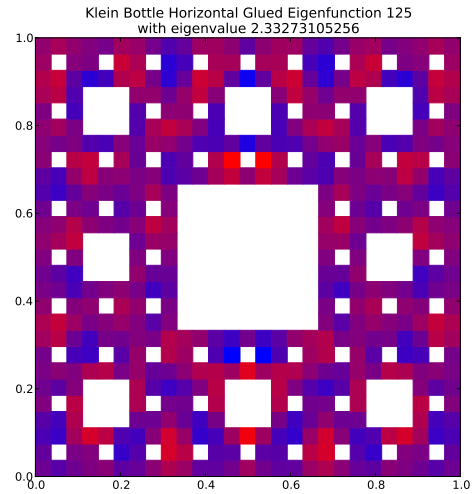
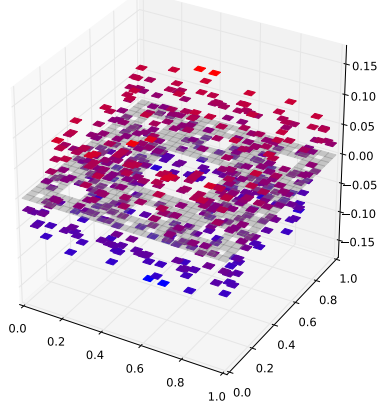
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 1.16466782857$

Dot Value: 1.1102230246251565e-16

# 126 $M = 3$ Eigenfunction 125

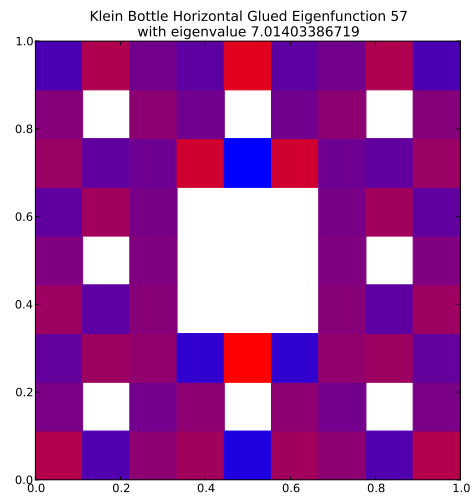
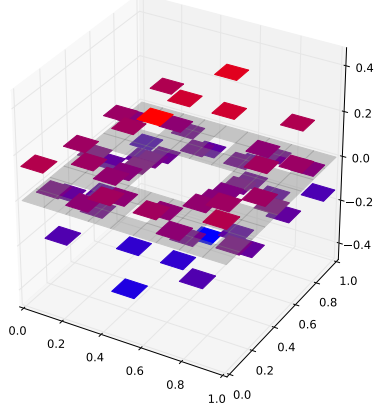
$M = 3$  Eigenfunction 125 has eigenvalue 2.33273105256

Klein Bottle Horizontal Glued Eigenfunction 125  
with eigenvalue 2.33273105256



Compare to  $m = 2$  eigenspace with eigenvalue 7.01403386719

Klein Bottle Horizontal Glued Eigenfunction 57  
with eigenvalue 7.01403386719

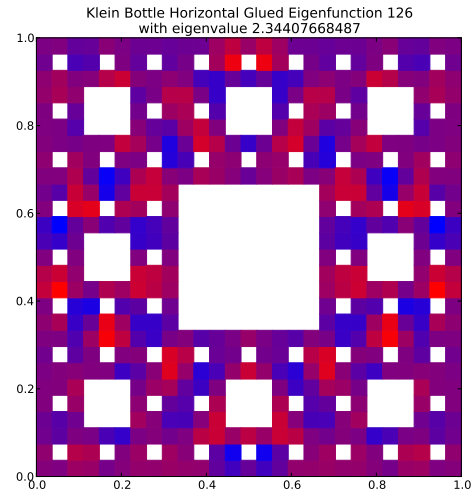
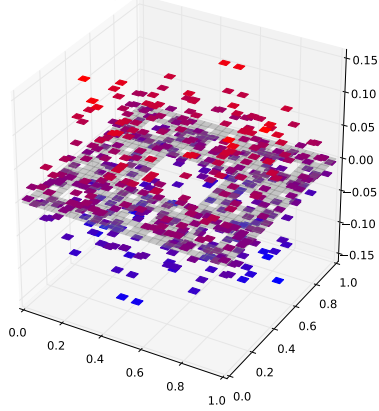


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.332580523095$   
Dot Value: 0.37247301262776855

# 127 $M = 3$ Eigenfunction 126

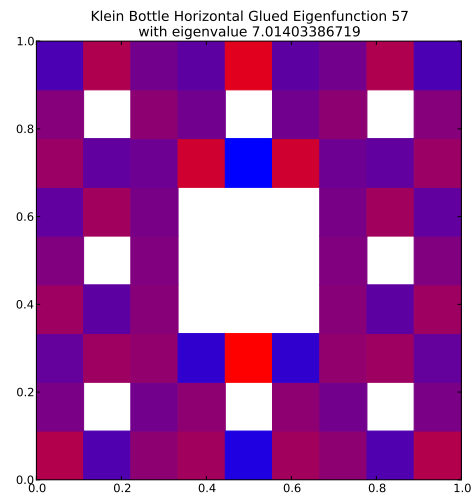
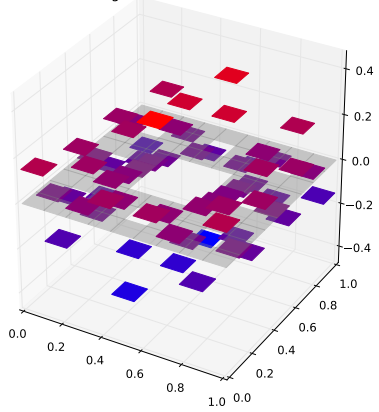
$M = 3$  Eigenfunction 126 has eigenvalue 2.34407668487

Klein Bottle Horizontal Glued Eigenfunction 126  
with eigenvalue 2.34407668487



Compare to  $m = 2$  eigenspace with eigenvalue 7.01403386719

Klein Bottle Horizontal Glued Eigenfunction 57  
with eigenvalue 7.01403386719

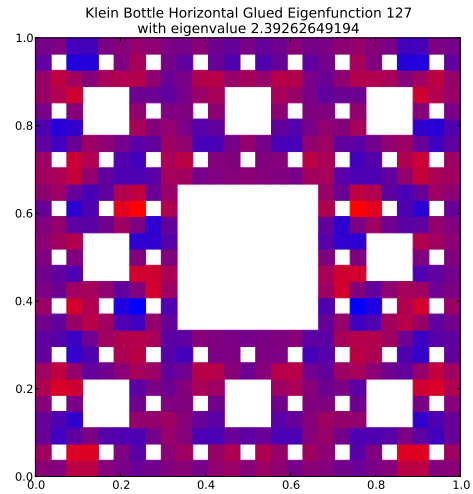
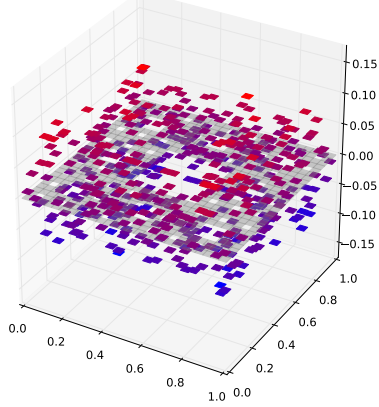


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.334198084762$   
Dot Value: 0.2510433393000855

# 128 $M = 3$ Eigenfunction 127

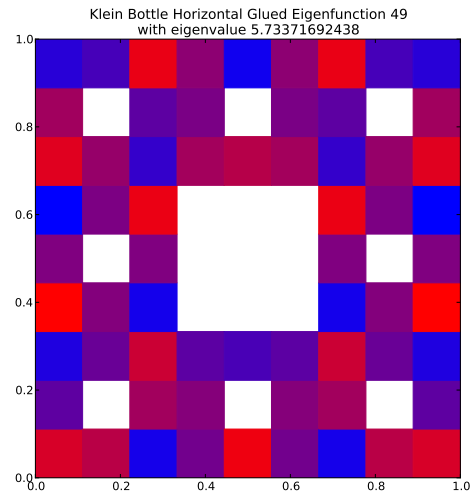
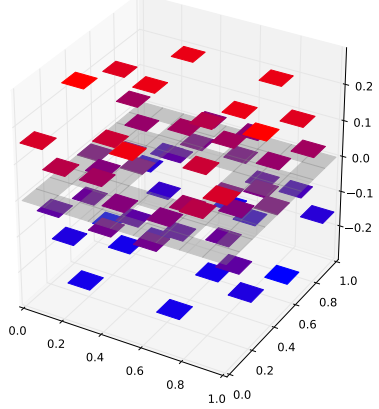
$M = 3$  Eigenfunction 127 has eigenvalue 2.39262649194

