# Some Optimization Problems

**General Strategy:**

1. **Read** the problem carefully.
2. Make a **sketch**, and **label** the involved quantities with variables.
3. **Record** what you know, and **identify** what quantity you want to maximize or minimize.
4. Using your diagram and the information given, write the quantity you want to maximize/minimize as a function of **just one variable**.
5. Identify the **domain** of the function you want to maximize/minimize.
6. Use **calculus** (or more general mathematical knowledge) to maximize or minimize your function on the given domain.
7. Write a **concluding statement**, including your complete answer, with units.
8. **Check** that your answer makes sense in the given situation, (e.g., length should not be a negative number).

## Problem 1.

A rectangular animal enclosure is to be constructed having one side along an existing long wall and the other three sides fenced. If 100 m of fence are available, what is the largest possible area for the enclosure?

## Problem 2. (page 302, example 1)

An open-top box is to be made by cutting small congruent squares from the corners of a 12 inch by 12 inch sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

## Problem 3.

Find the length of the shortest ladder that can extend from a vertical wall, over a fence 2 m high located 1 m away from the wall, to a point on the ground outside the fence.

## Problem 4. (page 302, example 2)

You have been asked to design a 1 liter can shaped like a right circular cylinder. What dimensions will use the least material?

## Problem 5.

A man can run twice as fast as he can swim. He is standing at a point $A$ on the edge of a circular swimming pool 40 m in diameter, and he wishes to get to the diametrically opposite point $B$ as quickly as possible. He can run around the edge to point $C$, then swim directly from $C$ to $B$. Where should $C$ be chosen to minimize the total time taken to get from $A$ to $B$?

## Problem 6. (page 304, example 3)

A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have, and what are its dimensions?

## Problem 7. (page 305, example 4)

Fermat's principle in optics states that light travels from one point to another along a path for which the time of travel is a minimum. In terms of angles of incidence, find the path that a ray of light will follow in going from a point $A$ in a medium where the speed of light is $c_1$ to a point $B$ in a second medium where its speed is $c_2$. (See Figure 4.39, page 305.)

## Problem 8. (page 307, example 5)

Suppose that the revenue from selling $x$ thousand units is $r(x) = 9x$, while the cost of producing $x$ thousand units is $c(x) = x^3 - 6x^2 + 15x$. Is there a production level that maximizes the profit, $p(x) = r(x) - c(x)$? If so, what is it?