Math 2220 Exam 1

Tuesday, February 21, 2012

Name: _____

Show all work and explain all answers except as noted.

1. a. Find the equation of the tangent plane to the surface defined by

$$\ln(x^2 + z^4 - 1) - x^2(y - 2) - z = 0$$

at the point $(1, 3, -1)$.

b. Find all values of (a, b) for which the tangent plane to $z = x^2y - xy^2 - x + y + xy$ at $(x, y) = (a, b)$ parallel to the plane $2x - 2y + 2z = 5$.

2. Find all local maxima, local minima, and saddle points of $f(x, y) = x^4 - xy + \frac{1}{4}y^4$. (You must indicate which are local maxima, which are local minima, and which are saddle points.)

3. Define $f(x,y) = xy/(x^2+y^2)$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Determine the set of all (x,y) such that $\frac{\partial f}{\partial x}$ is defined and find the value when possible. Is $\frac{\partial f}{\partial x}$ continuous?

4. Find the absolute maximum and minimum value of the function $f(x, y) = x^2 + y^2 + 2y$ on the set $D = \{(x, y) : x^2 + y^2 \leq 2\}$.

5. a. Define

$$\mathbf{r}(t) = \langle \frac{1}{2}\cos(2t) + \frac{1}{2}, \frac{1}{2}\sin(2t), \sin(t) \rangle.$$

Compute the velocity vector as a function of t.

b. Find the equation of the line tangent to the curve parametrized by $\mathbf{r}(t)$ at $(1, 0, 0)$.

c. Show that $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$ for all t. (The following identities may be helpful: $\sin(2t) = 2\sin(t)\cos(t)$ and $\cos(2t) = 2\cos^2(t) - 1 = \cos^2(t) - \sin^2(t)$.)

6. a. Show that there is a local solution $z = f(x, y)$ to $xy^2z^5 - 2x^3yz + 4x^2y = 3$ at the point $(1, 1, 1)$. Compute $\frac{\partial z}{\partial x}|_{(1,1,1)}$.

b. Let U be a subset of $\mathbb{R}^n$. Define what is meant by the phrase "U is open."

c. If $D = \{(x, y) : (x^2 + y^2 > 1) \text{ and } (y > 0)\}$, what is the boundary of D? Your answer should both contain a sketch and a clear description of the set. You do not have to justify your answer.