Math 2220 Final Exam

Friday, May 11, 2012

Name: ______________________________

Show all work and explain all answers except as noted.

1. If $x = \sin\phi\cos\theta$, $y = \sin\phi\sin\theta$, and $z = \cos\phi$, compute $dx \wedge dy$, $dz \wedge dx$, and $dy \wedge dz$ in terms of $d\phi$, $d\theta$, and $d\phi \wedge d\theta$. (Your answer should not include $d\theta \wedge d\phi$.)

2. Find the volume of the solid bounded by $y = x^3$, $x + y = 2$, $y = 0$, $z = 0$, and $z = y^2$. Include a sketch of the intersection of this solid with $z = 0$.

3. Find the location of the point on the surface

$$\frac{1}{x} + \frac{2}{y} + \frac{3}{z} = 1$$

which is closest to the origin.

4. Let T be the surface of revolution obtained by revolving the circle $(x-2)^2+y^2=1$ (in the xy-plane) about the y-axis, with the outward orientation. Let T_0 be the portion of T (with the same orientation) for which $z \leq 0$. Compute the flux of the constant vector field $\mathbf{F} = \langle 2, -1, 3 \rangle$ through T_0.

5. Find the flux of $\mathbf{F} = y\mathbf{j}$ through the portion of the sphere $x^2 + y^2 + z^2 = 1$ on which $0 \leq x \leq y/\sqrt{3}$ and $z \geq 0$ (the sphere is oriented outward).

6. Let T be the surface $\{(x, y, z) : x^2+z^2 = y \text{ and } y \leq 1\}$ oriented so that $\mathbf{n}(0, 0, 0) = -\mathbf{j}$. Compute

$$\iint_T (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F} = \langle -z, y, x\rangle$.

7. Find the equation of the tangent plan to $z^3 - xy^2 - yz^2 = 9$ at $(3, -1, 2)$.

8. Compute the following, where ρ and $\mathbf{e}_\rho$ are the magnitude and direction, respectively, of the vector $\langle x, y, z\rangle$ (you may express your answer either in terms of ρ and $\mathbf{e}_\rho$ or x, y, z:

a. $\nabla \rho$.

b. $\nabla \times (y\mathbf{i} + z\mathbf{j} + x\mathbf{k})$.

c. $\nabla \cdot (\rho \mathbf{e}_\rho)$.

d. $\displaystyle\int_{\mathbf{c}} \rho \mathbf{e}_\rho \cdot d\mathbf{s}$ where $\mathbf{c}(t) = \langle \sin(\pi t), \cos(\pi t^2), t^3\rangle$ with $t \in [0, 1]$.

9. Let D be the region in the first quadrant enclosed by the curves $y = 1$, $y = 3$, $y = 1/x$, and $y = 3/x$ (in units of meters). Let $\mathbf{c}$ be the curve which traces out the boundary of D, oriented *clockwise*. If a particle moves along $\mathbf{c}$ subject to the force $\mathbf{F} = (y/x)\mathbf{i} + (x/y)\mathbf{j}$ (with units of Newtons), compute the work done.