An example of a transformed sine curve

Question 1. (problem 84 from Section 1.3)
Sketch the graph of the function

$$y = \frac{1}{2}\sin(\pi x - \pi) + \frac{1}{2}.$$

Solution to Question 1. The first thing we want to do is rearrange the above equation so that it is in the form of a plain sine curve ($y = \sin x$) which has been shifted and stretched. With a little algebraic manipulation, we obtain the equation

$$2\left(y - \frac{1}{2}\right) = \sin\Big(\pi(x-1)\Big).$$

Now we will see how to build this equation up starting from the standard sine curve.

- **standard sine curve:**
$$y = \sin(x)$$
- **contract horizontally** by a factor of π: (new period $= 2\pi/\pi = 2$)
$$y = \sin(\pi x)$$
- **contract vertically** by a factor of 2:
$$2y = \sin(\pi x)$$
- **shift right** by 1:
$$2y = \sin\Big(\pi(x-1)\Big)$$
- **shift up** by 1/2:
$$2\left(y - \frac{1}{2}\right) = \sin\Big(\pi(x-1)\Big)$$

Therefore, the graph of the given function looks like the graph of sine, except that its **amplitude** is 1/2, its **period** is 2, and it is **shifted** right 1 unit and up 1/2 units. This graph is pictured in Figure 1.

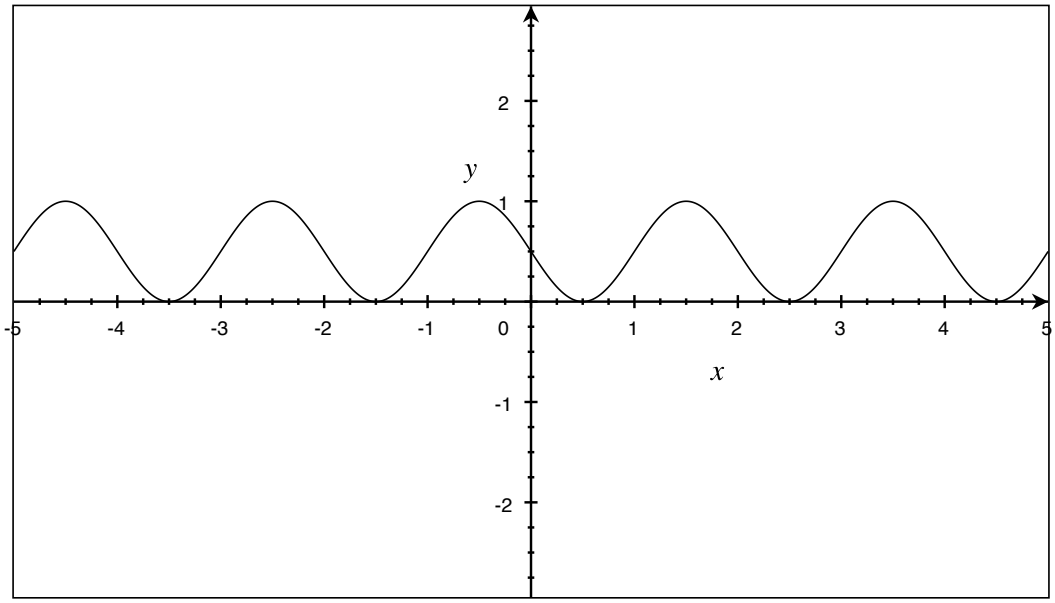

Figure 1: The graph of $y = \frac{1}{2}\sin(\pi x - \pi) + \frac{1}{2}$.