1. For the form $Q(x, y) = x^2 + xy + y^2$ do the following things:

(a) Draw enough of the topograph to show all the values less than 100 that occur in the topograph. You do not need to draw parts of the topograph that are symmetric with other parts.

(b) Make a list of the primes less than 100 that occur in the topograph, and a list of the primes less than 100 that do not occur.

(c) Characterize the primes in the two lists in part (b) in terms of congruence classes modulo $|\Delta|$ where Δ is the discriminant of *Q*.

(d) Characterize the nonprime values in the topograph in terms of their factorizations into primes in the lists in part (b).

(e) Summarize the previous parts by giving a simple criterion for which numbers are representable by the form Q, i.e., the numbers n such that Q(x, y) = n has an integer solution (x, y), primitive or not. The criterion should say something like n is representable if and only if $n = m^2 p_1 \cdots p_k$ where each p_i is a prime such that ... (e) Check that all forms having the same discriminant as Q are equivalent to Q.

2. Do the same things for the form $x^2 + xy - y^2$. This form is hyperbolic and it takes the same negative values as positive values, so you can just ignore all the negative values.

3. Do the same things for the form $x^2 + xy + 2y^2$, except that this time you only need to consider values less than 50 instead of 100.

4. For discriminant $\Delta = -24$ do the following:

(a) Verify that the class number is 2 and find two quadratic forms Q_1 and Q_2 of discriminant -24 that are not equivalent.

(b) Draw topographs for Q_1 and Q_2 showing all values less than 100. (You don't have to repeat parts of the topographs that are symmetric.)

(c) Divide the primes less than 100 into three lists: those represented by Q_1 , those represented by Q_2 , and those represented by neither Q_1 nor Q_2 . (No primes are represented by both Q_1 and Q_2 .)

(d) Characterize the primes in the three lists in part (c) in terms of congruence classes modulo $|\Delta| = 24$.

(e) Characterize the nonprime values in the topograph of Q_1 in terms of their factor-

izations into primes in the lists in part (c), and then do the same thing for Q_2 . Your answers should be in terms of whether there are an even or an odd number of prime factors from certain of the lists.

(f) Summarize the previous parts by giving a criterion for which numbers are representable by the form Q_1 and which are representable by Q_2 .

5. This problem will show how things can be more complicated than in the previous problems.

(a) Show that the class number for discriminant -23 is 2 and find forms Q_1 and Q_2 of discriminant -23 that are not equivalent.

(b) Draw the topographs of Q_1 and Q_2 up to the value 70. (Again you don't have to repeat symmetric parts.)

(c) Find a number *n* that occurs in both topographs, and find the *x* and *y* values that give $Q_1(x_1, y_1) = n = Q_2(x_2, y_2)$. (This sort of thing never happens in the previous problems.)

(d) Find a prime p_1 in the topograph of Q_1 and a different prime p_2 in the topograph of Q_2 such that p_1 and p_2 are congruent modulo $|\Delta| = 23$. (This sort of thing also never happens in the previous problems.)