Klein Bottle Horizontal Glued Eigenfunction 127  
with eigenvalue 2.39262649194



Compare to  $m = 2$  eigenspace with eigenvalue 5.73371692438

Klein Bottle Horizontal Glued Eigenfunction 49  
with eigenvalue 5.73371692438

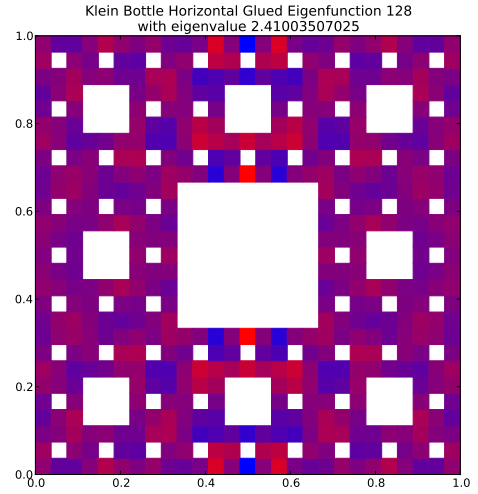
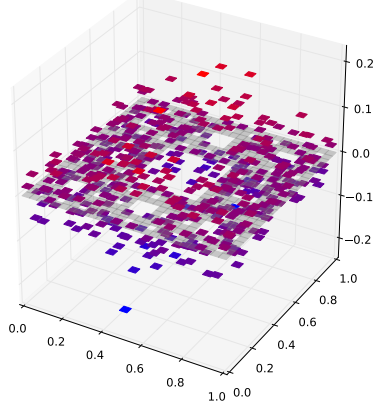


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.417290655171$   
Dot Value: 0.12385086320486172

# 129 $M = 3$ Eigenfunction 128

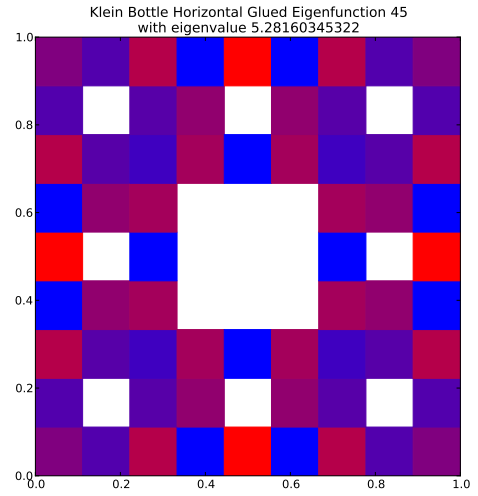
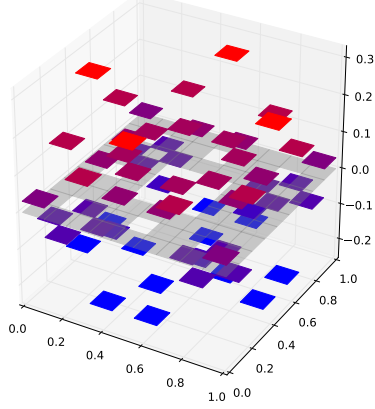
$M = 3$  Eigenfunction 128 has eigenvalue 2.41003507025

Klein Bottle Horizontal Glued Eigenfunction 128  
with eigenvalue 2.41003507025



Compare to  $m = 2$  eigenspace with eigenvalue 5.28160345322

Klein Bottle Horizontal Glued Eigenfunction 45  
with eigenvalue 5.28160345322

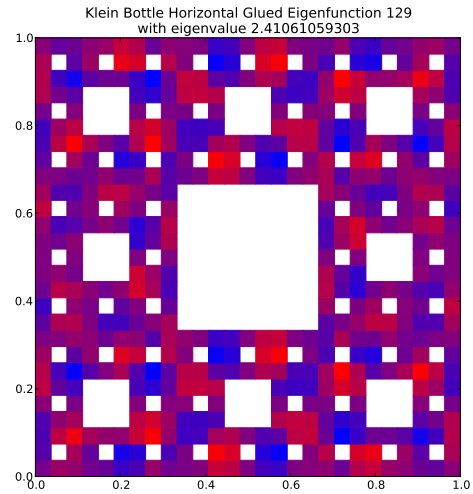
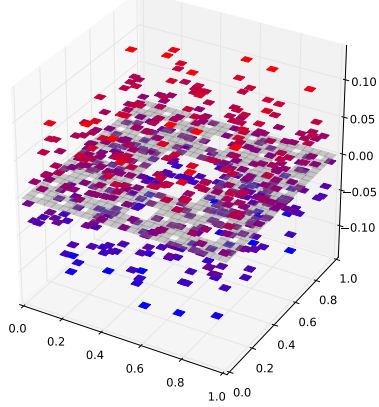


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.45630746261$   
Dot Value: 0.3946890653005306

# 130 $M = 3$ Eigenfunction 129

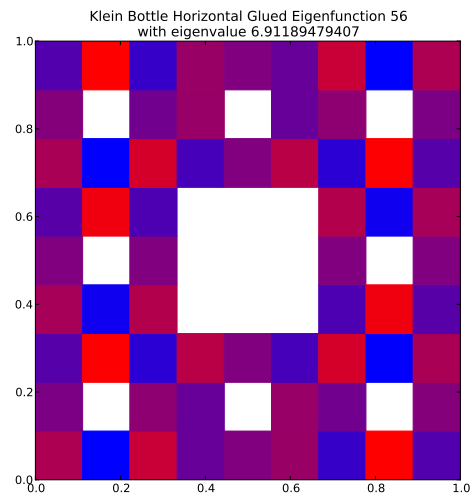
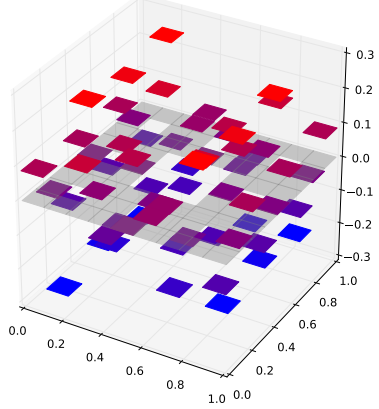
$M = 3$  Eigenfunction 129 has eigenvalue 2.41061059303

Klein Bottle Horizontal Glued Eigenfunction 129  
with eigenvalue 2.41061059303



Compare to  $m = 2$  eigenspace with eigenvalue 6.91189479407

Klein Bottle Horizontal Glued Eigenfunction 56  
with eigenvalue 6.91189479407



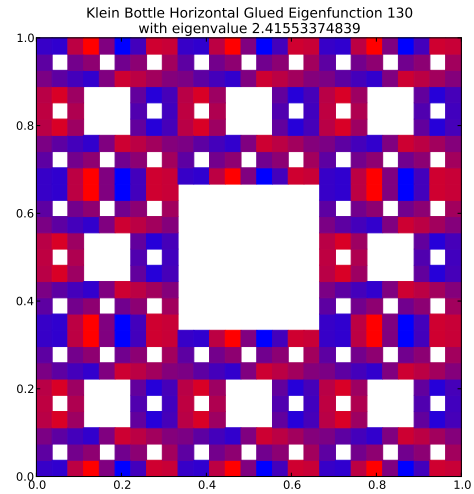
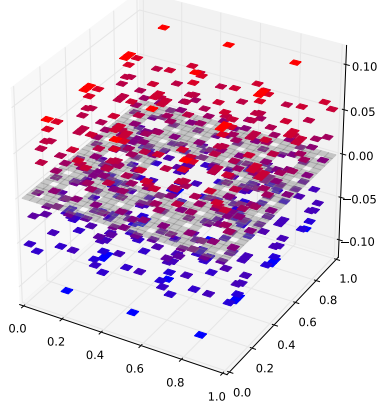
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.348762628027$

Dot Value: 0.2551095289678029

# 131 $M = 3$ Eigenfunction 130

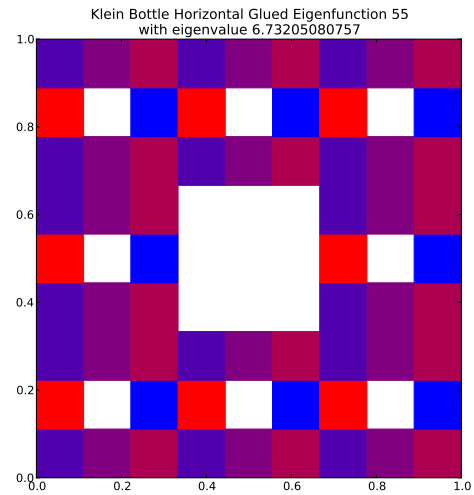
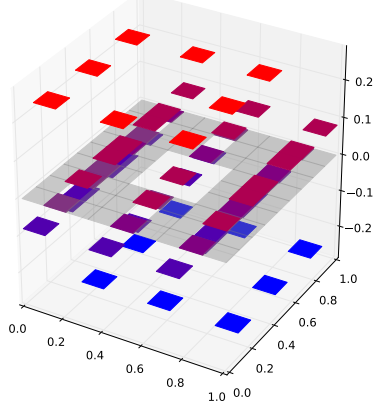
$M = 3$  Eigenfunction 130 has eigenvalue 2.41553374839

Klein Bottle Horizontal Glued Eigenfunction 130  
with eigenvalue 2.41553374839



Compare to  $m = 2$  eigenspace with eigenvalue 6.73205080757

Klein Bottle Horizontal Glued Eigenfunction 55  
with eigenvalue 6.73205080757

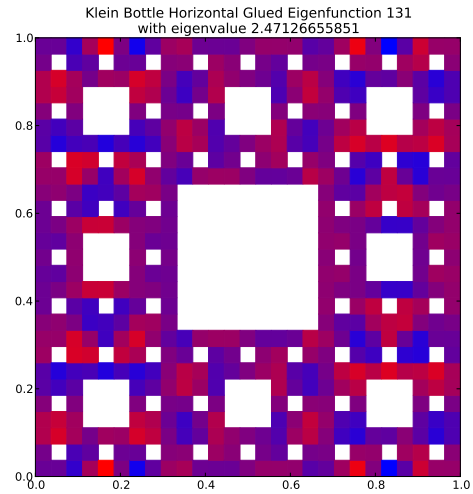
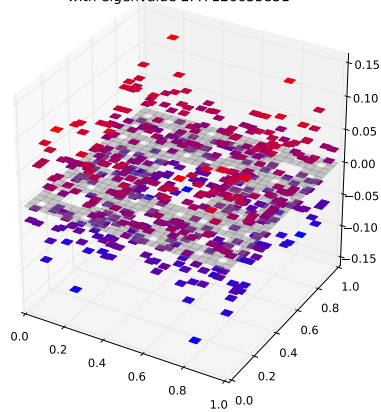


