

## Motion in a line

Suppose an object moves in a straight line. For convenience, let's slap an  $x$ -axis onto this line of motion, and at time  $t$  denote the **position** of the object on this axis by  $x(t)$ . The **velocity** of the object at time  $t$  is

$$v(t) := x'(t),$$

and the **acceleration** of the object at time  $t$  is

$$a(t) := v'(t) = x''(t).$$

Thus, the velocity is the rate of change of position with respect to time, and the acceleration is the rate of change of velocity with respect to time, which is the *rate of change of the rate of change* of position with respect to time. If this  $x$ -axis is in its standard position, with  $+\infty$  to the right and  $-\infty$  to the left, then the signs of  $x(t)$ ,  $v(t)$ , and  $a(t)$  can be interpreted as shown in the table on the following page.



$x(t) > 0$	object is to the right of the origin
$x(t) < 0$	object is to the left of the origin
$x(t) = 0$	object is at the origin (at this instant)
$v(t) > 0$	object is moving to the right
$v(t) < 0$	object is moving to the left
$v(t) = 0$	object is not moving (at this instant)
$a(t) > 0$	object's motion to the right is speeding up object's motion to the left is slowing down
$a(t) < 0$	object's motion to the left is speeding up object's motion to the right is slowing down
$a(t) = 0$	object is moving at constant speed (at this instant)

The object is **speeding up** if it is speeding up in the same direction in which it is moving. Therefore, the object is speeding up at time  $t$  if  $v(t)$  and  $a(t)$  have the same sign, (i.e.  $v(t) > 0$  and  $a(t) > 0$ , or  $v(t) < 0$  and  $a(t) < 0$ ).

The object is **slowing down** if it is slowing down in the same direction in which it is moving. Therefore, the object is slowing down at time  $t$  if  $v(t)$  and  $a(t)$  have different signs, (i.e.  $v(t) > 0$  and  $a(t) < 0$ , or  $v(t) < 0$  and  $a(t) > 0$ ).