Dominant strategies and Nash equilibria

A 2×2 **ordinal game** consists of two players, which we call ROW and COLUMN, each of whom can choose between two **strategies**, which we call C (cooperate) and N (noncooperate). There are four possible results of such a game, which we label CC, CN, NC, and NN. (Remember our convention that we list ROW's strategy first, so that the symbol "CN" means ROW cooperates and COLUMN noncooperates.)

Each player ranks the possible results by assigning from 1 to 4 points to each possibility, with no ties and with more points assigned to more preferable results. Each combination of strategies leads to an **outcome**, which we represent by the ordered pair of (ROW's point assignment, COLUMN's point assignment).

For example, if C stands for "silent" and "N' stands for "confess", then the following matrix represents the possible outcomes in the **Prisoner's Dilemma**.

		COLUMN	
		C	N
ROW	C	$(3, 3)$	$(1, 4)$
	N	$(4, 1)$	$(2, 2)$

Recall the following terms from the textbook and class.

Definition 1. Fix a particular 2×2 ordinal game.

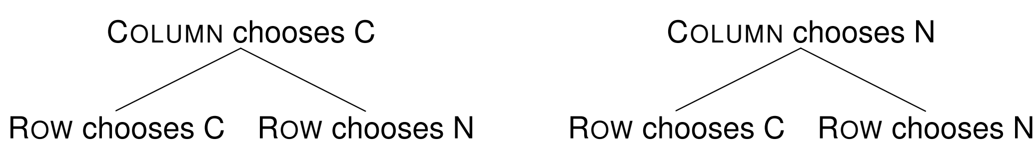
A player's strategy is called **dominant** for that player if, no matter what the other player does, this strategy is more beneficial than the other strategy.

A particular outcome is called a **Nash Equilibrium** if neither player will do better by *unilaterally* changing his/her strategy. (If a player changes his/her strategy *unilaterally*, then this player switches strategies, but the other player does not.)

Is there a dominant strategy?

The following steps give a method to determine whether the player Row has a dominant strategy in a given 2×2 ordinal game. (To determine whether Column has a dominant strategy, just reverse the rolls of Row and Column in the steps.)

1. Make two trees, each with one root node connected to two nodes below it.
2. Label one tree "Column chooses C", and the other "Column chooses N".
3. In each tree, label one bottom node "Row chooses C", and other "Row chooses N".
4. Next to each node of each tree, write the point value that Row assigned to that result. For example, for the "Row chooses N" node of the "Column chooses C" tree, you would write the point value that Row assigned to the NC result.
5. Considering *one tree at a time*, circle the bottom node that has the higher number written next to it.
6. - If you circled "Row chooses C" in *both trees*, then C is a dominant strategy for Row.
   - If you circled "Row chooses N" in *both trees*, then N is a dominant strategy for Row.
   - If you circled differently labeled nodes in the different trees, then Row has no dominant strategy.



SUMMARY:
1. Make one tree for each of Column's choices.
2. Give each tree a terminal node for each of Row's choices.
3. If a single strategy for Row is better in *both trees*, then it is dominant.

Is this a Nash Equilibrium?

The following steps give a method to check whether a particular outcome of a given 2×2 ordinal game is a Nash Equilibrium.

1. Make two trees, each with one root node connected to two nodes below it.
2. Label one tree "ROW" and the other "COLUMN".
3. In each tree, label one bottom node "stay", and other "switch".
4.
    - For the "stay" node of the ROW tree, write down the point value assigned by ROW to the outcome you are checking. (This is just the first number of the ordered pair.)
    - In the table, put your finger on the outcome you are testing. Find the outcome if ROW switches strategies by moving up or down. Next to the "switch" node of the ROW tree, write the point value assigned by ROW to this outcome. (Again, this is the first number of this ordered pair.)
5. Repeat the previous step for the COLUMN tree, except use the second number of the ordered pairs instead of the first number, and move left and right instead of up and down.
6. Considering *one tree at a time*, circle the bottom node that has the higher number written next to it.
7. If you circled the "stay" node for *both trees*, then the outcome you are testing IS a Nash Equilibrium. Otherwise, it is NOT.

SUMMARY:
1. Make two trees, one for ROW and one for COLUMN.
2. Give each tree two terminal nodes, one for "stay" and one for "switch".
3. If "stay" is a better option in *both trees*, then the outcome you are testing is a Nash Equilibrium.