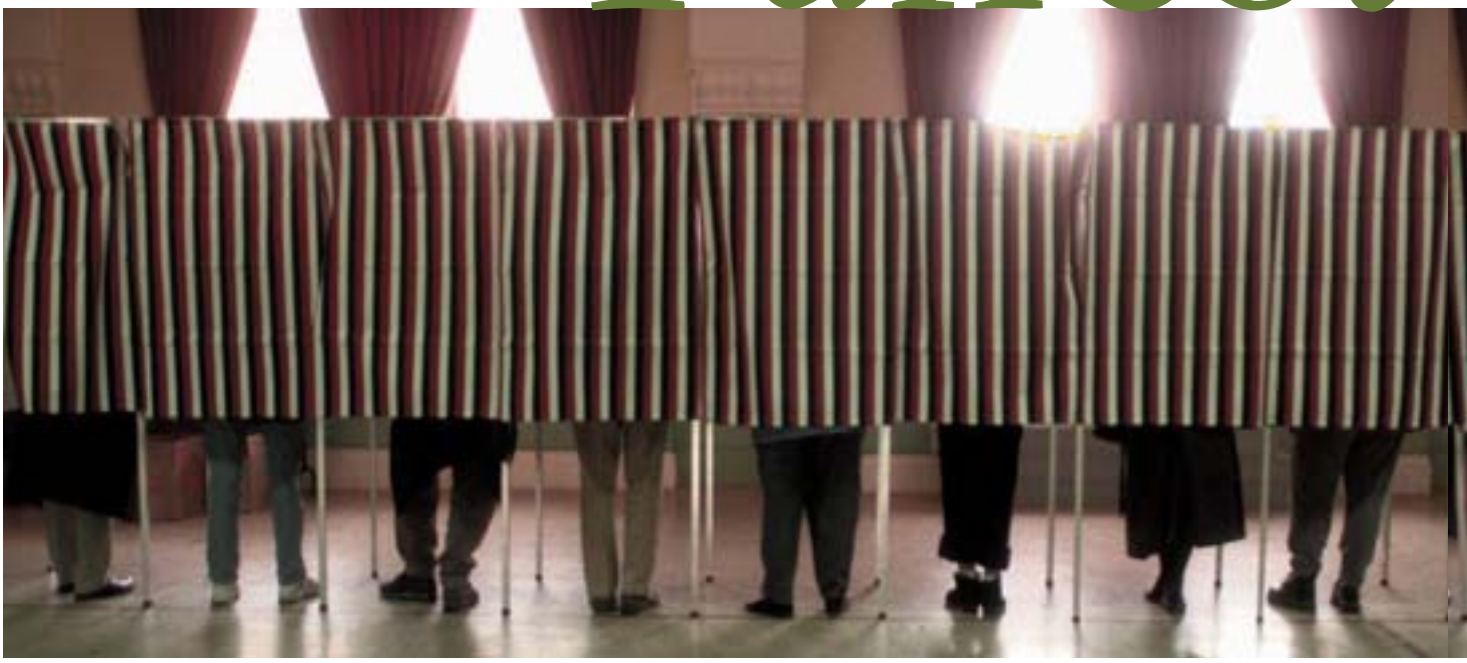
The Fairest



All VOTING SYSTEMS have drawbacks. But by taking into rank candidates, one system gives the

By Partha Dasgupta and Eric Maskin

Most American and French citizens—indeed, those of democracies the world over—spend little time contemplating their voting systems. That preoccupation is usually left to political and electoral analysts. But in the past few years, a large segment of both these countries' populations have found themselves utterly perplexed. People in France wondered how a politician well outside the political mainstream made it to the final two-candidate runoff in the presidential election of 2002. In the U.S., many voters asked why the most popular candidate lost the election of 2000.

We will leave discussions of hanging chads, butterfly ballots, the electoral college and the U.S. Supreme Court to political commentators. But based on research by ourselves and colleagues, we can address a more fundamental issue: What kinds of systems, be they for electing national leaders or student council

Vote of all

account how voters truest REFLECTION OF THE ELECTORATE'S VIEWS

presidents, go furthest toward truly representing the wishes of the voters? We argue that one particular system would be best in this sense—and it would be simple and practical to implement in the U.S., France and myriad other countries.

The Importance of Being Ranked

IN MOST NATIONAL presidential electoral systems, a voter chooses only his or her favorite candidate rather than ranking them all. If just two candidates compete, this limitation makes no difference. But with three or more candidates, it can matter a great deal.