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.358810980106$   
Dot Value: 0.011440672902887639

# 132 $M = 3$ Eigenfunction 131

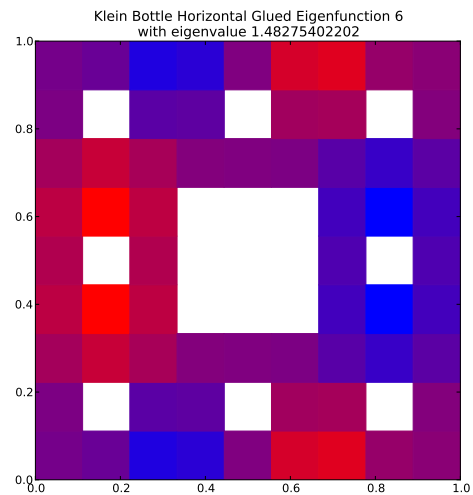
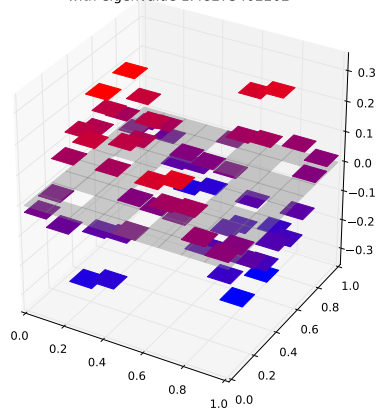
$M = 3$  Eigenfunction 131 has eigenvalue 2.47126655851

Klein Bottle Horizontal Glued Eigenfunction 131  
with eigenvalue 2.47126655851



Compare to  $m = 2$  eigenspace with eigenvalue 1.48275402202

Klein Bottle Horizontal Glued Eigenfunction 6  
with eigenvalue 1.48275402202

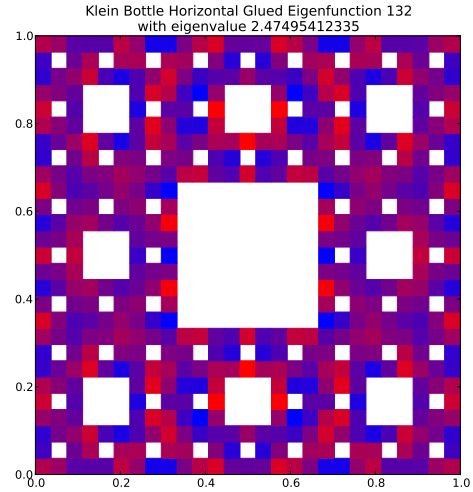
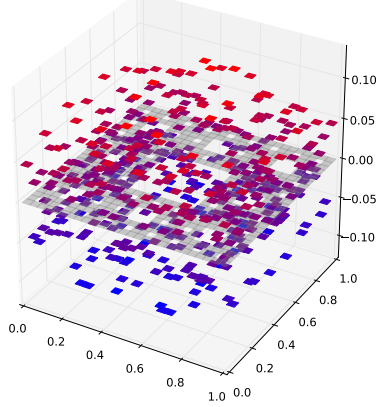


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 1.66667331317$   
Dot Value: 0.4789538942939531

# 133 $M = 3$ Eigenfunction 132

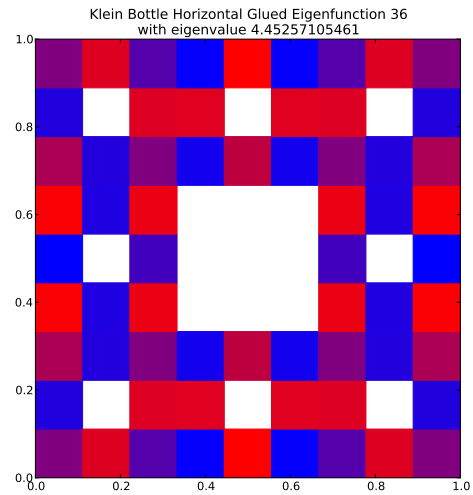
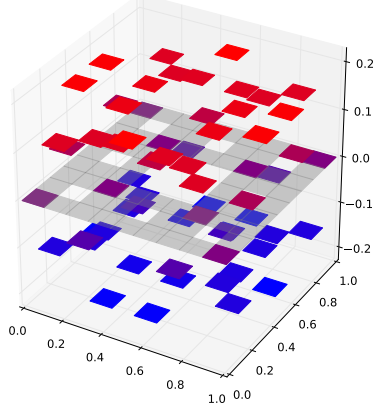
$M = 3$  Eigenfunction 132 has eigenvalue 2.47495412335

Klein Bottle Horizontal Glued Eigenfunction 132  
with eigenvalue 2.47495412335



Compare to  $m = 2$  eigenspace with eigenvalue 4.45257105461

Klein Bottle Horizontal Glued Eigenfunction 36  
with eigenvalue 4.45257105461



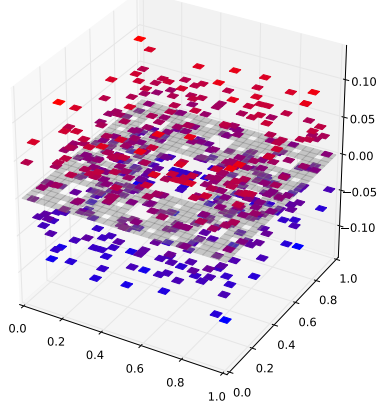
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.555848316175$

Dot Value: 0.5220985892896857

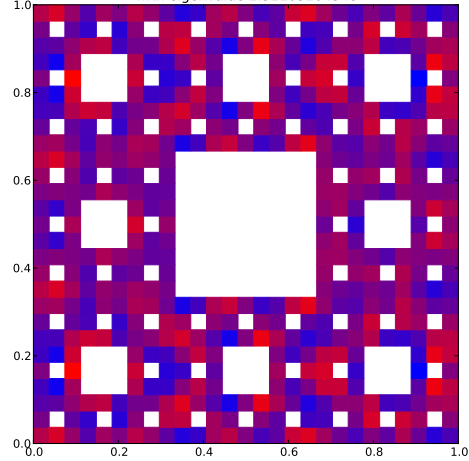
# 134 $M = 3$ Eigenfunction 133

$M = 3$  Eigenfunction 133 has eigenvalue 2.52103204348

Klein Bottle Horizontal Glued Eigenfunction 133  
with eigenvalue 2.52103204348

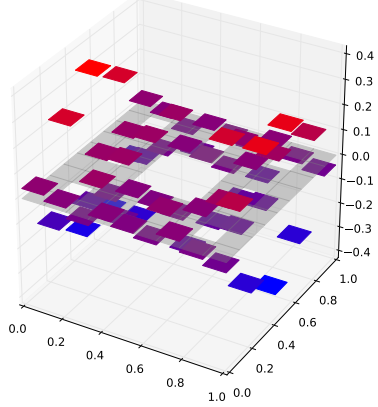


Klein Bottle Horizontal Glued Eigenfunction 133  
with eigenvalue 2.52103204348

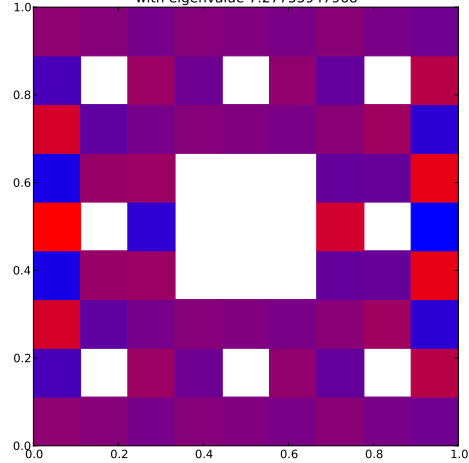


Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



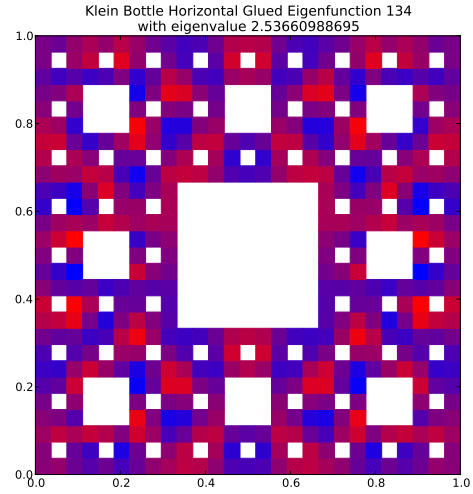
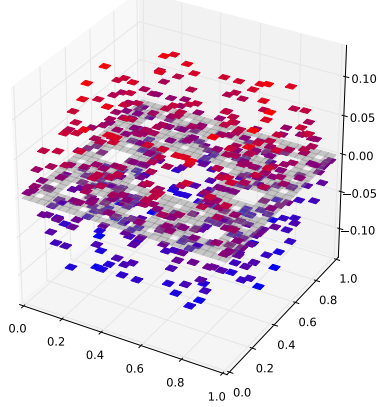
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.346411740161$

