When a problem asks you to show that some statement is true, this means that you should give a logical mathematical argument why the statement is always true.

1. (a) Make a list of the 16 primitive Pythagorean triples (a,b,c) with $c \le 100$, regarding (a,b,c) and (b,a,c) as the same triple.

(b) How many more would there be if we allowed nonprimitive triples?

(c) How many triples (primitive or not) are there with $c = 65$?

2. (a) Find all the positive integer solutions of $x^2 - y^2 = 512$ by factoring $x^2 - y^2$ as $(x+y)(x-y)$ and considering the possible factorizations of 512.

(b) Show that the equation $x^2 - y^2 = n$ has only a finite number of integer solutions for each value of n.

(c) Find a value of n for which the equation $x^2 - y^2 = n$ has at least 100 different positive integer solutions.

3. (a) Show that there are only a finite number of Pythagorean triples (a,b,c) with a equal to a given number n.

(b) Show that there are only a finite number of Pythagorean triples (a,b,c) with c equal to a given number n.

4. Find an infinite sequence of primitive Pythagorean triples where two of the numbers in each triple differ by 2.

5. Find a right triangle whose sides have integer lengths and whose acute angles are close to 30 and 60 degrees by first finding the irrational value of r that corresponds to a right triangle with acute angles exactly 30 and 60 degrees, then choosing a rational number close to this irrational value of r.

6. Find a right triangle whose sides have integer lengths and where one of the nonhypotenuse sides is approximately twice as long as the other, using a method like the one in the preceding problem. (One possible answer might be the $(8,15,17)$ triangle, or a triangle similar to this, but you should do better than this.)

7. Find a rational point on the sphere $x^2+y^2+z^2 = 1$ whose x, y, and z coordinates are nearly equal.

8. (a) Derive formulas that give all the rational points on the circle $x^2 + y^2 = 2$ in terms of a rational parameter m, the slope of the line through the point $(1,1)$ on the circle.

(b) Using these formulas, find five different rational points on the circle in the first quadrant, and hence five solutions of $a^2 + b^2 = 2c^2$ with positive integers a, b, c.

(c) The equation $a^2 + b^2 = 2c^2$ can be rewritten as $c^2 = (a^2 + b^2)/2$, which says that c^2 is the average of a^2 and b^2, or in other words, the squares a^2, c^2, b^2 form an arithmetic progression. One can assume $a < b$ by switching a and b if necessary. Find four such arithmetic progressions of three increasing squares where in each case the three numbers have no common divisors.

9. (a) For integers x, what are the possible values of x^2 modulo 8?

(b) Show that the equation $x^2 - 2y^2 = \pm 3$ has no integer solutions by considering this equation modulo 8.

(c) Show that there are no primitive Pythagorean triples (a, b, c) with a and b differing by 3.

10. Show that for every Pythagorean triple (a, b, c) the product abc must be divisible by 60. (It suffices to show that abc is divisible by 3, 4, and 5.)