1) Given two points $P = (x, y, z)$ and $Q = (x', y', z')$, find the coordinates of the midpoint of segment PQ.

2) Show that the line segment joining the midpoints of two sides of a triangle is parallel to and has half the length of the third side.

3) Show that the diagonals of a rectangle are perpendicular if and only if the rectangle is a square.

4) Find the line through $(3, 1, -2)$ that intersects and is perpendicular to the line

$$x = -1 + t$$

$$y - -2 + t$$

$$z = -1 + t$$

5) Vectors v and w are sides of an equilateral triangle with side length 1. Find $v \cdot w$.

6) Find the equation for the plane that passes through the point $(1, 2, -3)$ and is perpendicular to the line $v = (0, -2, 1) + t(1, -2, 3)$.

7) Find the equation for the plane containing the two parallel lines:

$$v_1 = (0, 1, -2) + t(2, 3, -1)$$

$$v_2 = (2, -1, 0) + t(2, 3, -1)$$

8) Find the distance to the point $(6, 1, 0)$ from the plane through the origin that is perpendicular to $i - 2j + k$.

9) Describe the surfaces $\rho =$ constant, $\theta =$ constant and $\phi =$ constant in the spherical coordinate system.

10) Let $v, w \in \mathbb{R}^n$. If $\|v\| = \|w\|$ show that $v + w$ and $v - w$ are orthogonal.

11) Describe the level curves of $f(x, y) = xy$.

12) Compute

$$\lim_{(x,y,z)\to(0,0,0)} \frac{2x^2 y \cos z}{x^2 + y^2}$$

13) Compute

$$\lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{x^2y^2}$$

14) Compute

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{\sqrt{x^2+y^6}}$$

15) Compute

$$\lim_{(x,y)\to(0,0)} \frac{x^3+2y^4}{2x^3+y^3}$$

16) Give a function in two variables which is continuous at only one point.

17) At which points of this function do the partial derivatives exist?

$$f(x,y)=\begin{cases} \dfrac{x^2y}{\sqrt{x^2+y^2}} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{otherwise} \end{cases}$$

18) Show that in this function the partial derivatives exist but the function is not continuous at $(0,0)$.

$$f(x,y)=\begin{cases} \dfrac{x^2y^4}{x^4+6y^8} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{otherwise} \end{cases}$$

19) Give a parametrization for the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$.

20) If we attach a thin carbon to the edge of a cylinder and we make it roll parallel to a wall, will the figure drawn by the carbon autointersect or not?