Dot Value: 0.46807199559314705

# 135 $M = 3$ Eigenfunction 134

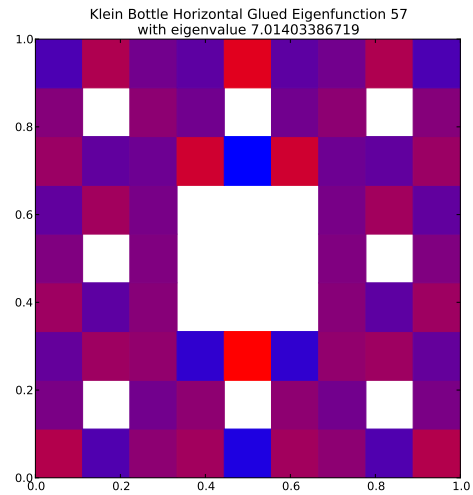
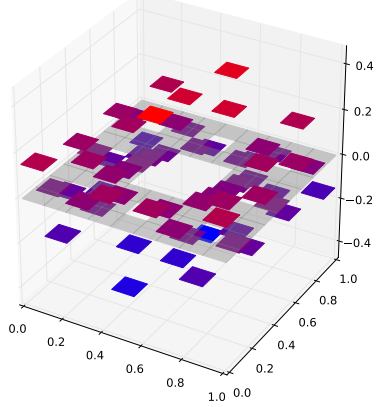
$M = 3$  Eigenfunction 134 has eigenvalue 2.53660988695

Klein Bottle Horizontal Glued Eigenfunction 134  
with eigenvalue 2.53660988695



Compare to  $m = 2$  eigenspace with eigenvalue 7.01403386719

Klein Bottle Horizontal Glued Eigenfunction 57  
with eigenvalue 7.01403386719

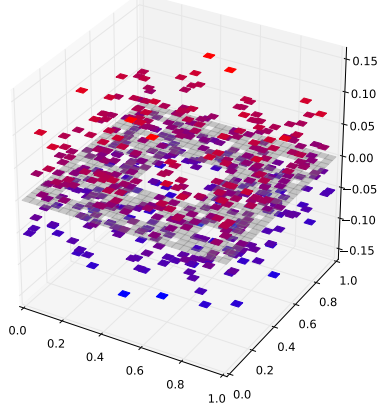


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.361647795688$   
Dot Value: 0.1430173019390647

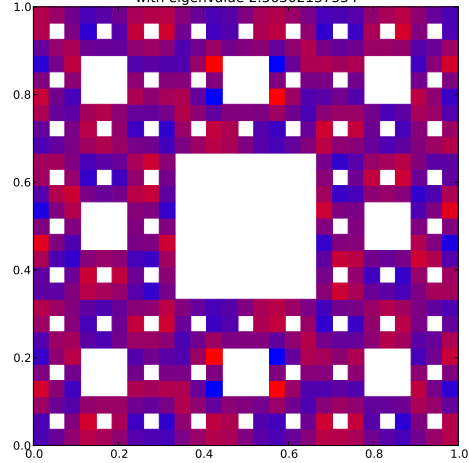
# 136 $M = 3$ Eigenfunction 135

$M = 3$  Eigenfunction 135 has eigenvalue 2.56302137554

Klein Bottle Horizontal Glued Eigenfunction 135  
with eigenvalue 2.56302137554

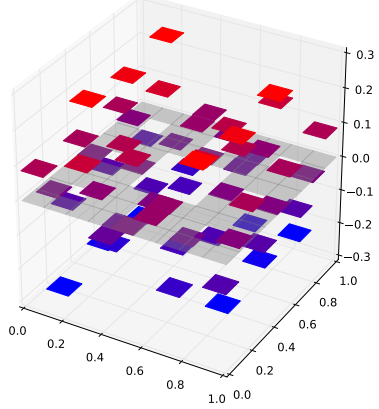


Klein Bottle Horizontal Glued Eigenfunction 135  
with eigenvalue 2.56302137554

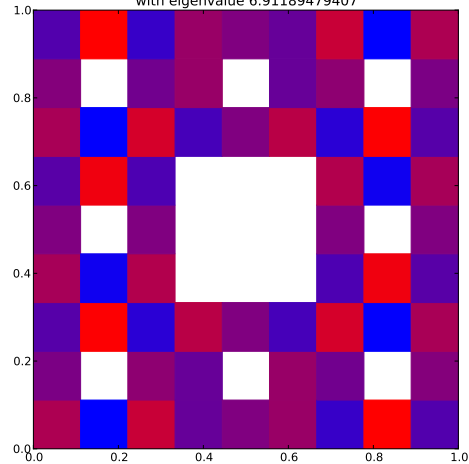


Compare to  $m = 2$  eigenspace with eigenvalue 6.91189479407

Klein Bottle Horizontal Glued Eigenfunction 56  
with eigenvalue 6.91189479407



Klein Bottle Horizontal Glued Eigenfunction 56  
with eigenvalue 6.91189479407



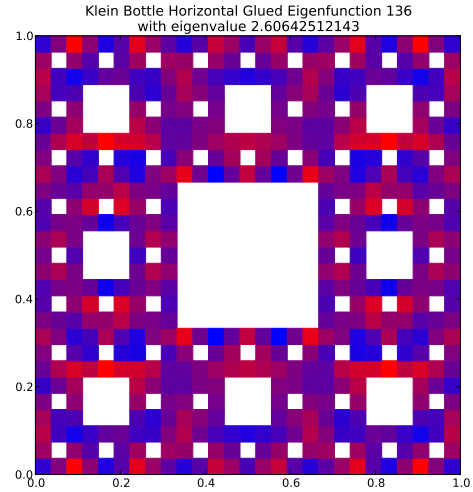
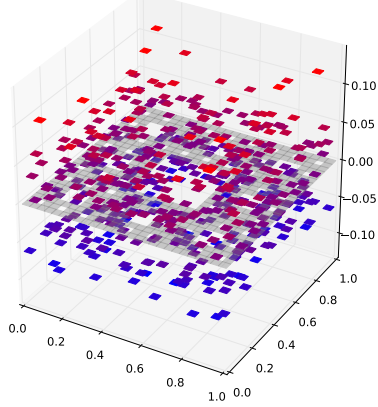
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.370813134734$

Dot Value: 0.0526000656521749

# 137 $M = 3$ Eigenfunction 136

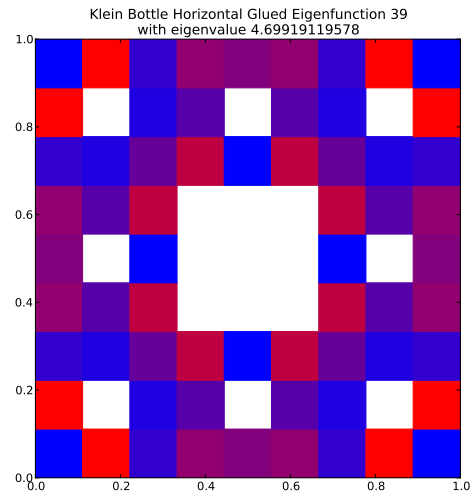
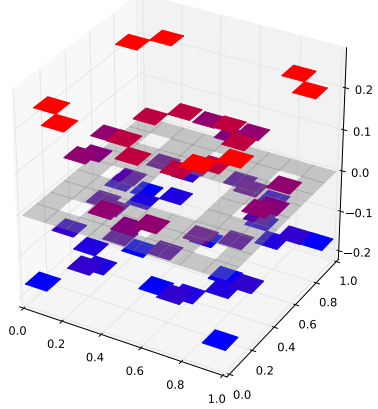
$M = 3$  Eigenfunction 136 has eigenvalue 2.60642512143

Klein Bottle Horizontal Glued Eigenfunction 136  
with eigenvalue 2.60642512143



Compare to  $m = 2$  eigenspace with eigenvalue 4.69919119578

Klein Bottle Horizontal Glued Eigenfunction 39  
with eigenvalue 4.69919119578

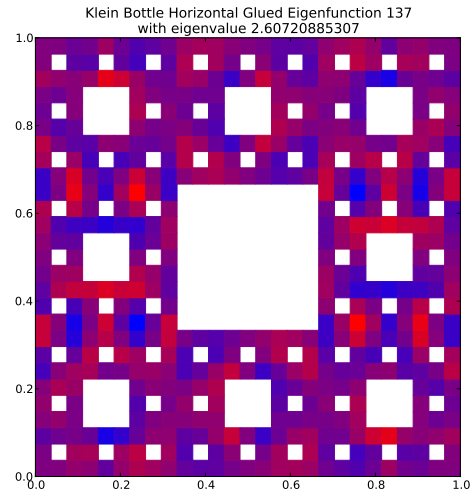
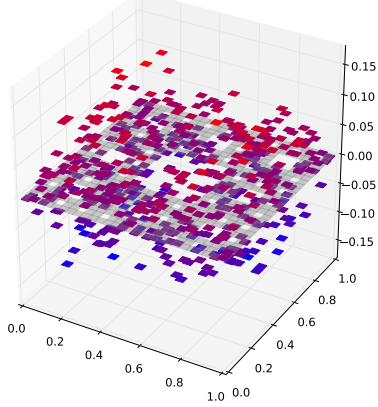


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.554653984662$   
Dot Value: 0.35084014041006273

