1) The midpoint is

$$P + \frac{1}{2}PQ = \left(\frac{x + x'}{2}, \frac{y + y'}{2}, \frac{z + z'}{2} \right)$$

2) Simple computation using 1).

3) Let a and b be the sides of the rectangle. Then one diagonal is $a + b$ and the other $a - b$. We have that

$$(a + b) \cdot (a - b) = 0 \Leftrightarrow a \cdot a - b \cdot b = 0 \iff \|a\|^2 = \|b^2\|$$

4) Let $P = (-1 + t, -2 + t, -2 + t)$ be a generic point of the line and let Q be the given point. Then since the vector of the line is $(1, 1, 1)$, PQ is going to be perpendicular to the line iff

$$PQ \cdot (1, 1, 1) = 0 \iff (-4 + t, -3 + t, 1 + t) \cdot (1, 1, 1) = 0$$

This gives $t = 2$ and $P = (1, 0, 1)$. The line is then $(P + t \cdot PQ = (1, 0, 1) + t(-2, -1, 3)$.

5) $v \cdot w = \|v\|\|w\| \cos\theta = 1 \cdot 1 \cdot 1/2 = 1/2$.

6) The plane is perpendicular to $(1, -2, 3)$ and passes through $(1, 2, -3)$, so its equation is $(1, -2, 3) \cdot (x - 1, y - 2, z + 3) = 0$.

7) The plain contains the vector $v = (2, 3, 1)$ and points $P = (0, 1, -2)$ and $Q = (2, -1, 0)$. Hence it contains vector $w = Q - P = (2, -2, 2)$. Then this problem is reduced to compute the plane that passes through a point (for instance P) and is orthogonal to $v \times w$.

8) After finding the equation of the plane it just remains to plug the point in the distance point-plane formula.

9) Respectively a sphere, a cone, and half plane.

10) Same as 3)

11) Level curves hare hyperbolas given by $xy = c$ or $y = c/x$.

12) In cylindrical coordinates this limit is the same as

$$\lim_{(r,z)\to(0,0)} \frac{r^3\cos^2\theta\sin\theta\cos z}{r^2} = \lim_{(r,z)\to(0,0)} r\cos^2\theta\sin\theta\cos z = 0$$

13) The Taylor expansion of $\cos(t)$ is $\cos(t) = 1 - t^2/2 + r_3(t)$, where $r_3(t)/t^2$ goes to 0 as t approaches 0. Hence

$$\lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{x^2y^2} = \lim_{(x,y)\to(0,0)} \frac{[1-(xy)^2/2]-1}{x^2y^2} = -\frac{1}{2}$$

14) Since $|x| < \sqrt{x^2+y^6}$ and we have that

$$\left|\frac{xy^4}{\sqrt{x^2+y^6}}\right| =< \frac{y^4\sqrt{x^2+y^6}}{\sqrt{x^2+y^6}} = y^4$$

The result follows.

15) Taking paths $x = 0$ and $y = 0$ give different limits at 0. Hence the limit doesn't exist.

16) Take

$$f(x,y) = \begin{cases} x^2+y^2 & \text{if } x \text{ and } y \text{ are both rational} \\ 0 & \text{otherwise} \end{cases}$$

Then this function is continuous only at $(0,0)$ [think about it!].

17) When the denominator is nonzero, the composition, product, addition and division properties of the derivative allow us to ensure that the partial derivatives exist. However at $(0,0)$ we have to use the limit definition and we have

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

Hence partial derivatives exist everywhere.

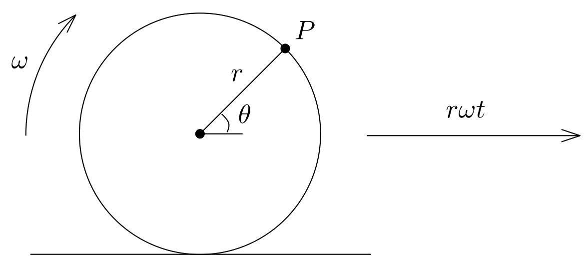
18) An analogous reasoning to 17) shows that both partials exist at $0, 0$. However approaching by $x = 0$ and by $x = y^2$ gives different limits at $0, 0$, so the function is not continuous at $(0, 0)$.

19) We have since both terms are positive, that $x^2/9 \leq 1$ and then that $|x| \leq 3$. This means that we can write $x = 3\cos t$ for some $t \in [0, 2\pi)$. Then

$$\frac{y^2}{25} = 1 - \frac{(3\cos t)^2}{9} = 1 - \cos^2 t = \sin^2 t$$

This shows that $y^2 = 25\sin^2 t$ and hence $y = \pm 5\sin t$. We can just left the $+$ sign since $(\cos t, -\sin t) = (\cos(-t), \sin(-t))$ and then we are already covering both cases. Conversely a straightforward computation shows that for each $t \in [0, 2\pi)$ the point $(3\cos t, 5\sin t)$ satisfies the given equation, so all the points are in this parametrization.

20) The position of the point P as the cylinder spins at speed $\theta = \omega t$ respect the cylinder is $(r\cos\theta, r\sin\theta) = (r\cos\omega t, r\sin\omega t)$, but the cylinder is itself moving at speed $r\omega t$ parallel to the x axis. Hence the position of the particle respect to the wall is $(r\omega t + r\cos\omega t, r\sin\omega t)$



Taking the derivative (over t), we find that the speed of the particle is

$$v(t) = r\omega - r\omega\sin\omega t, r\omega\cos\omega t$$

. Thus, the first component of v is $r\omega(1 - \cos\omega t)$ which is always non-negative, and shows that the particle never goes backwards in its path, and hence this never self-intersect.