Quiz 3

Name: ____

Question from Class. A man climbs a mountain one day, and climbs down the next. Show that there is a time of day such that his height above ground at that time on the first day was the same as his height at that time on the second day.

Question 1. Given that

$$\lim_{x\to 1} \frac{3x^2+1}{g(x)} = \infty,$$

find

$$\lim_{x\to 1} g(x) =$$

What can you say about the value of $g(1)$? How do you know?

Question 2. Calculate:

$$\lim_{x\to -\infty} \tan^{-1} x =$$

Question 3. Matching:

tangent line	a line you cannot cross
horizontal asymptote	has a slope equal to the limit of the difference quotient
vertical asymptote	occurs when $\lim_{x\to\pm\infty} f(x)$ is a constant
oblique asymptote	occurs when the degree of the numerator is one greater than the degree of the denominator

Solutions to Quiz 3

Solution to Question from Class. For $0 \leq t \leq 24$, let $f(t)$ be the difference between the man's height t hours into the first day and his height t hours into the second day. Then

$$f(0) = 0 - (\text{height of the mountain}) < 0$$

and

$$f(24) = (\text{height of the mountain}) - 0 > 0.$$

Assuming that the man's altitude changed continuously with time, (i.e. that the man did not teleport), the function $f(t)$ must be continuous, so the **Intermediate Value Theorem** implies that there must be a number c in the interval $[0, 24]$ such that $f(c) = 0$. Therefore, the man's altitude was the same c hours into each day.

Solution to Question 1. As $x \to 1$, the numerator $3x^2 + 1$ approaches $3(1)^2 + 1 = 4 \neq 0$. In order for the limit of $(3x^2 + 1)/g(x)$ to approach ∞, the denominator must approach 0, (and because the limit of the quotient is *positive* ∞, it must approach 0 through *positive numbers*). Therefore $\boxed{\lim_{x \to 1} g(x) = 0}$, (or 0^+, to be precise).

The value of the limit as $x \to 1$ has nothing to do with the value of the function at $x = 1$, so we can say $\boxed{\text{nothing}}$ about the value of $g(1)$ — not even whether or not it is defined.

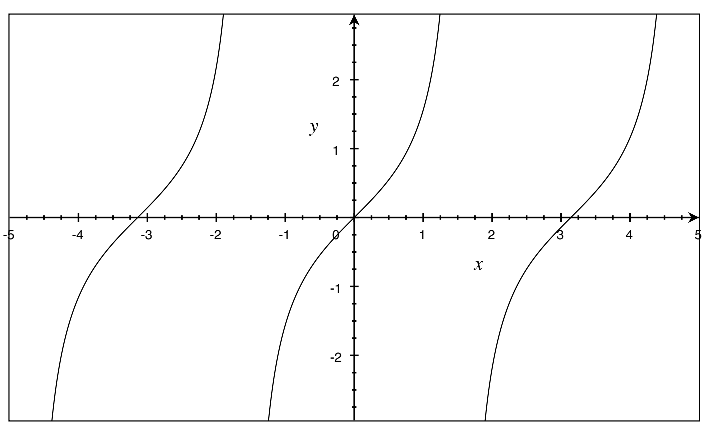

Figure 1: The graph of $y = \tan x$.

Solution to Question 2. The graph of $\tan x$ is pictured in Figure 1. It has an infinite number of vertical asymptotes, $\{x = \frac{\pi}{2} + 2\pi k \mid k = 0, \pm 1, \pm 2, \ldots\}$. Recall that $\tan^{-1} x$ is the inverse of $\tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. The given limit is asking what value of x we need to approach in order to make $\tan x$ approach $-\infty$. From the graph, we see that the answer is $\boxed{\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}}$.

Solution to Question 3.

- A **vertical asymptote** is a line you cannot cross.
- A **tangent line** has a slope equal to the limit of the difference quotient.
- A **horizontal asymptote** occurs when $\lim_{x \to \pm\infty} f(x)$ is a constant.
- An **oblique asymptote** occurs when the degree of the numerator is one greater than the degree of the denominator.