Math 2220
Prelim 1
February 19th 2013 Duration - 1.5 hours

Last Name: ______________________________

Given Name: ______________________________

NetID : ______________________________

Please circle your instructor/lecture section and your TA/discussion section.

Bruce Fontaine	MWF 10:10am - 11:00am
Allen Knutson	MWF 12:20pm - 1:10pm

Zhexiu Tu	W 1:25pm - 2:15pm (MRL 106)
Chenxi Wu	W 2:30pm - 3:20pm (RCK 102)
Zhexiu Tu	W 2:30pm - 3:20pm (MRL 106)
Zhexiu Tu	W 3:35pm - 4:25pm (MRL 106)
Chenxi Wu	W 3:35pm - 4:25pm (RCK 102)

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	20	
6	20	
Total:	100	

INSTRUCTIONS

1. This exam has 6 questions worth 100 points on 7 pages (including this one).
2. Please enter your name and circle your lecture/discussion sections now.
3. Please show your work and when possible indicate your final answer by putting it in a box.
4. You are allowed a single, one-sided letter sized page of notes in the exam. No other aids are allowed (i.e. no calculators).

1. (15 points) Define a function

$$f(x,y) = \begin{cases} \frac{x}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

a) Determine the set of points where f is continuous.
b) Compute the partial derivatives where they exist.
c) Are the partial derivatives continuous at all points in $\mathbb{R}^2$?

2. (15 points) Let $f : U \to \mathbb{R}$ where f is a continuous function and $U \subset \mathbb{R}^n$ is an open set.
Let $\boldsymbol{x} \in U$ such that $f(\boldsymbol{x}) > 0$.
Show that there is a nonempty open set $V \subset \mathbb{R}^n$ such that $f(\boldsymbol{y}) > 0$ for all $\boldsymbol{y} \in V$.

3. (15 points) A subset $U \subset \mathbb{R}^n$ is called **path-connected** if for every $\boldsymbol{x}, \boldsymbol{y} \in U$, there exists a smooth function $\boldsymbol{c} : \mathbb{R} \to \mathbb{R}^n$ (the "path") such that $\boldsymbol{c}(0) = \boldsymbol{x}$ and $\boldsymbol{c}(1) = \boldsymbol{y}$.
Let $f : U \subset \mathbb{R}^n \to \mathbb{R}$ be a continuous function on an open path-connected set U. If $\nabla f(\boldsymbol{x}) = 0$ for all $\boldsymbol{x} \in U$, then show that f is constant.
Hint: show that $f(\boldsymbol{x}) = f(\boldsymbol{y})$ for all $\boldsymbol{x}, \boldsymbol{y} \in U$.

4. (15 points) Find the equation of the tangent plane at the point $(1, 0, 0)$ to the surface $z + 1 = xe^y \cos(z)$.

5. (20 points) Consider the function

$$f(x,y) = \begin{cases} & 1\text{if } x > 0 \text{ and } y > \sqrt{x} \\ 0 & \text{otherwise.} \end{cases}$$

a) Along each line $y = mx$ through the origin, of slope m, calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$.
b) Is $f(x,y)$ continuous at $(0,0)$? Prove or disprove.
Hint: it may help to draw the regions where $f(x,y)$ is 0 and 1.

6. (20 points) A rectangular box without a lid (i.e. it only has 5 sides) is constructed from 27 m^2 of cardboard. We want to find the maximum possible volume.
a) First, set up this problem as a maximization problem you can solve.
b) Then, solve.