The French presidential election of 2002 provides a case in point. In the first round, voters could vote for one of nine candidates, the most prominent being the incumbent Jacques Chirac of the Gaullist party, the Socialist leader Lionel Jospin and the National Front candidate Jean-Marie Le Pen. The rules dictated that

if no candidate obtained an outright majority, the two candidates with the largest numbers of votes would face each other in a runoff. Chirac finished first (with 19.9 percent of the vote). The real surprise, however, lay in second place: the far-right-winger Le Pen took it (with 16.9 percent), while Jospin—who, with Chirac, had been heavily favored to reach the runoff—finished third (with 16.2 percent). In the second round, Chirac handily defeated Le Pen.

Despite Jospin's third-place finish, most available evidence suggests that in a one-to-one contest against Le Pen, he would have easily won. It is even plausible that he could have defeated Chirac had he made it to the second round. Yet by having voters submit only their top choice, the French electoral system cannot take account of such important information. Furthermore, it permits extremist candidates such as Le Pen—candidates who have no real chance of winning—to have an appreciable effect on the outcome.

Overview/*Getting Voting Right*

- There is no such thing as a perfect voting system: every kind has one flaw or another.
- Nevertheless, one method could solve some of the problems that arose during recent elections in France and the U.S. Called true majority rule, this system incorporates information about the ranking of candidates, permitting a more accurate representation of voters' views.
- Our theoretical work shows that true majority rule more often avoids the flaws that arise for other voting methods. And, significantly, it could be easily implemented in countries the world over.

True majority rule and rank-order voting result in DRAMATICALLY DIFFERENT outcomes.

The 2000 U.S. presidential election exposed similar shortcomings. To make this point most clearly, we will pretend that the election procedure was simpler than it actually was. We will consider just the four main candidates, and we will assume that there is no difference between the popular vote and the electoral college vote. (There have been many complaints about the electoral college, but even if it were replaced by popular vote, serious problems would remain.) We will also assume that there are only four kinds of voters: those who prefer Ralph Nader to Al Gore, Gore to George W. Bush, and Bush to Pat Buchanan (the "Nader" voters); those with the ranking Gore, Bush, Nader, Buchanan (the "Gore" voters); those with the ranking Bush, Buchanan, Gore, Nader (the "Bush" voters); and those with the ranking Buchanan, Bush, Gore, Nader (the "Buchanan" voters).

To be concrete, suppose that 2 percent of the electorate are Nader voters, 49 percent Gore voters, 48 percent Bush voters, and 1 percent Buchanan voters. If voters each choose one candidate, Gore will receive 49 percent and Bush 48 percent of the total (the actual percentages were 48.4 percent and 47.9 percent, respectively). Given that no candidate receives a majority (that is, more than 50 percent), how is the winner to be determined? Gore receives a plurality (the most votes short of 50 percent), so perhaps he should win.

On the other hand, the American Constitution stipulates that, absent a majority of the electoral votes, the House of Representatives should determine the winner. With a Republican majority in 2000, the House would presumably have gone for Bush. Clearly, having U.S. voters name solely their favorite candidate does not result in an outcome that is obviously right.

As in the French election, such ambiguity can be resolved by having voters submit complete rankings. Even though Gore is the favorite of only 49 percent of the electorate, the rankings show that a clear majority of 51 percent—the Gore and Nader voters combined—prefer Gore to either Bush or Buchanan. So Gore is the winner according to an electoral system called true majority rule (or simple majority rule), in which voters submit rankings of all the candidates and the winner is the one who beats each opponent in head-to-head competition based on these rankings.

Rankings can also be used in other electoral systems. Consider, for instance, "rank-order voting"—a procedure often used to elect committee officers that has been proposed to solve the problems inherent in the American and French presidential electoral systems. If four candidates are running, each voter assigns four points to his or her favorite, three to the next favorite, two to the next, and one to the least favorite. The winner is the candidate with the biggest total. The method appears to have been invented by Jean-Charles Borda, an 18th-century French engineer, and is sometimes known as the Borda count.

Imagine that 100 million people vote in the U.S. election. Based on our earlier assumptions, we know that 49 million of them will rank Gore first. So Gore will receive 196 million points—that is, 49 million times four points—from the Gore voters. The Nader voters place him second, so he picks up six million points from them. Finally, the Bush and Buchanan voters place him third, for an additional 98 million points. His grand total is 300 million points. If we make the corresponding computations for the others, we find that Nader gets 155 million points and Buchanan 199 million. Strikingly, Bush gets 346 million, even though a majority of the electorate prefer Gore [*see scenario A in box on opposite page*]. Only 2 percent of the electorate ranks Bush lower than second place, which is good enough to elect him under rank-order voting.

