

Description of the “Dot Value” in Eigenfunction Comparisons

SPUR 2016

The Comparison Algorithm

Say we have an eigenfunction f on level $M+1$. By taking averages we obtain a function \bar{f} on level M . Of course \bar{f} is not necessarily an eigenfunction, but if it is close to an eigenfunction, then we can guess the eigenvalue to be around

$$v = \frac{\langle \bar{f}, \Delta_m \bar{f} \rangle}{\langle \bar{f}, \bar{f} \rangle}.$$

From there, if $\lambda \approx v$ is a true eigenvalue of Δ_m , we may project \bar{f} onto the eigenspace associated to λ . We can then guess that f is a refinement of \bar{f}_λ .

Description of the Dot Value

The dot value is in general a way to measure how well \bar{f} and \bar{f}_λ match. It measures how close Cauchy-Schwarz is to equality. For most eigenfunctions f , the dot value is

$$\min \left\{ \left| 1 - \frac{\langle \bar{f}, \bar{f}_\lambda \rangle}{\|\bar{f}\| \|\bar{f}_\lambda\|} \right| : \text{eigenvalues } \lambda \approx v \right\}.$$

(For the calculations in these plots, the minimum was over the nearest 20 eigenvalues above v and the nearest 20 below v .) There are two corner cases in which the dot value is artificially set:

- (i) If $\bar{f} = 0$, then the dot value I artificially set to 2.
- (ii) If $\bar{f}_\lambda = 0$, then the dot value is set to 3.

Key to Scatter Plots

Dot Value	Color
$\in [0, 1]$	linear color spectrum from 0 (blue, good match) to 1 (red, bad match)
$= 2$	green
$= 3$	cyan

The vertical positions of these green and cyan dots in the scatter plots are meaningless, because we should not expect a match. They are included only to see that they occur.