# 138 $M = 3$ Eigenfunction 137

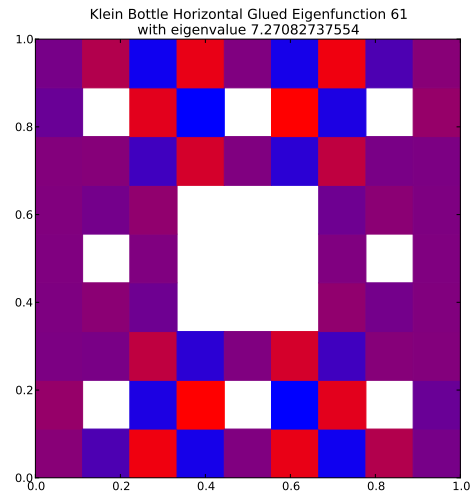
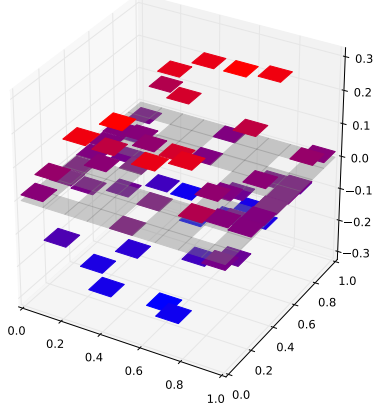
$M = 3$  Eigenfunction 137 has eigenvalue 2.60720885307

Klein Bottle Horizontal Glued Eigenfunction 137  
with eigenvalue 2.60720885307



Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554

Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554



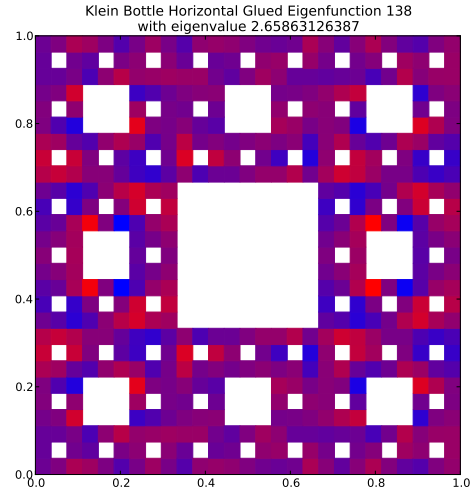
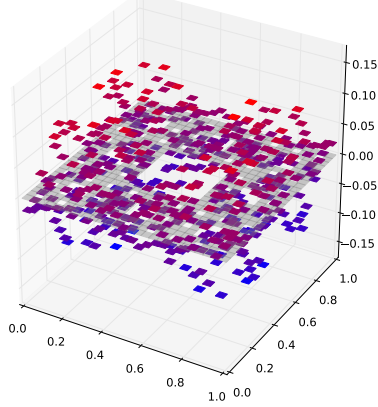
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.358584892531$

Dot Value: 0.25406226619885486

# 139 $M = 3$ Eigenfunction 138

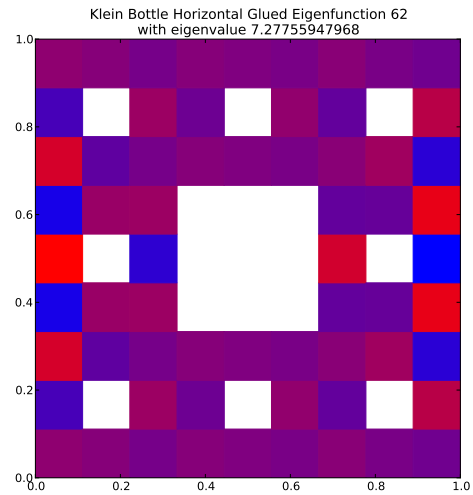
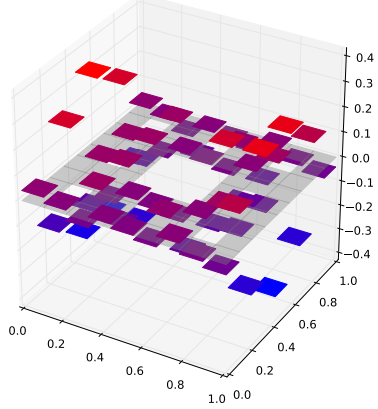
$M = 3$  Eigenfunction 138 has eigenvalue 2.65863126387

Klein Bottle Horizontal Glued Eigenfunction 138  
with eigenvalue 2.65863126387



Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



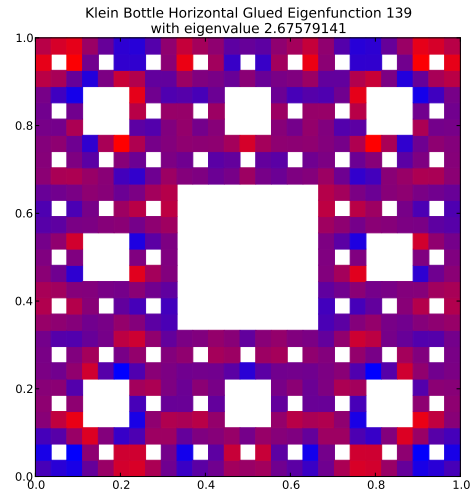
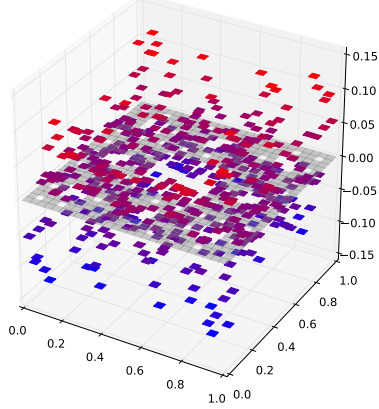
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.365319070396$

Dot Value: 0.3340349438377168

# 140 $M = 3$ Eigenfunction 139

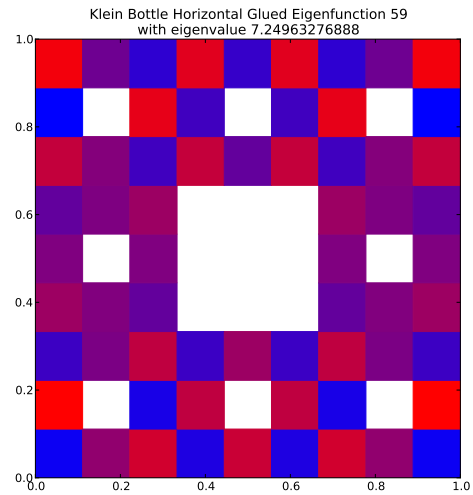
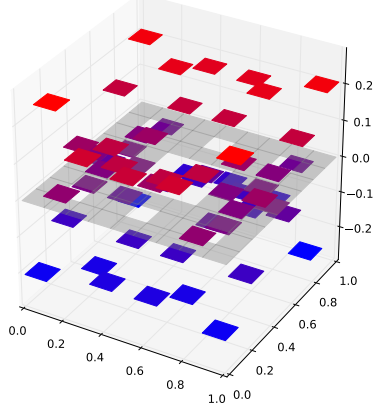
$M = 3$  Eigenfunction 139 has eigenvalue 2.67579141

Klein Bottle Horizontal Glued Eigenfunction 139  
with eigenvalue 2.67579141



Compare to  $m = 2$  eigenspace with eigenvalue 7.24963276888

Klein Bottle Horizontal Glued Eigenfunction 59  
with eigenvalue 7.24963276888

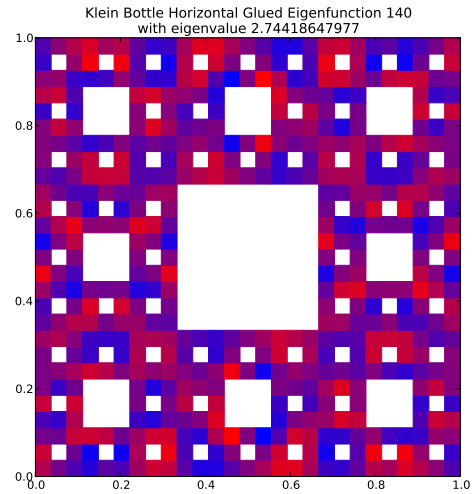
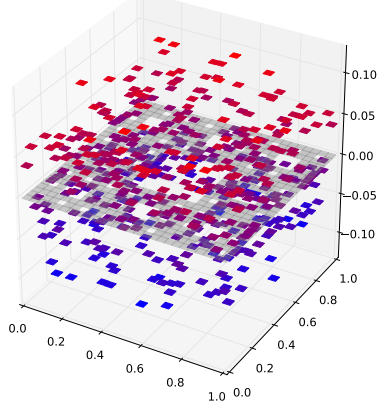


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.369093372769$   
Dot Value: 0.25104058339701396

# 141 $M = 3$ Eigenfunction 140

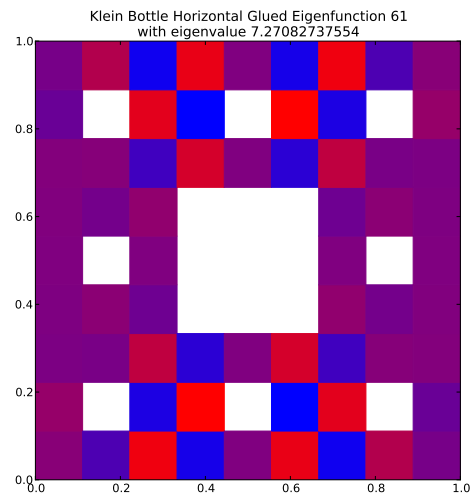
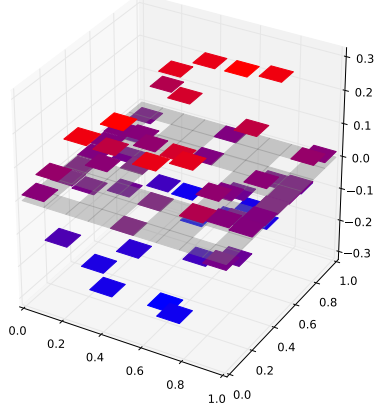
$M = 3$  Eigenfunction 140 has eigenvalue 2.74418647977