Thus, true majority rule and rank-order voting result in dramatically different outcomes. Considering this sharp contrast, it may seem hard to say which method is better at capturing the essence of voters' views. But we propose to do just that. We can evaluate these two systems—and any other—according to some

TOBY TALBOT *AP Photo*; PHOTOILLUSTRATION BY JOHNNY JOHNSON (*preceding pages*)

RANK-ORDER VOTING: SAMPLE SCENARIOS

IN THIS ELECTORAL SYSTEM, candidates are ranked and the corresponding points are tallied. Interestingly, even if a candidate were the true majority winner, as Gore is in scenario A, he or she would not necessarily win the rank-order vote. But a slight change in ranking, as happens with the Bush voters in scenario B, can make an enormous difference. In this case, it would lead to Gore winning.

Scenario A

CANDIDATE RANKING	POINTS ASSIGNED	VOTE TOTALS (in millions)
GORE VOTERS 49% (of 100 million votes)		
Gore	4	4 X 49 = 196
Bush	3	3 X 49 = 147
Nader	2	2 X 49 = 98
Buchanan	1	1 X 49 = 49
NADER VOTERS 2%		
Nader	4	4 X 2 = 8
Gore	3	3 X 2 = 6
Bush	2	2 X 2 = 4
Buchanan	1	1 X 2 = 2
BUSH VOTERS 48%		
Bush	4	4 X 48 = 192
Buchanan	3	3 X 48 = 144
Gore	2	2 X 48 = 96
Nader	1	1 X 48 = 48
BUCHANAN VOTERS 1%		
Buchanan	4	4 X 1 = 4
Bush	3	3 X 1 = 3
Gore	2	2 X 1 = 2
Nader	1	1 X 1 = 1
		Gore Total: 300
		Bush Total: 346

Scenario B

BUSH VOTERS 48%		
Bush	4	4 X 48 = 192
Gore	3	3 X 48 = 144
Buchanan	2	2 X 48 = 96
Nader	1	1 X 48 = 48
		Gore Total: 348
		Bush Total: 346

fundamental principles that any electoral method should satisfy. Kenneth J. Arrow of Stanford University originated this axiomatic approach to voting theory in a 1951 monograph, a work that has profoundly shaped the voting literature.

Most voting analysts would agree that any good electoral method ought to satisfy several axioms. One is the consensus principle, often called the Pareto principle after Italian sociologist Vilfredo Pareto. It states that if everyone agrees that candidate A is better than B, then B will not be elected. This axiom does not help discriminate between true majority rule and rank-order voting, however, because both methods satisfy it—that is, both will end up with B losing. Moreover, the principle does not apply very often: in our U.S. election example, there is no unanimous preference for any one candidate over another.

Another important axiom holds that all voters should count equally—the "one-person, one-vote," or equal-treatment, principle. Voting theorists call it the principle of anonymity: who you are should not determine your influence on the election. True majority rule and rank-order voting also both satisfy anonymity.

A third criterion, however, does differentiate between the two. Neutrality, as this axiom is called, has two components. The first is symmetry, which means that the electoral rules should not favor one candidate over the other. The second requires that the voters' choice between candidates A and B should not depend on their views about some third candidate C. What would happen in our U.S. example if the Bush voters' ranking shifted to become Bush, Gore, Buchanan, Nader (instead of Bush, Buchanan, Gore, Nader)? From the standpoint of true majority rule, nothing important would change: the majority still prefer Gore to Bush. But look at what happens under rank-order voting: Gore now receives 348 million points, while Bush's total remains 346 million [*see scenario B in box at left*]. Gore now wins instead of Bush.

Obviously, rank-order voting can violate neutrality. Voters' preferences between Gore and Buchanan, a candidate who stands no chance of getting elected, determine the choice between Bush and Gore—and the outcome of the election. In contrast, true majority rule always satisfies neutrality. This last assertion may puzzle those readers who recall that in the actual election, discussion abounded about whether votes for Nader would affect the race between Bush and Gore. Indeed, in retrospect it appears that Nader—perhaps with help from the infamous butterfly ballot in Florida and even from Buchanan—may have siphoned off enough Gore votes to tip the election to Bush. But this effect was possible only because the U.S. election system is not actually true majority rule but its own unique system.

