STUDY GUIDE

Note: This list is not meant to be complete nor comprehensive. It is merely a guide to some of the main points from each section.

Section 1.1: *Functions and Their Graphs*

- definition of ***functions***
- ***domain*** and ***range*** of a function
- ***natural domain*** of a function given by a formula
- ***graph*** of a function
- ***vertical line test***
- defining functions with graphs and tables
- ***piecewise functions***
- $|x|$ as a piecewise function
- ***floor*** and ***ceiling functions***

Section 1.2: *Identifying Functions; Mathematical Models*

- ***linear***, ***polynomial***, ***rational***, ***power***, ***trigonometric***, ***exponential***, and ***logarithmic functions***
- recognizing types of functions from their formulas
- recognizing types of functions from their graphs
- ***increasing*** and ***decreasing functions***
- ***even functions*** — symmetry about the y-axis
- ***odd functions*** — symmetry about the origin
- proportional quantities

Section 1.3: *Combining Functions; Shifting and Scaling Graphs*

- adding, subtracting, multiplying, and dividing functions
- ***composition*** of functions
- finding the domain of a composition, sum, product, etc. of functions
- ***shifting*** the graph of a function vertically or horizontally
- ***scaling*** (stretching or compressing) the graph of a function vertically or horizontally

- ***reflecting*** the graph of a function (across the x-axis, across the y-axis, through the origin)
- an ellipse as a scaled circle
- example: shifting, scaling, and reflecting a sine curve

Section 1.5: *Exponential Functions*

- the ***exponential function*** with base $a > 0$, $y = a^x$, and how to evaluate it
- $y = a^x$ if $a > 1$ or $a < 1$
- the number $e = 2.718281828459\ldots$
- the ***natural exponential function***, $y = e^x$, and why it is special (tangent line at $(0, 1)$)
- ***exponential growth*** ($k > 1$) and ***decay*** ($0 < k < 1$), $y(t) = y_0 e^{kt}$
- yearly interest compounded n times per year, $y(t) = y_0 \left(1 + \dfrac{r}{n}\right)^{nt}$
- yearly interest compounded continuously, $y(t) = y_0 e^{rt}$

Section 1.6: *Inverse Functions and Logarithms*

- ***one-to-one functions***
- ***horizontal line test*** for one-to-one functions
- ***inverse functions***
- relations between f and f^{-1}: domain, range, cancellation
- f is one-to-one $\iff$ f has an inverse
- finding inverses from the function's formula, from the function's graph
- the ***logarithmic function*** with base $a > 0$, $a \neq 1$
- the ***natural logarithmic function***, $\ln x = \log_e(x)$
- properties of logarithms
- change of base formulas for exponential and logarithmic functions
- ***restricted domains*** for trigonometric functions (to make them one-to-one)
- inverse trigonometric functions: $\sin^{-1}(x) = \arcsin(x)$, $\cos^{-1}(x) = \arccos(x)$, etc.
- evaluating inverse trigonometric functions

Section 2.1: *Rates of Change and Limits*

- ***average speed*** over a time interval, instantaneous speed
- ***average rate of change*** and ***secant line*** of a function over an interval
- ***instantaneous rate of change*** of a function
- ***limits*** of functions (i.e. limits of the values of the function)
- estimating limits using tables of values
- finding a limit visually using the function's graph
- limits of constant functions, polynomial functions, and other functions whose graphs are connected
- Limits only depend on the function's behavior **near** the point, not **at** the point.
- using algebraic manipulation to evaluate limits
- examples of when limits don't exist: function approaches more than one value, oscillation, etc.