Klein Bottle Horizontal Glued Eigenfunction 140  
with eigenvalue 2.74418647977



Compare to  $m = 2$  eigenspace with eigenvalue 7.27082737554

Klein Bottle Horizontal Glued Eigenfunction 61  
with eigenvalue 7.27082737554



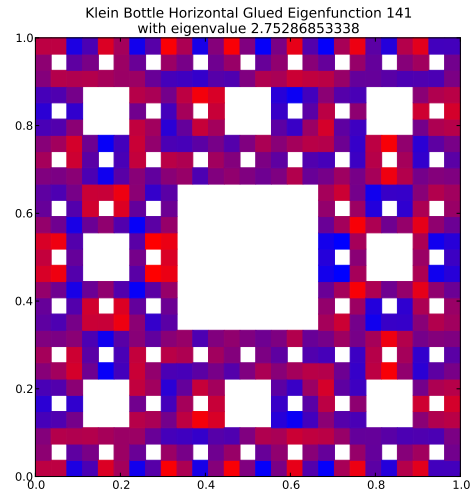
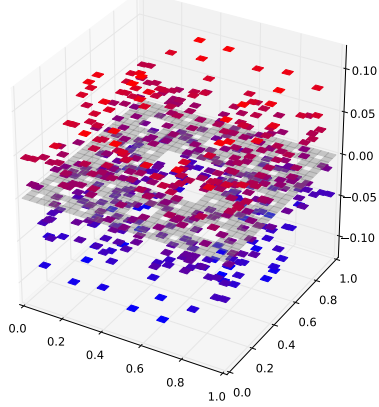
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.377424237714$

Dot Value: 0.09808883394953816

# 142 $M = 3$ Eigenfunction 141

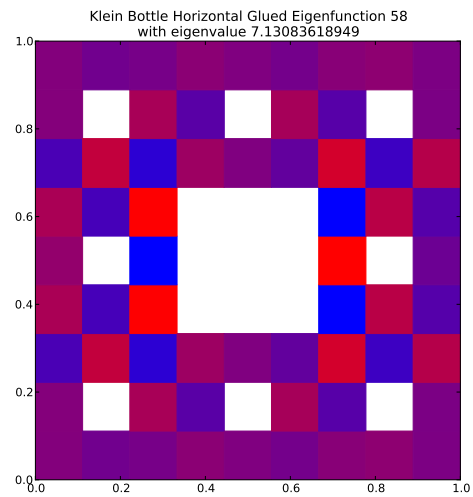
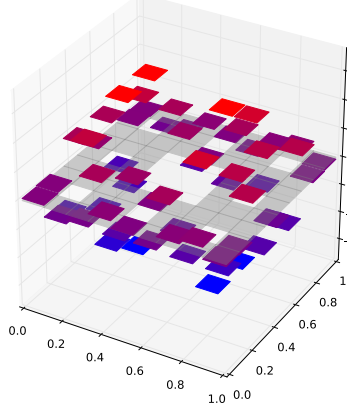
$M = 3$  Eigenfunction 141 has eigenvalue 2.75286853338

Klein Bottle Horizontal Glued Eigenfunction 141  
with eigenvalue 2.75286853338



Compare to  $m = 2$  eigenspace with eigenvalue 7.13083618949

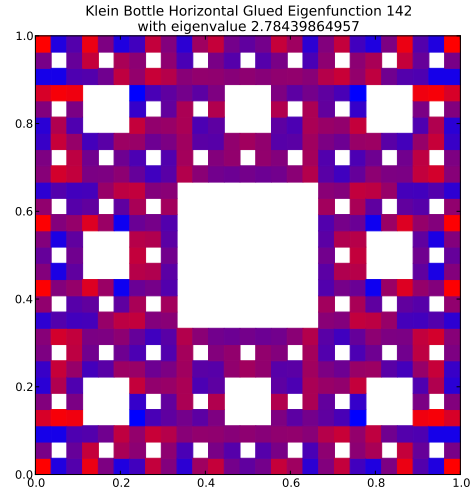
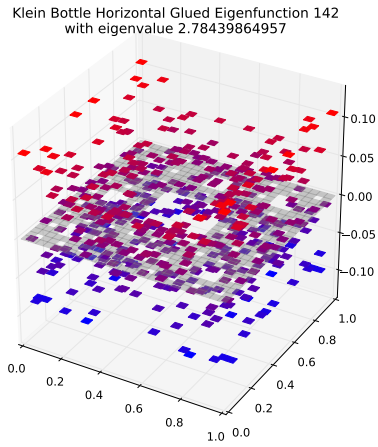
Klein Bottle Horizontal Glued Eigenfunction 58  
with eigenvalue 7.13083618949



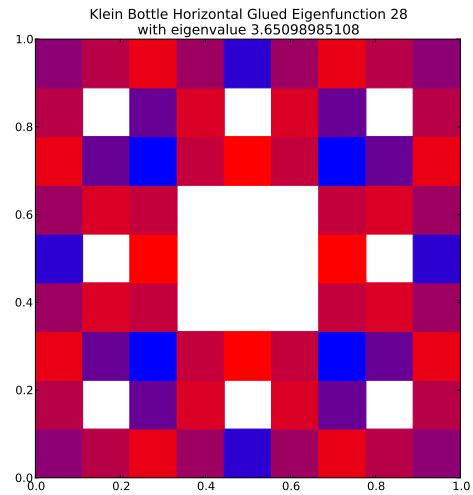
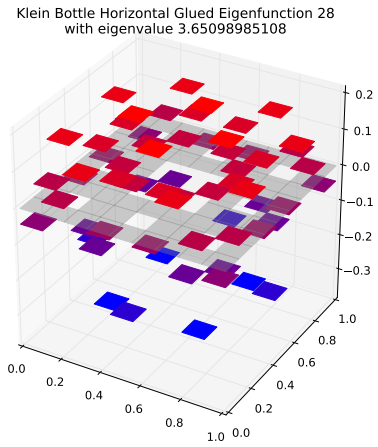
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.386051293316$   
Dot Value: 0.27497284663653754

# 143 $M = 3$ Eigenfunction 142

$M = 3$  Eigenfunction 142 has eigenvalue 2.78439864957



Compare to  $m = 2$  eigenspace with eigenvalue 3.65098985108

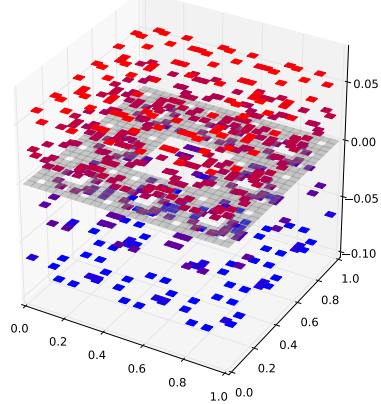


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.762642122586$   
Dot Value: 0.2487592287138587

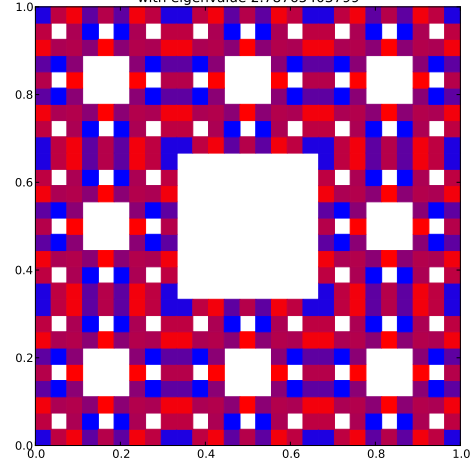
# 144 $M = 3$ Eigenfunction 143

$M = 3$  Eigenfunction 143 has eigenvalue 2.78765403799

Klein Bottle Horizontal Glued Eigenfunction 143  
with eigenvalue 2.78765403799

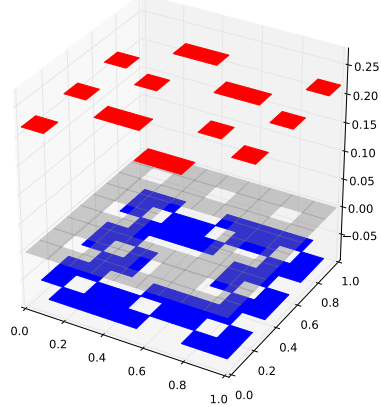


Klein Bottle Horizontal Glued Eigenfunction 143  
with eigenvalue 2.78765403799

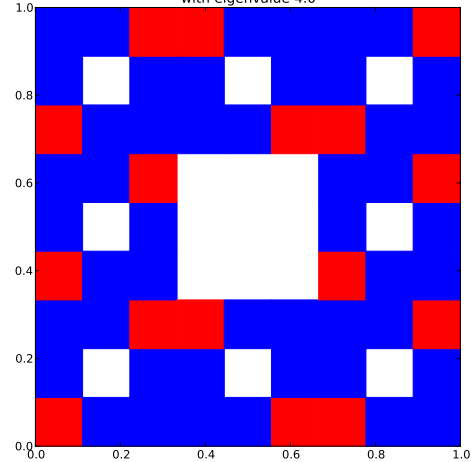


Compare to  $m = 2$  eigenspace with eigenvalue 4.0  
(Note: Eigenspace Dimension  $> 1$ )