Majority Rule and the French Election

LET'S LOOK AT WHAT would happen to the French election of 2002 under true majority rule—which, for simplicity's sake, we will henceforth refer to as majority rule. Imagine Chirac, Jospin and Le Pen are the only candidates, and the electorate divides into three groups. Everyone in the first group, 30 percent of voters, has the ranking Jospin, Chirac, Le Pen. In the second group, 36 percent of the electorate, the ranking is Chirac, Jospin, Le Pen. In the remaining 34 percent, voters rank Le Pen over Jospin over Chirac. Chirac and Le Pen—with 36

THE AUTHORS

PARTHA DASGUPTA and *ERIC MASKIN* frequently collaborate in their research, including recent work on auction theory. Dasgupta is Frank Ramsey Professor of Economics at the University of Cambridge and past president of the Royal Economic Society. Maskin is Albert O. Hirschman Professor of Social Science at the Institute for Advanced Study in Princeton, N.J., and past president of the Econometric Society.

SLIM FILMS

FRENCH ELECTION OF 2002

CANDIDATE RANKING	PERCENTAGE OF VOTERS CHOOSING THIS RANKING
Jospin Chirac Le Pen	30
Chirac Jospin Le Pen	36
Le Pen Jospin Chirac	34


Jospin


Chirac


Le Pen

VOTERS' PREFERENCES BY PERCENT

Prefer Jospin to Chirac	64
Prefer Jospin to Le Pen	66
Prefer Chirac to Jospin	36
Prefer Chirac to Le Pen	66
Prefer Le Pen to Chirac	34
Prefer Le Pen to Jospin	34

and 34 percent of the vote, respectively—would move forward into a runoff, where Chirac would easily prevail because 66 percent of voters prefer him to Le Pen.

The same outcome would result under yet another system, called instant-runoff voting (IRV), which is practiced in Ireland and Australia and which, like rank-order voting, has been advocated as an alternative to the French and U.S. systems. In IRV, simply put, rankings are used by election officials to successively eliminate the lowest-ranking candidates (and to incorporate their percentages into the voters' next-ranked choices) until only two candidates remain.

But the French and IRV systems conflict with majority rule. If you examine the configuration of voters' rankings, you see that Jospin actually commands an enormous majority: 64 percent of the electorate prefer him to Chirac, and 66 percent prefer him to Le Pen. Majority rule dictates that Jospin should win by a landslide [*see box above*].

Recall that under majority rule a voter can make a political statement without harming the chances of any electable candidate. Someone who preferred Jospin to Chirac and knew that Le Pen had no chance of winning but wished to rank him first as a gesture of protest could do so without fear of knocking Jospin out of the race. (Except, of course, in the highly unlikely event that a majority of other voters made the same gesture.) The analogous point can be made about a voter who preferred Gore to Bush but wished to lend symbolic support to Nader.

Yet despite these virtues, majority rule has a flaw. It can violate another well-accepted voting principle: transitivity. Transitivity requires that if candidate A is chosen over B, and B is chosen over C, then A should be chosen over C. Now, ignoring Buchanan, pretend that 35 percent of the electorate prefer Gore to Bush to Nader, 33 percent rank Bush above Nader above Gore, and 32 percent go for Nader above Gore above Bush. Sixty-seven percent of voters rank Gore above Bush, 68 percent rank Bush above Nader, and 65 percent rank Nader above Gore. In other words, no matter which candidate is chosen, at least 65 percent of voters prefer somebody else! In this case, majority rule produces no winner.

This possibility, called the Condorcet paradox, was identified in the late 18th century by Marie-Jean-Antoine-Nicholas de Caritat, the Marquis de Condorcet, a colleague and archcritic of Borda. The three rankings—Gore over Bush over Nader, Bush over Nader over Gore, and Nader over Gore over Bush—are collectively called a Condorcet cycle.

Our comparison of majority rule and rank-order voting appears to have resulted in a dead heat: majority rule satisfies every principle on our list except transitivity, and rank-order voting satisfies all but neutrality. This conundrum leads us to consider whether some other electoral system exists that satisfies all the principles. Arrow's celebrated impossibility theorem says no. It holds that any electoral method must sometimes violate at least one principle [see "Rational Collective Choice," by Douglas H. Blair and Robert A. Pollak; SCIENTIFIC AMERICAN, August 1983].

Beyond Impossibility

BUT ARROW'S THEOREM is unduly negative. It requires that an electoral method must satisfy a given axiom, no matter what voters' rankings turn out to be. Yet some rankings are quite unlikely. In particular, the Condorcet paradox—the bugaboo of majority rule—may not always be a serious problem in practice. After all, voters' rankings do not come out of thin air. They often derive from ideology.

To see what implications ideology holds for majority rule, think about each candidate's position on a spectrum ranging from the political left to the right. If we move from left to right, we presumably encounter the 2000 presidential candidates in the order Nader, Gore, Bush, Buchanan. And if ideology drives voters' views, then any voter who ranks Nader above Gore is likely to rank Gore above Bush and Bush above Buchanan. Similarly, any voter who ranks Bush above Gore can be anticipated to rank Gore above Nader. We would not expect to find a voter with the ranking Bush, Nader, Gore, Buchanan.

