

Torus Random Walk Recurrence: 500 Walks

SPUR 2016

August 12, 2016

At most 1000000 steps
Starting Cell Cell('0', '0')

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Total number of recurrent walks: 331

Lengths of recurrent walks: [410, 4, 1620, 8, 576, 2, 22316, 2, 26077, 6, 88944, 2, 234926, 1585, 34, 2, 119, 487, 186, 49, 2, 10, 2, 24, 2, 2, 2, 20, 2, 231, 682131, 230, 400, 8, 6, 38, 5979, 165, 5173, 24, 2, 568, 10, 2, 2, 12, 2, 2, 2, 96, 2, 122999, 27, 82, 2, 2, 4, 20, 61, 4, 380, 234, 2, 2, 10, 36, 4, 1369, 2, 2, 1628, 16, 2, 2, 2568, 2, 6, 4, 2, 2, 2, 748849, 616, 4, 2, 2, 672, 34, 2, 2, 2, 2, 32, 2, 11, 10722, 2, 224780, 2, 2, 1106, 8, 2, 10, 2, 2, 4, 238641, 2, 2, 2, 4, 38844, 2, 7, 96, 76849, 2, 2, 4, 101, 8, 6, 266, 10, 4, 2, 11, 387, 2, 2, 4063, 2, 4, 2, 2, 4, 2, 2, 125, 8, 10, 2, 2, 4843, 2, 2, 110, 6, 21121, 2, 2, 2, 2, 4, 13, 7, 2, 2, 13, 409, 84, 2, 8847, 2, 29, 20, 2, 393, 2, 271, 116, 2, 32564, 4, 2, 2713, 217, 2, 2, 12, 409, 11, 314, 3542, 2, 8646, 2074, 68, 2, 28551, 60, 6, 2, 2, 15, 2244, 2249, 2, 2, 8, 2, 2, 2, 265, 4, 2, 1139, 2, 4, 2, 252794, 2, 19, 409256, 340, 18, 2, 2, 4, 1270, 133, 2, 55, 17, 33149, 2, 454, 2, 53006, 2, 2, 6, 21, 2, 4919, 2, 240, 19, 100, 2, 2, 10, 2, 2, 2, 2, 4, 2, 4, 2, 8, 2, 2, 11, 4, 2, 2, 357, 2, 6925, 2, 2, 2, 6, 2, 2, 2, 21, 4, 2, 2, 12, 2, 8788, 32, 2, 2, 2, 75, 6, 795, 683, 2, 2, 14, 14, 40, 2, 14, 2, 2, 4, 4, 2, 2, 4, 2, 2, 6, 2, 2, 70238, 162, 4, 2, 2, 2, 6, 2655, 4, 100, 6, 4, 15679, 8, 4, 19, 333, 34, 6, 2, 2, 2, 253053, 39, 2, 656, 6, 7]

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