Klein Bottle Horizontal Glued Eigenfunction 33  
with eigenvalue 4.0



Klein Bottle Horizontal Glued Eigenfunction 33  
with eigenvalue 4.0

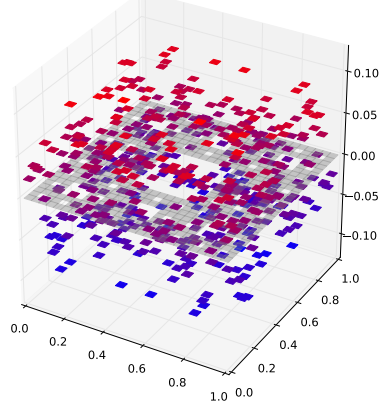


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.696913509497$   
Dot Value: 1.1102230246251565e-16

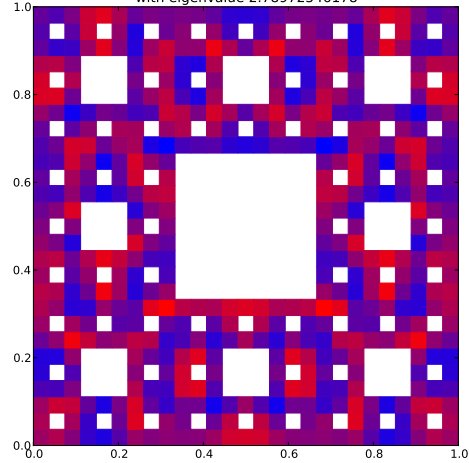
# 145 $M = 3$ Eigenfunction 144

$M = 3$  Eigenfunction 144 has eigenvalue 2.78972546178

Klein Bottle Horizontal Glued Eigenfunction 144  
with eigenvalue 2.78972546178

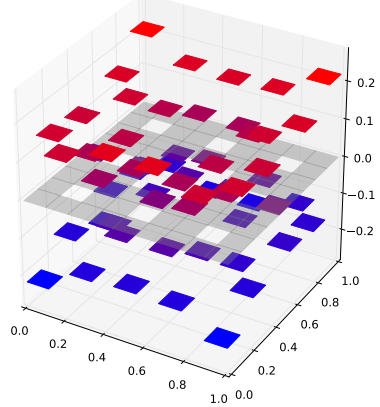


Klein Bottle Horizontal Glued Eigenfunction 144  
with eigenvalue 2.78972546178

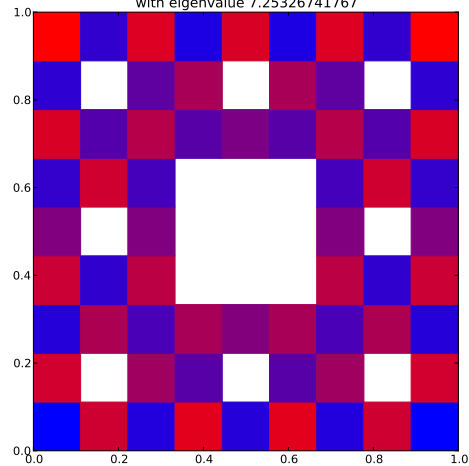


Compare to  $m = 2$  eigenspace with eigenvalue 7.25326741767

Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



Klein Bottle Horizontal Glued Eigenfunction 60  
with eigenvalue 7.25326741767



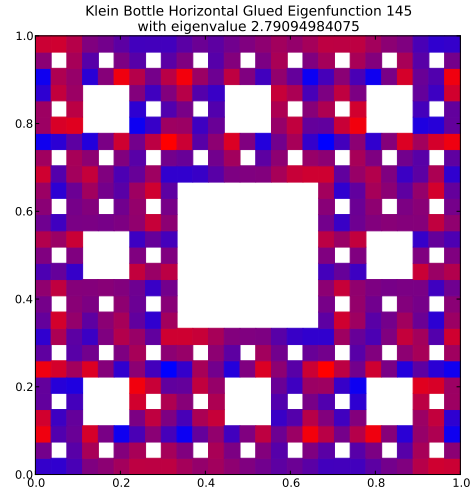
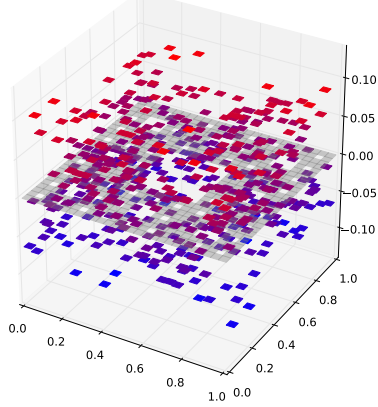
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.384616380609$

Dot Value: 0.20904162213424915

# 146 $M = 3$ Eigenfunction 145

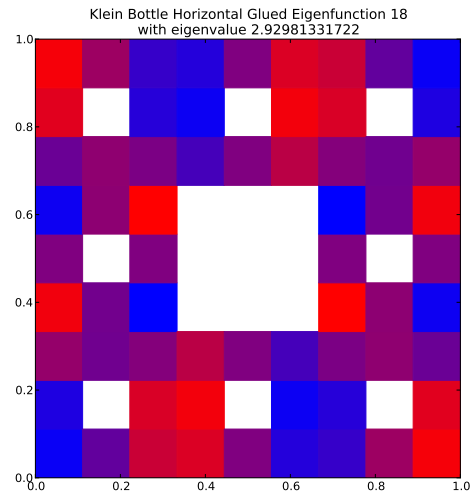
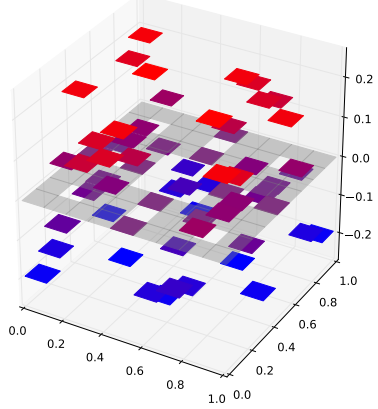
$M = 3$  Eigenfunction 145 has eigenvalue 2.79094984075

Klein Bottle Horizontal Glued Eigenfunction 145  
with eigenvalue 2.79094984075



Compare to  $m = 2$  eigenspace with eigenvalue 2.92981331722

Klein Bottle Horizontal Glued Eigenfunction 18  
with eigenvalue 2.92981331722



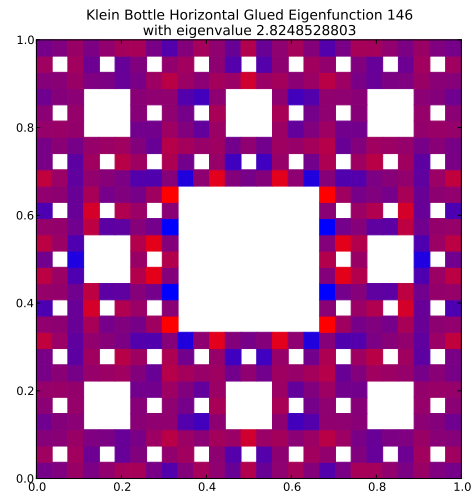
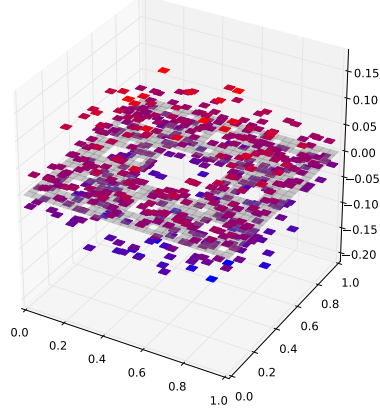
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.952603302179$

Dot Value: 0.13753054338875959

# 147 $M = 3$ Eigenfunction 146

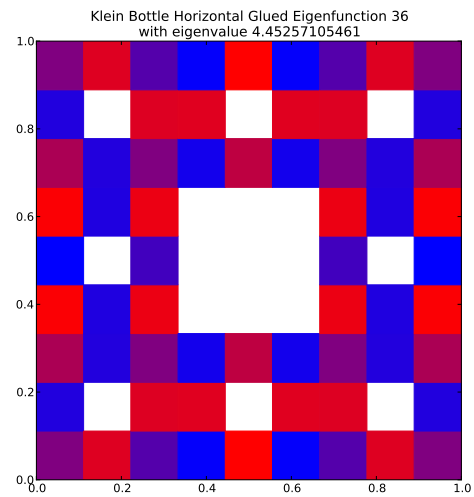
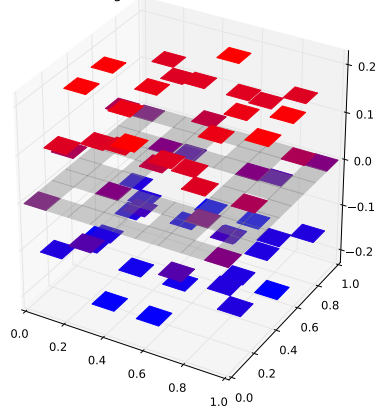
$M = 3$  Eigenfunction 146 has eigenvalue 2.8248528803

