**1.** Draw the topograph for the form  $Q(x, y) = 2x^2 + 5y^2$ , showing all the values of  $Q(x, y) \le 60$  in the topograph, with the associated fractional labels x/y. If there is symmetry in the topograph, you only need to draw one half of the topograph and state that the other half is symmetric.



This is just the upper half of the topograph, and there is a symmetric lower half, starting with the value 7 at x/y = -1/1.

**2.** Do the same for the form  $Q(x, y) = 2x^2 + xy + 2y^2$ , in this case displaying all values  $Q(x, y) \le 40$  in the topograph.



In this case there is a symmetric half to the right, starting with the value 2 at x/y = 0/1.

**3.** Do the same for the form  $Q(x, y) = x^2 - y^2$ , showing all the values between +30 and -30 in the topograph, but omitting the labels x/y this time.



There is a symmetric lower half in this case.

**4.** For the form  $Q(x, y) = 2x^2 - xy + 3y^2$  do the following:

(a) Draw the topograph, showing all the values  $Q(x, y) \le 30$  in the topograph, and including the labels x/y.  $\frac{3}{2}$   $\frac{2}{3}$ 



(b) List all the values  $Q(x, y) \le 30$  in order, including the values when the pair (x, y) is not primitive.

These are the values in the topograph, and also all the numbers obtained from these

values by multiplying by squares  $4, 9, 16, \cdots$ , so we get

$$2, 3, 4, 6, 8 = 4 \times 2, 9, 12, 13, 16, 18, 24, 26, 27 = 9 \times 3, 29$$

(c) Find all the integer solutions of Q(x, y) = 24, both primitive and nonprimitive. (And don't forget that quadratic forms always satisfy Q(x, y) = Q(-x, -y).)

These are  $(x, y) = \pm (3, 2), \pm (-3, 1)$ , and  $\pm (-2, 2)$  where this last pair comes from the 6 in the topograph.

**5.** Determine the periodic separator line in the topograph for each of the following quadratic forms (you do not need to include the fractional labels x/y).

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$$x^{2} - 7y^{2}$$
  
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**6.** Using your answers in the preceding problem, write down the continued fraction expansions for  $\sqrt{7}$ ,  $2\sqrt{3}/3$ , and  $(-1 + \sqrt{5})/2$ .

(a)  $\sqrt{7}$  is the positive root of  $x^2 - 7 = 0$  so we are moving toward the right along the periodic separator line. The sequence of left and right side roads starting from the the edge of the separator line separating the 1/0 and 0/1 regions is  $R^2 \overline{LRLR^4}$  so we have  $\sqrt{7} = 2 + \overline{1/1} + \frac{1}{1} +$ 

(b)  $2\sqrt{3}/3 = \sqrt{4/3}$  is the positive root of  $3x^2 - 4 = 0$  and the side road sequence along the separator line toward the right is  $R\overline{L^6R^2}$  so  $2\sqrt{3}/3 = 1 + \frac{1}{6} + \frac{1}{2}$ .

(c)  $(-1 + \sqrt{5})/2$  is the positive root of  $x^2 + x - 1$  and the side road sequence traveling to the right is  $\overline{LR}$  so  $(-1 + \sqrt{5})/2 = \overline{\frac{1}{1}}$ .

7. For the following quadratic forms, draw enough of the topograph, starting with the

edge separating the 1/0 and 0/1 regions, to locate the periodic separator line, and include the separator line itself in your topograph.

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$$x^{2} + 3xy + y^{2}$$
  
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