In a pioneering paper published in the 1940s, the late Duncan Black of the University College of North Wales showed that if voters' rankings are ideologically driven in the above manner—or at least if there are not too many nonideological voters—majority rule will satisfy transitivity. This discovery made possible a great deal of work in political science because, by posit-

When more than two choices present themselves, voters should SUBMIT A RANKING of candidates.

JACK GUEZ *AFP/Corbis (Jospin)*; DANIEL JANIN *AFP/Corbis (Chirac)*; FRANÇOIS GUILLOT *Reuters NewMedia/Corbis (Le Pen)*

ing ideological rankings of candidates on the part of voters, researchers could circumvent the Condorcet paradox and make clear predictions about the outcome of majority rule.

Of course, voters may not always conform to such a tidy left-right spectrum. But other situations also ensure transitivity. For another example, look again at the French election. Although Chirac and Jospin led the two major parties, it seems fair to say that they did not inspire much passion. It was the extremist candidate, Le Pen, who aroused people's repugnance or enthusiasm: evidence suggests that a huge majority of voters ranked him third or first among the three top candidates; few ranked him second. One can argue about whether such polarization is good or bad for France. But it is unquestionably good for majority rule. If voters agree that one candidate of three is not ranked second, transitivity is guaranteed. This property, called value restriction, was introduced in 1966 by Amartya Sen of Harvard University.

In our research on voting, we say that a voting system works well for a particular class of rankings if it satisfies the four axioms when all voters' rankings belong to that class. For instance, majority rule works well when all rankings are ideologically driven. It also works well when all rankings are "value restricted." Indeed, we have found that whenever any voting system works well, so does majority rule. Furthermore, majority rule works well in some cases in which other systems do not. We call this the majority dominance theorem.

To illustrate, we will imagine a three-way race between Gore, Bush and Nader. Suppose that every voter in fact ranks the candidates as either Gore, Bush, Nader or Bush, Gore, Nader. With voters' rankings belonging to this two-element class, rank-order voting satisfies its nemesis: the principle of neutrality (because voters' views on Nader do not affect whether Bush or Gore wins a rank-order election). Yet majority rule also works well here, because it satisfies its nemesis, transitivity.

But rank-order voting no longer works well if the situation becomes slightly more complicated. If we add a third ranking—Gore, Nader, Bush—majority rule is still transitive. These three rankings together do not constitute a Condorcet cycle. Rank-order voting, however, no longer satisfies neutrality. Suppose 51 percent rank Bush above Gore above Nader. If the remaining 49 percent rank Gore above Nader above Bush, Gore will win. If the remainder instead have the ranking Gore, Bush, Nader, however, then Bush wins—even though this group of 49 percent has the same ranking of Gore and Bush in either case.

Majority rule still fails to work well sometimes, as the Condorcet paradox shows, though less often than other voting rules do. And in such cases, it has to be modified to identify a winner. There are many ways this can be done. Perhaps the simplest modification is as follows: If no one obtains a majority against all opponents, then among those candidates who defeat the most opponents in head-to-head comparisons, select as winner the one with the highest rank-order score.

BALLOT FROM SOUTH AFRICA'S first free elections in 1994. Sixty-two percent of the electorate chose Nelson Mandela and the African National Congress Party.

Improving Future Elections

THE WAY most countries pick their presidents is faulty. Both the 2000 U.S. and 2002 French presidential elections were appreciably affected—perhaps decisively—by candidates who had no realistic chance of winning. These candidates were able to wield influence because, in each case, only a voter's top-ranked candidate was counted. We believe that when more than two choices present themselves, voters should submit a ranking of candidates and that majority rule—as we have discussed it—should determine the winner. Such a method would not be perfect; no method is. But as the majority dominance theorem shows, it would come closer to an accurate representation of the voters' wishes than any other system does. SA

MORE TO EXPLORE

Social Choice and Individual Values. Kenneth J. Arrow. Wiley, 1951. (Yale University Press, 1990.)

The Theory of Committees and Elections. Duncan Black. Cambridge University Press, 1958. (Kluwer Academic Publishers, 1998.)

Collective Choice and Social Welfare. Amartya Kumar Sen. Holden-Day, 1970. (North-Holland, 1984.)

On the Robustness of Majority Rule and Unanimity Rule. Partha Dasgupta and Eric Maskin. Available at **www.sss.ias.edu/papers/papers/econpapers.html**