Klein Bottle Horizontal Glued Eigenfunction 146  
with eigenvalue 2.8248528803



Compare to  $m = 2$  eigenspace with eigenvalue 4.45257105461

Klein Bottle Horizontal Glued Eigenfunction 36  
with eigenvalue 4.45257105461

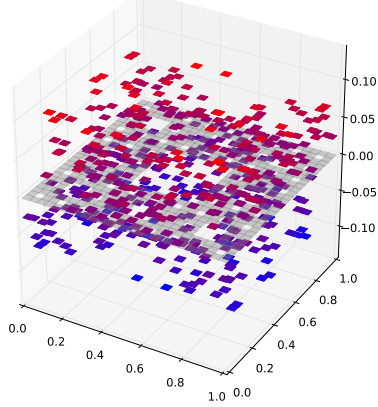


Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.634431847501$   
Dot Value: 0.3721420507782657

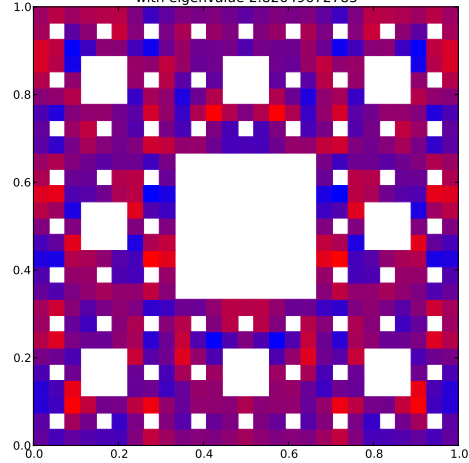
# 148 $M = 3$ Eigenfunction 147

$M = 3$  Eigenfunction 147 has eigenvalue 2.82649672783

Klein Bottle Horizontal Glued Eigenfunction 147  
with eigenvalue 2.82649672783

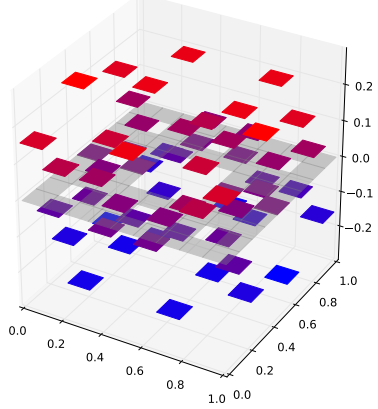


Klein Bottle Horizontal Glued Eigenfunction 147  
with eigenvalue 2.82649672783

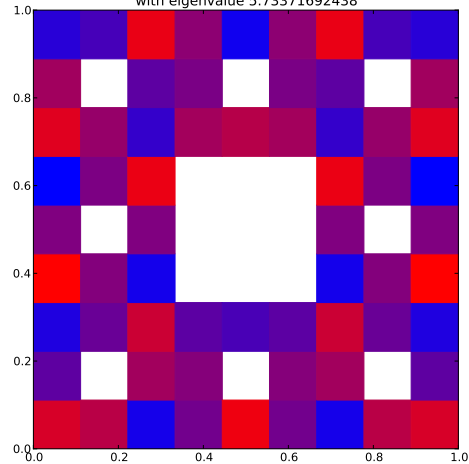


Compare to  $m = 2$  eigenspace with eigenvalue 5.73371692438

Klein Bottle Horizontal Glued Eigenfunction 49  
with eigenvalue 5.73371692438



Klein Bottle Horizontal Glued Eigenfunction 49  
with eigenvalue 5.73371692438



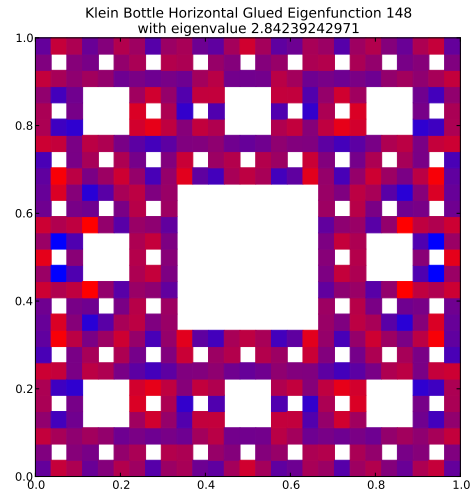
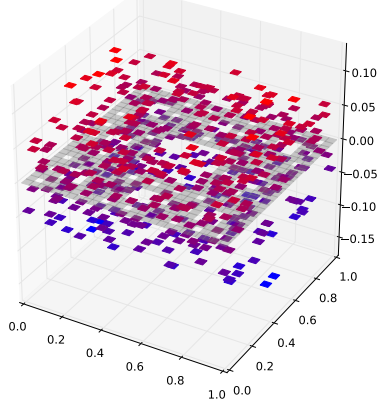
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.492960633583$

Dot Value: 0.2763740700589934

# 149 $M = 3$ Eigenfunction 148

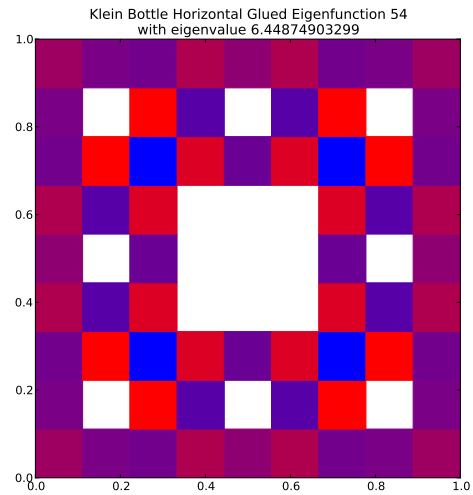
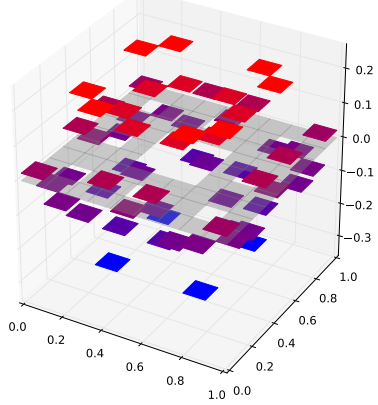
$M = 3$  Eigenfunction 148 has eigenvalue 2.84239242971

Klein Bottle Horizontal Glued Eigenfunction 148  
with eigenvalue 2.84239242971



Compare to  $m = 2$  eigenspace with eigenvalue 6.44874903299

Klein Bottle Horizontal Glued Eigenfunction 54  
with eigenvalue 6.44874903299



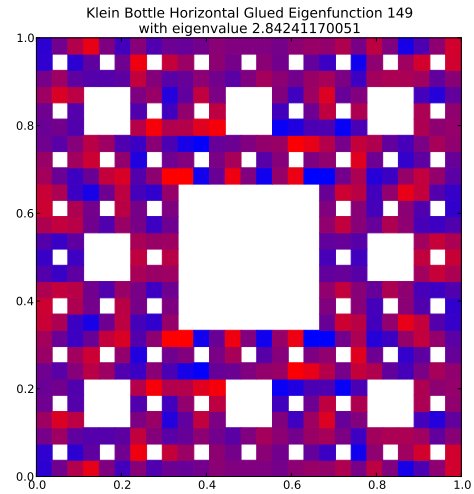
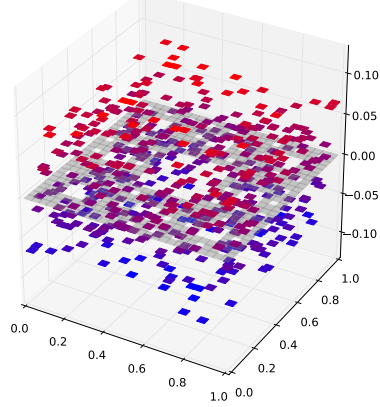
Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.440766482797$

Dot Value: 0.30754421870124726

# 150 $M = 3$ Eigenfunction 149

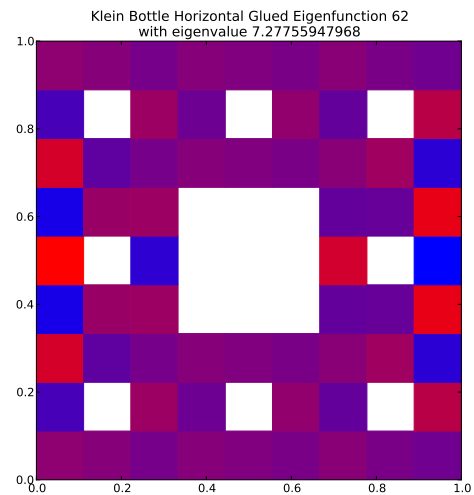
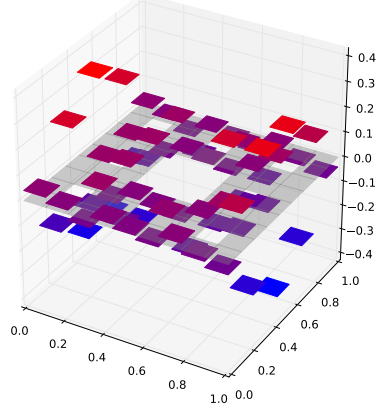
$M = 3$  Eigenfunction 149 has eigenvalue 2.84241170051

Klein Bottle Horizontal Glued Eigenfunction 149  
with eigenvalue 2.84241170051



Compare to  $m = 2$  eigenspace with eigenvalue 7.27755947968

Klein Bottle Horizontal Glued Eigenfunction 62  
with eigenvalue 7.27755947968



Eigenvalue Ratio:  $\lambda_3/\lambda_2 = 0.390572101603$   
Dot Value: 0.3178859928073874