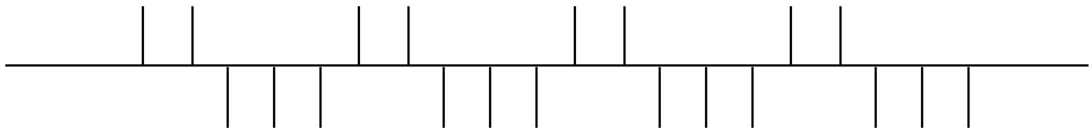
1. Find a hyperbolic quadratic form whose periodic separator line has the following pattern:



2. In this problem we consider only quadratic forms $Q(x,y) = ax^2 + cy^2$ with no xy term, for the sake of simplicity.

(a) Find two 0-hyperbolic forms $ax^2 + cy^2$ that have the same discriminant but take on different sets of values. Draw enough of the topographs of the two forms to make it apparent that they do not have exactly the same sets of values. (Remember that the topograph only shows the values $Q(x,y)$ for primitive pairs (x,y).)

(b) Do the same thing with two elliptic forms that take on positive values. Include the source vertex or source edge in the topographs.

(c) Do the same thing with two hyperbolic forms, drawing their separator lines.

3. (a) Show the quadratic form $Q(x,y) = 92x^2 - 74xy + 15y^2$ is elliptic by computing its discriminant.

(b) Find the source vertex or edge in the topograph of this form.

(c) Using the topograph of this form, find all the integer solutions of $92x^2 - 74xy + 15y^2 = 60$, and explain why your list of solutions is a complete list. (There are exactly four pairs of solutions $\pm(x,y)$, three of which will be visible in the topograph.)

4. (a) Show that if a quadratic form $Q(x,y) = ax^2 + bxy + cy^2$ can be factored as a product $(Ax + By)(Cx + Dy)$ with A, B, C, D integers, then Q takes the value 0 at some pair of integers $(x,y) \neq (0,0)$, hence Q must be either 0-hyperbolic or parabolic. Show also, by a direct calculation, that the discriminant of this form is a square.

(b) Find a 0-hyperbolic form $Q(x,y)$ such that $Q(1,5) = 0$ and $Q(7,2) = 0$ and draw a portion of the topograph of Q that includes the two regions where $Q = 0$.