1. As a sort of converse to Wilson's theorem, show that if n is not a prime then $(n-1)!$ is not congruent to -1 mod n. More precisely, when $n > 4$ and n is not prime, show that n divides $(n-1)!$, so $(n-1)! \equiv 0 \bmod n$. What happens when $n = 4$?

2. Determine the values of Δ for which there exists a quadratic form of discriminant Δ that represents 5, and also determine the discriminants Δ for which there does not exist a form representing 5.

3. Verify that the statement of quadratic reciprocity is true for the following pairs of primes (p,q): $(3,5)$, $(3,7)$, $(3,13)$, $(5,13)$, $(7,11)$, and $(13,17)$.

4. (a) In the book there is an example near the end of section 2.3 working out which primes are represented by some form of discriminant 13, using quadratic reciprocity for the key step. (This was also done in class.) Do the same thing for discriminant 17.
(b) Show that all forms of discriminant 17 are equivalent to the principal form $x^2 + xy - 4y^2$.
(c) Draw enough of the topograph of $x^2 + xy - 4y^2$ to show all values between -70 and 70, and verify that the primes that occur are precisely the ones predicted by your answer in part (a).

5. Using quadratic reciprocity as in part (a) of the previous problem, figure out which primes are represented by at least one form of discriminant Δ for the following values of Δ: -3, 8, -20, 21.

6. (a) Repeat the previous problem for $\Delta = 9$ where the answer may be rather surprising. Note that quadratic forms with $\Delta = 9$ are 0-hyperbolic, rather than the more usual hyperbolic or elliptic forms that we consider. (0-hyperbolic forms factor into linear factors with integer coefficients.)
(b) Draw enough of the topographs of all three equivalence classes of forms with $\Delta = 9$ to see why the answer you got in part (a) is correct.
(c) Show that in fact *every* integer n is represented primitively by at least one quadratic form of discriminant 9.