MATH 2220 Multivariable Calculus   Fall 2012   Prof.R.Rand

FINAL EXAM

December 14, 2012

RULES: Closed book exam. No calculators, computers, i-phones or other electro-mechanical devices. Show all work, explain what you are doing.

1. (10 points) Find the derivative of

$$u = xy^2 - 3z^2$$

in the direction of the normal to the surface

$$xy + xz + yz = -6$$

at the point $(1, -2, 4)$.

2. (10 points) The differential element of area for polar coordinates

$$x = r\cos\theta, \;\; y = r\sin\theta$$

is

$$dA = r dr d\theta.$$

Use differential forms to find an expression for the differential element of area dA for **parabolic coordinates,** given by

$$x = (u^2 - v^2)/2, \;\; y = uv.$$

3. (15 points) Let

$$g(x, y) = \sin(xy) - 3x^2 \log y + 1.$$

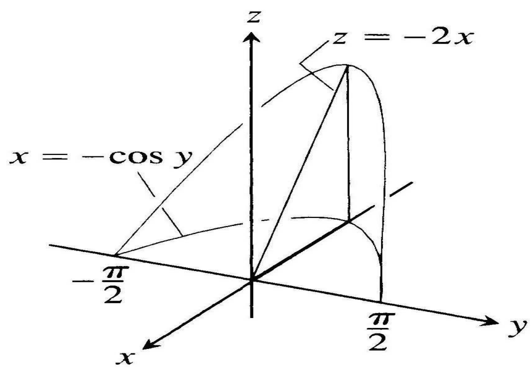
Develop $g(x, y)$ in a **Taylor series** about $x = \pi/2, \;\; y = 1$ up to and including quadratic terms.

EXAM CONTINUED ON THE OTHER SIDE OF THIS SHEET!

4. (15 points) Find the volume of the wedge-shaped region enclosed on the side by the cylinder

$$x = -\cos y, \ -\pi/2 \leq y \leq \pi/2,$$

on the top by the plane $z = -2x$, and below by the $x - y$ plane.



5. (15 points) Find the surface area of the following parametrized surface:

$$x = u, \ y = v, \ z = u^2/2, \ \text{for } 0 \leq u \leq 1, \ 0 \leq v \leq 1$$

6. (20 points) Let

$$\mathbf{F} = (2x^3 - y^3 + z^3, x^3 + y^3 + z^3, z^3 + xyz).$$

Evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dA,$$

where S is the surface defined by

$$x^2 + y^2 + z^2 = 1, \ z \geq 0$$

7. (15 points) Use **Lagrange multipliers** to find the point closest to the origin on the curve of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$.

Appendix

You may find the following integral helpful:

$$\int \sqrt{x^2 \pm a^2} \, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log(x + \sqrt{x^2 \pm a^2})$$